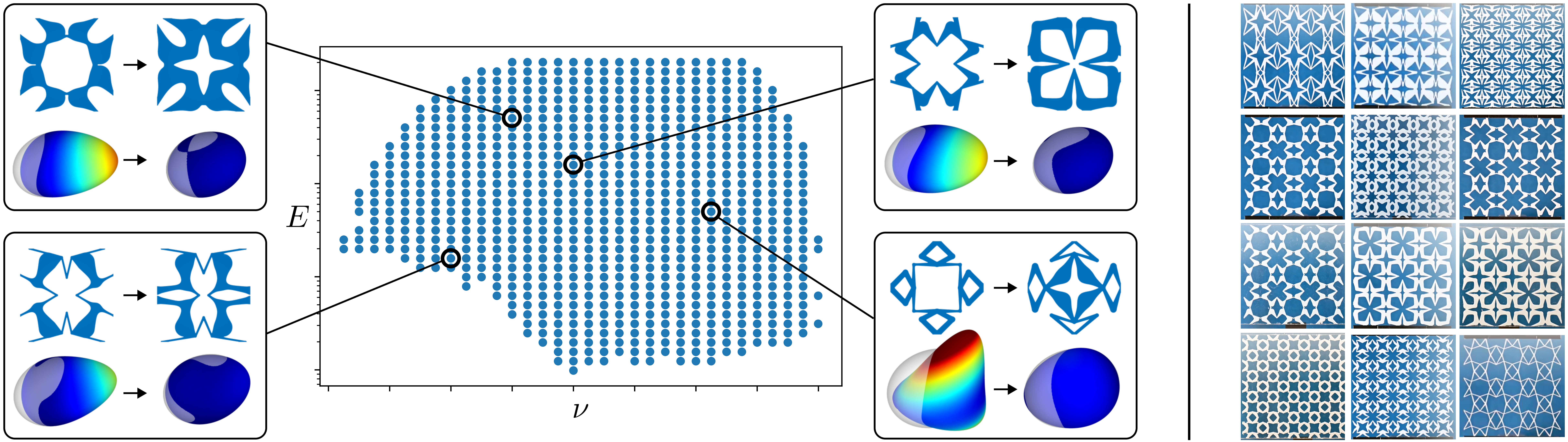
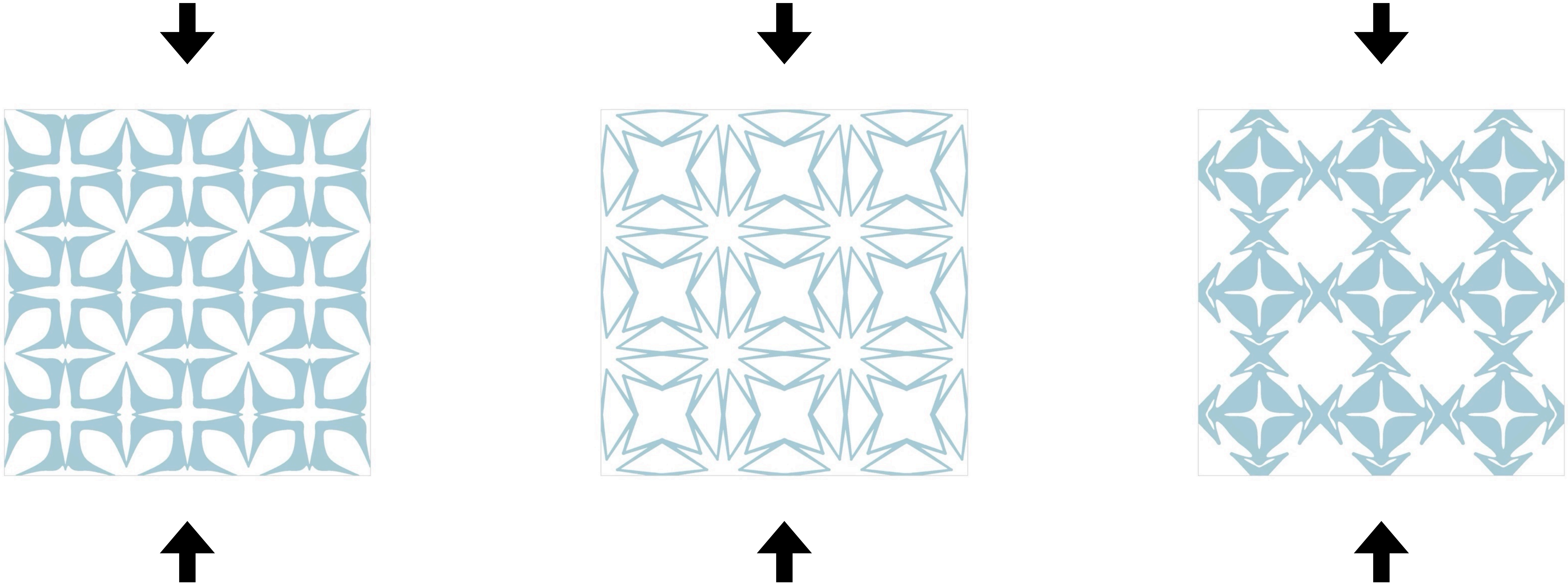


Computational Design of Flexible Planar Microstructures

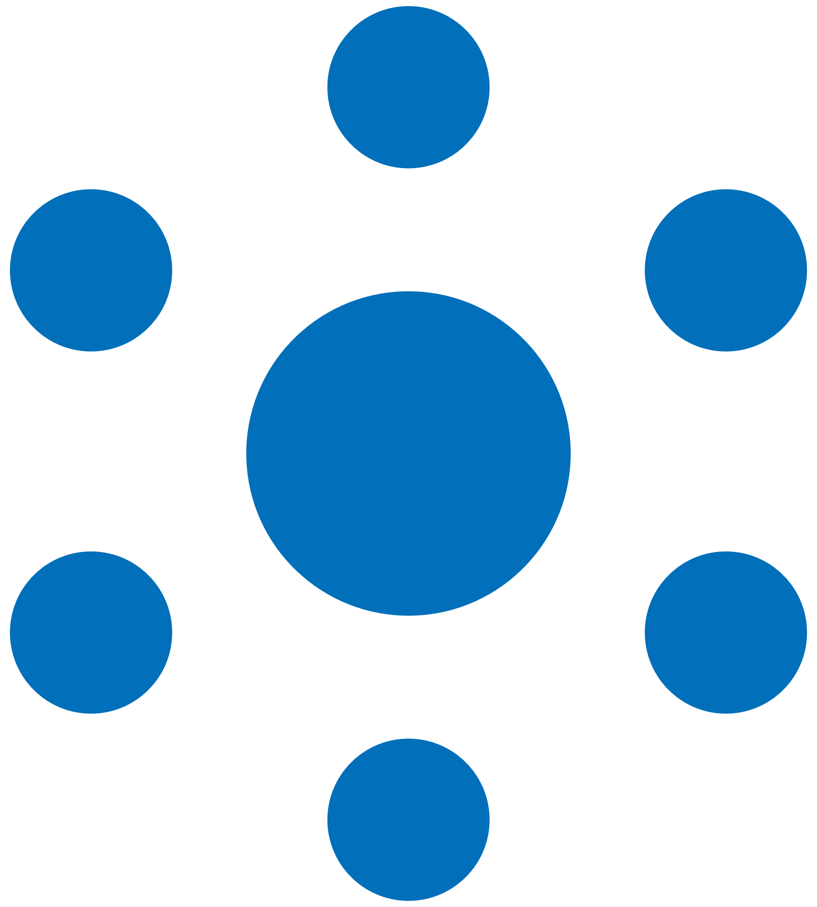
Zhan Zhang
Christopher Brandt
Jean Jouve
Yue Wang
Tian Chen
Mark Pauly
Julian Panetta

SIGGRAPH ASIA 2023 SYDNEY

METAMATERIALS

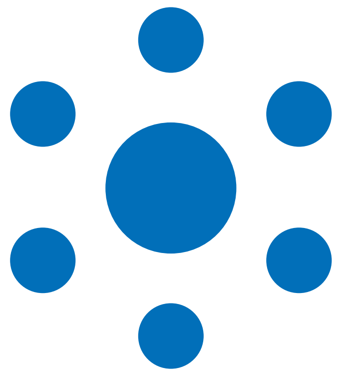


METAMATERIALS



Material

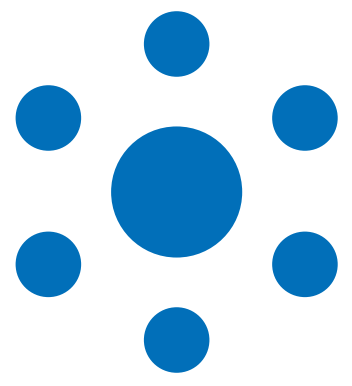
METAMATERIALS



Material

Microstructure

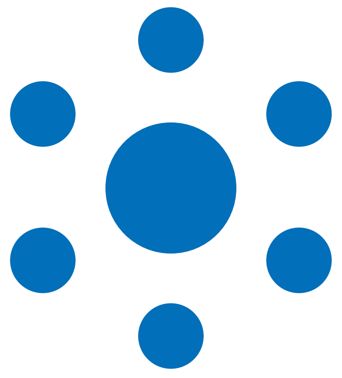
METAMATERIALS



Material

Microstructure

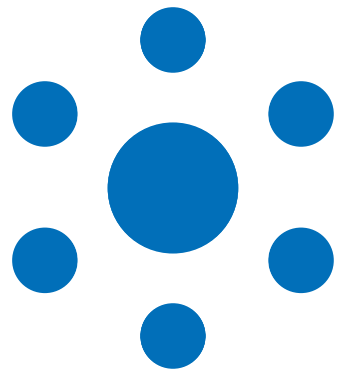
METAMATERIALS



Material

Microstructure

METAMATERIALS



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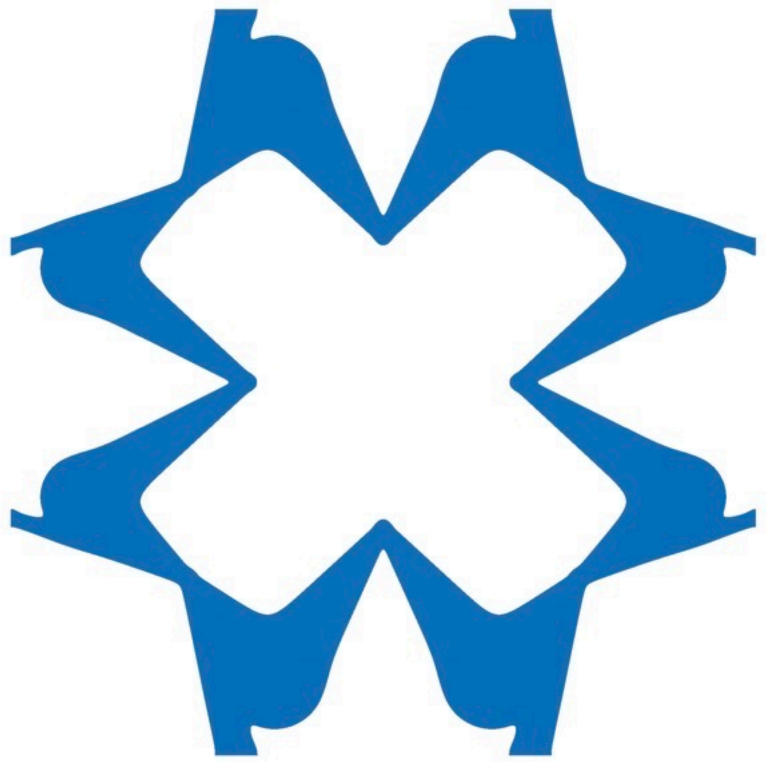
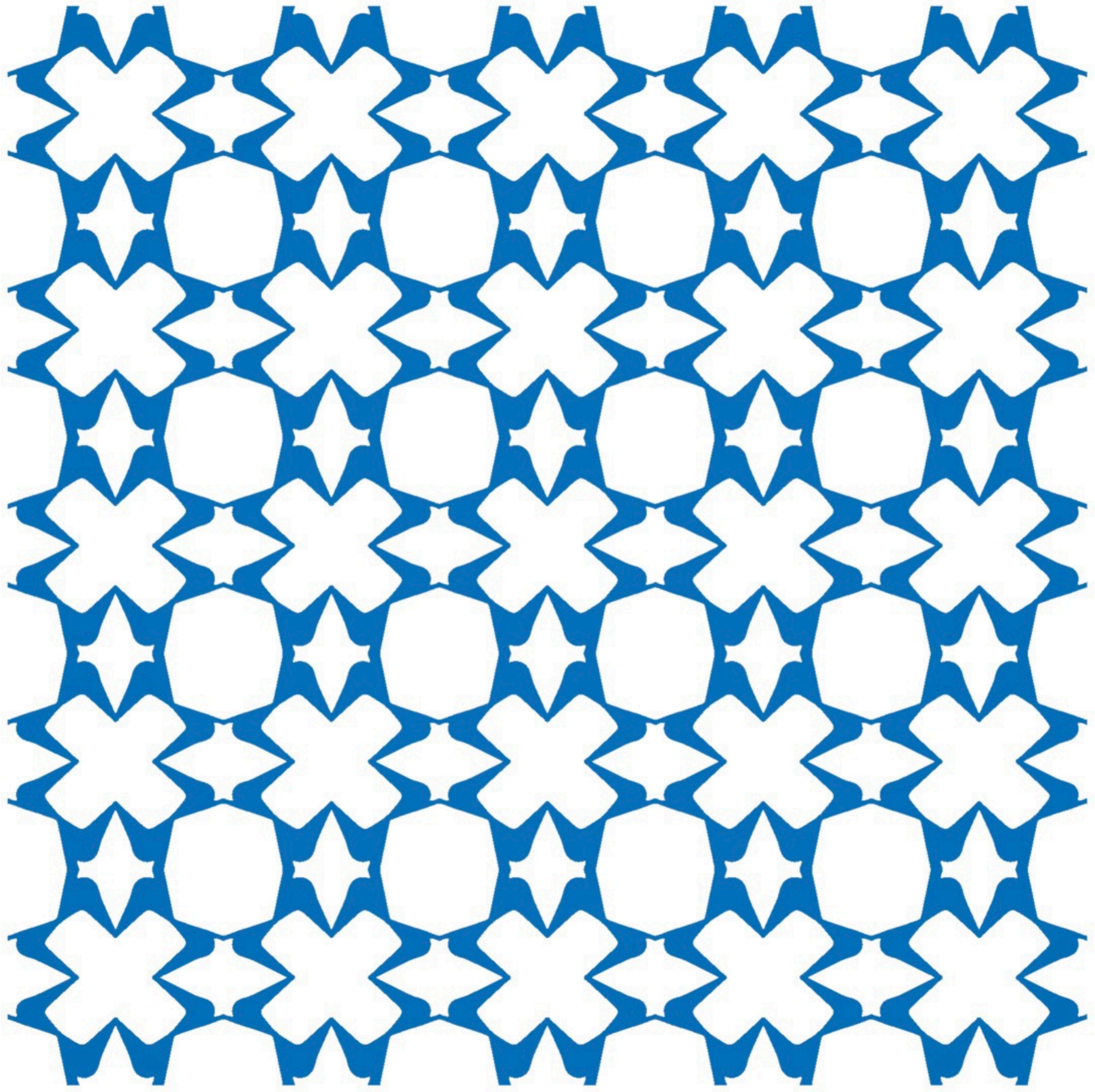
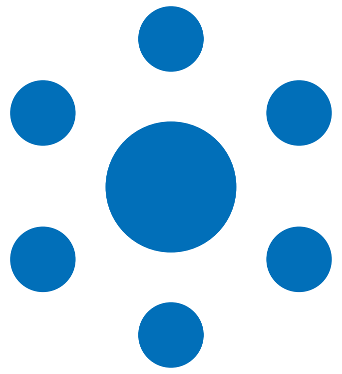


Microstructure



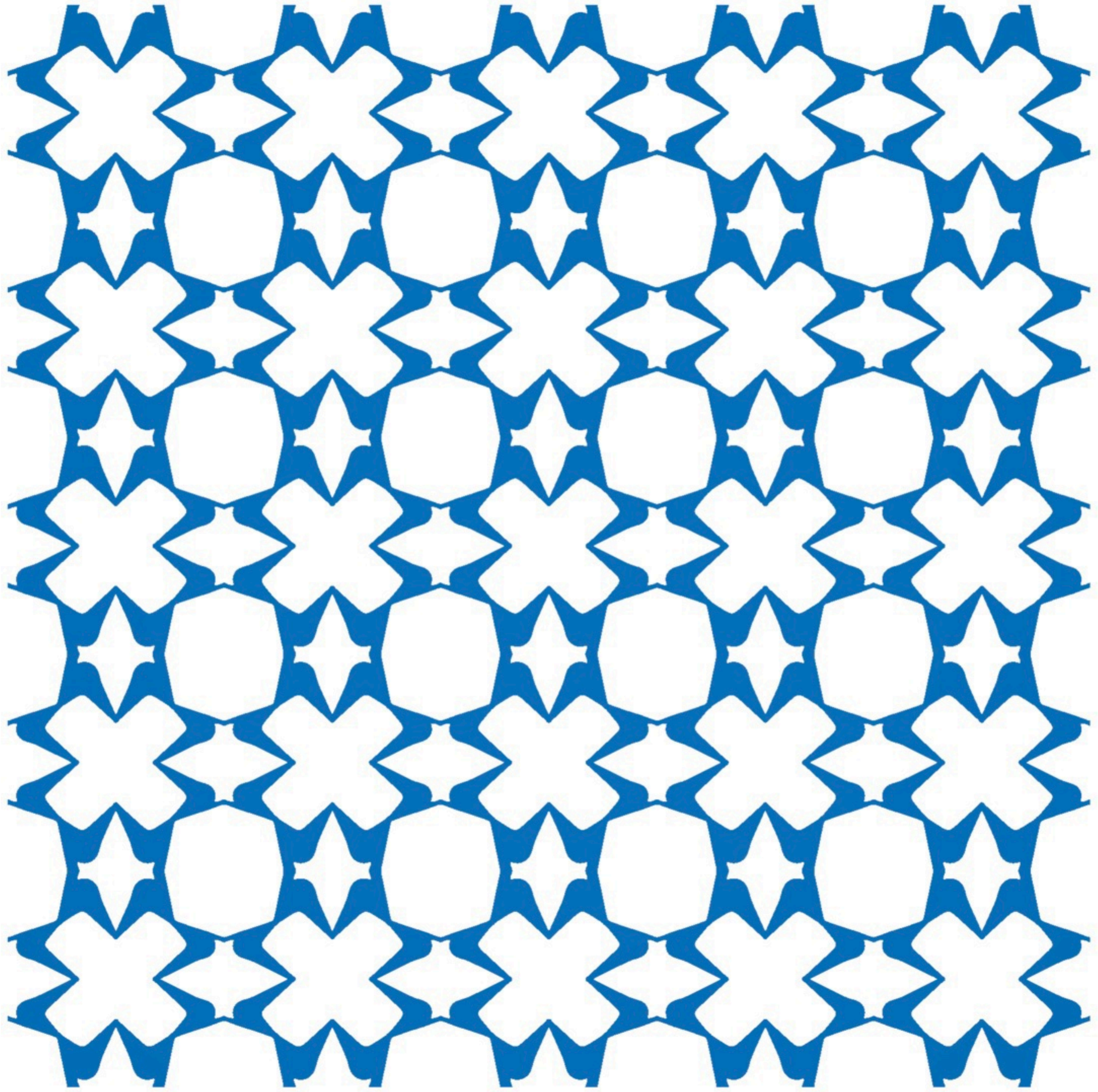
Metamaterial

METAMATERIALS



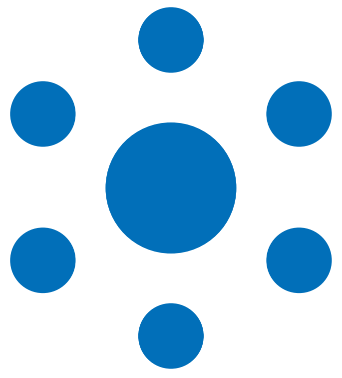
Material

Microstructure



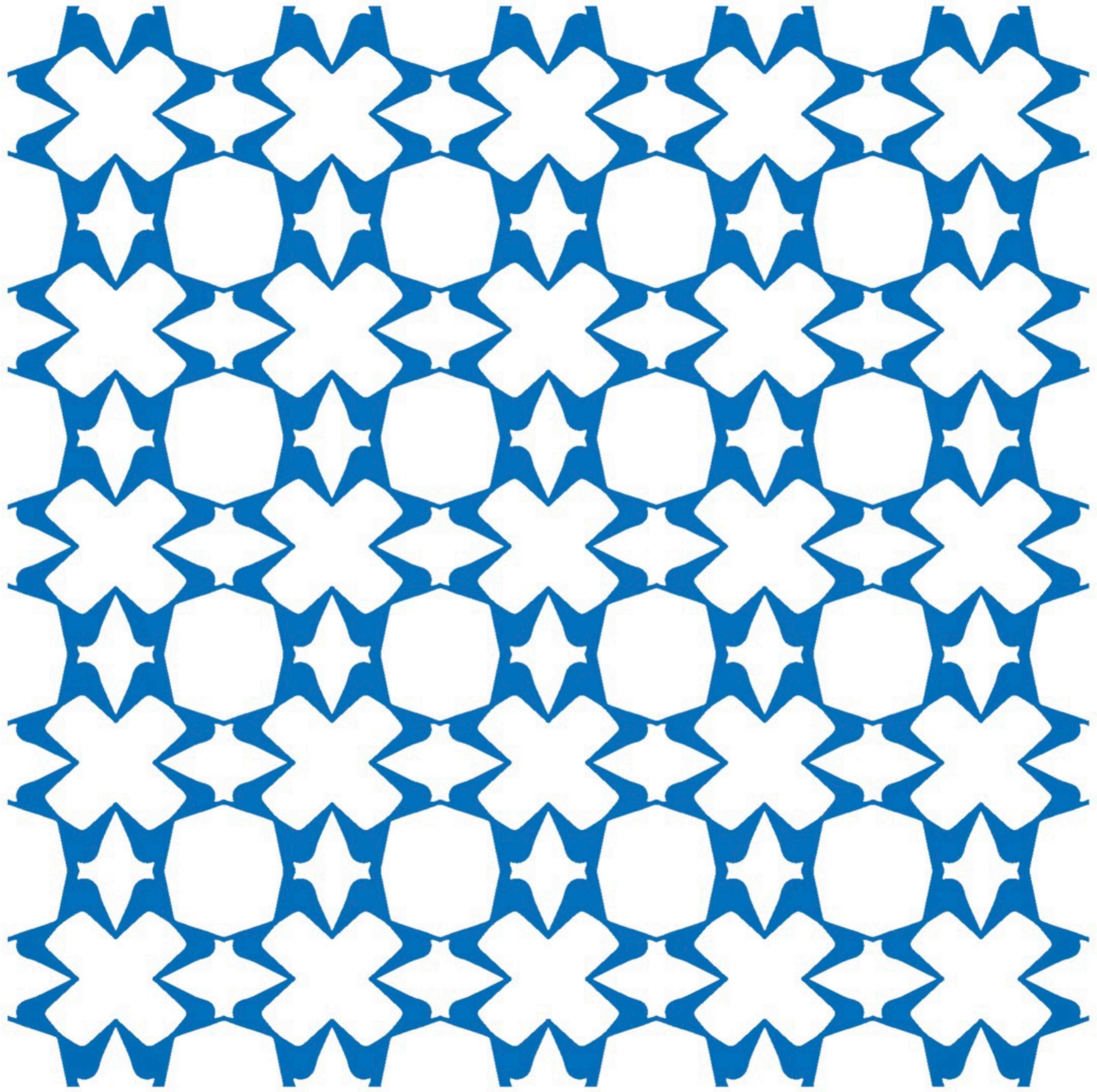
Metamaterial

METAMATERIALS



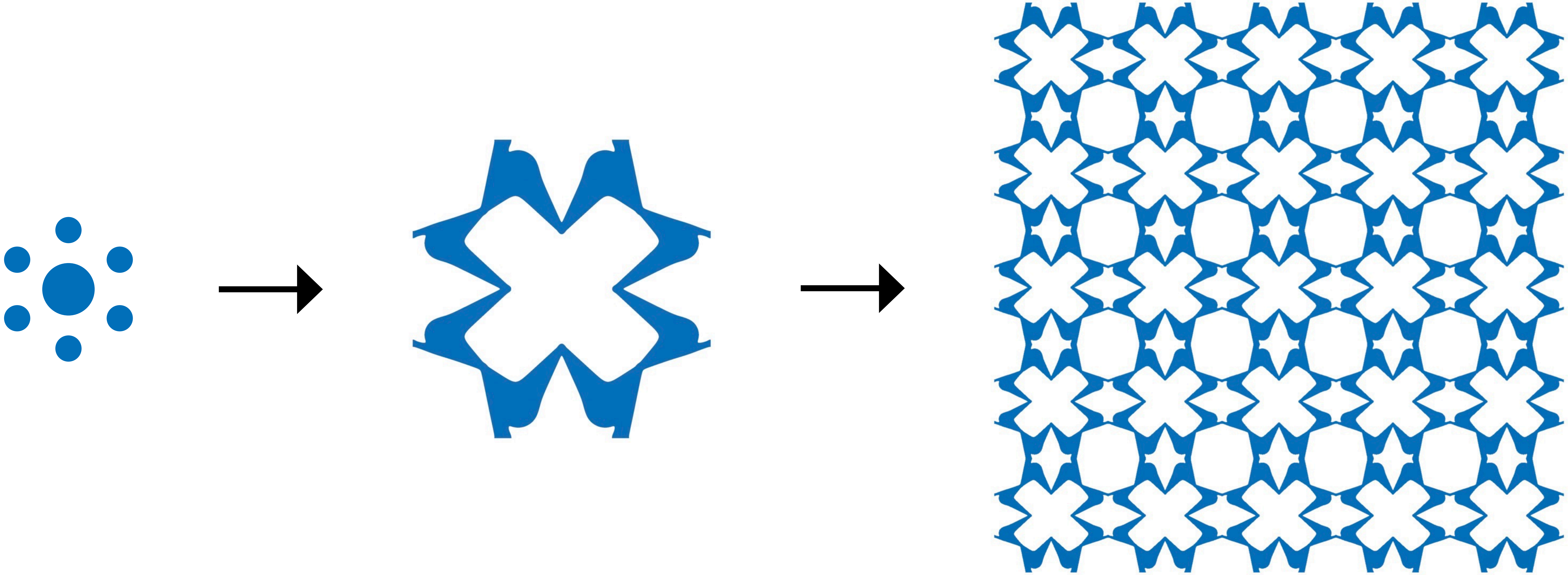
Material

Microstructure



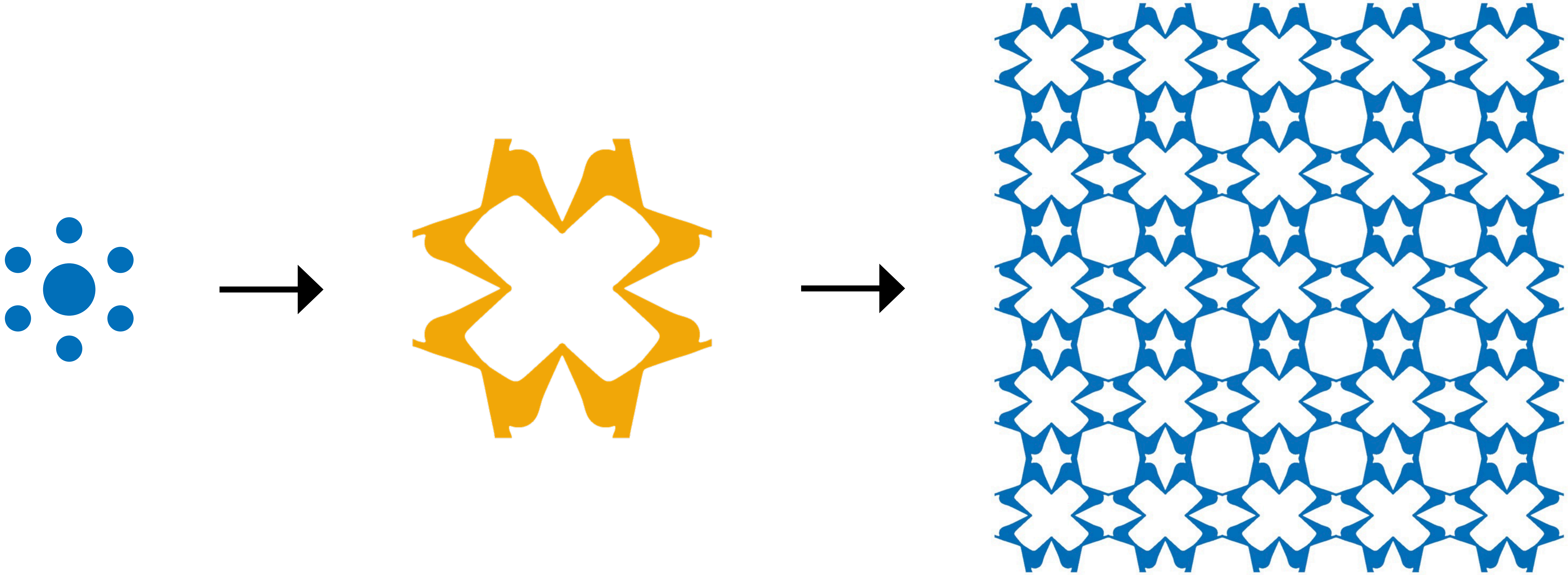
Metamaterial

METAMATERIALS



Material Microstructure Metamaterial

METAMATERIALS



Material

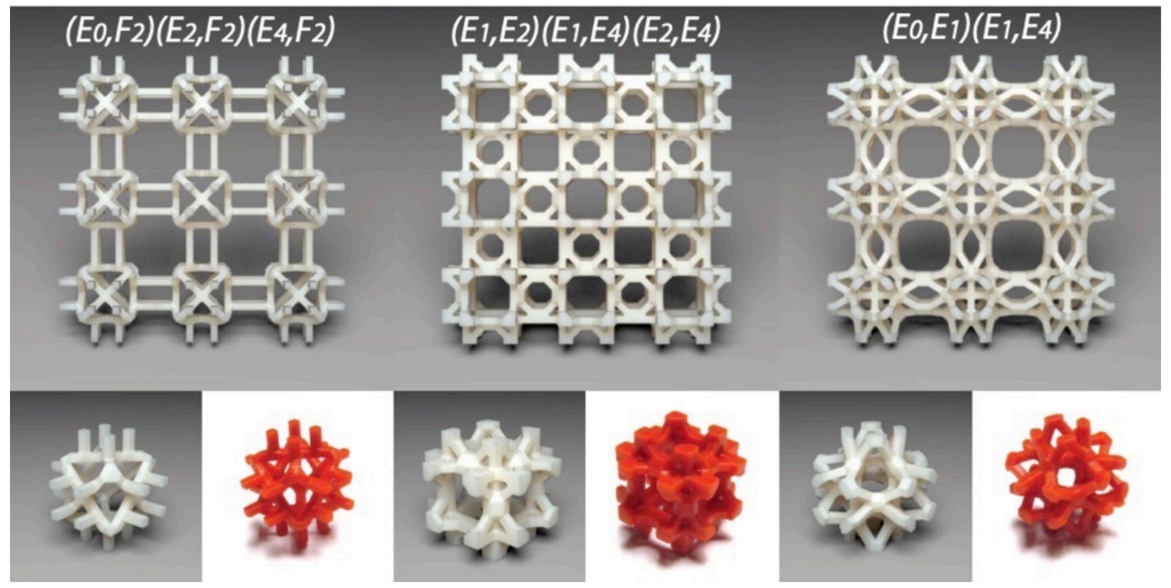
Microstructure

Metamaterial

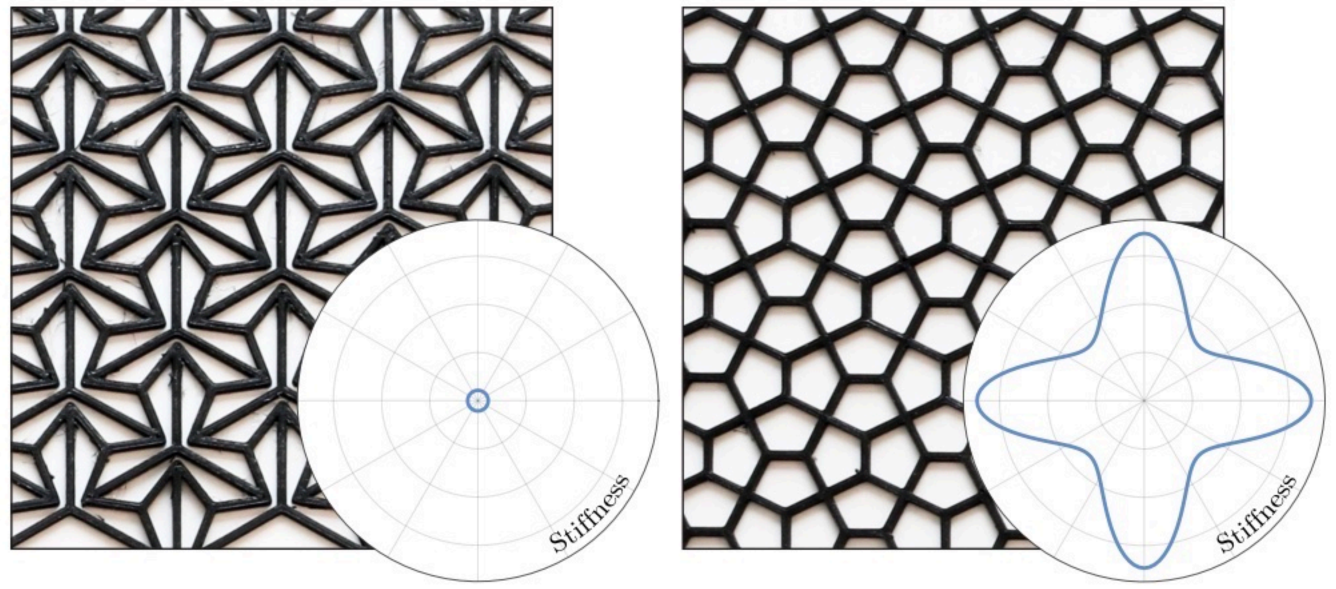
Zhan Zhang

UCDAVIS

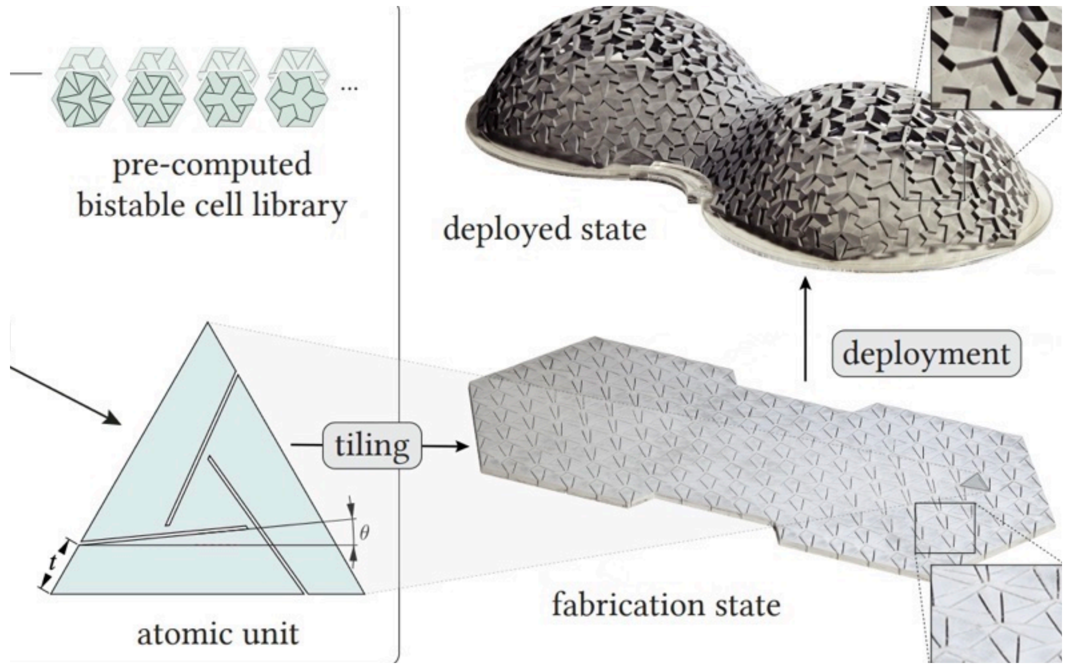
RELATED WORK



Elastic Textures for Additive Fabrication
[Panetta et al. 2015]

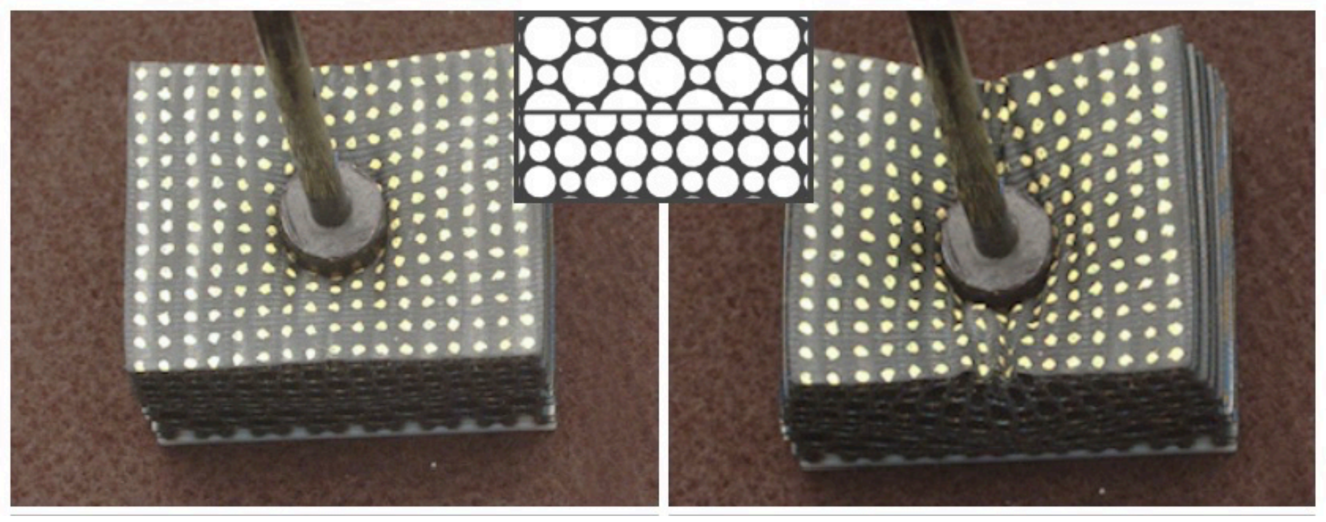


Structured Sheet Materials
[Schumacher et al. 2018]

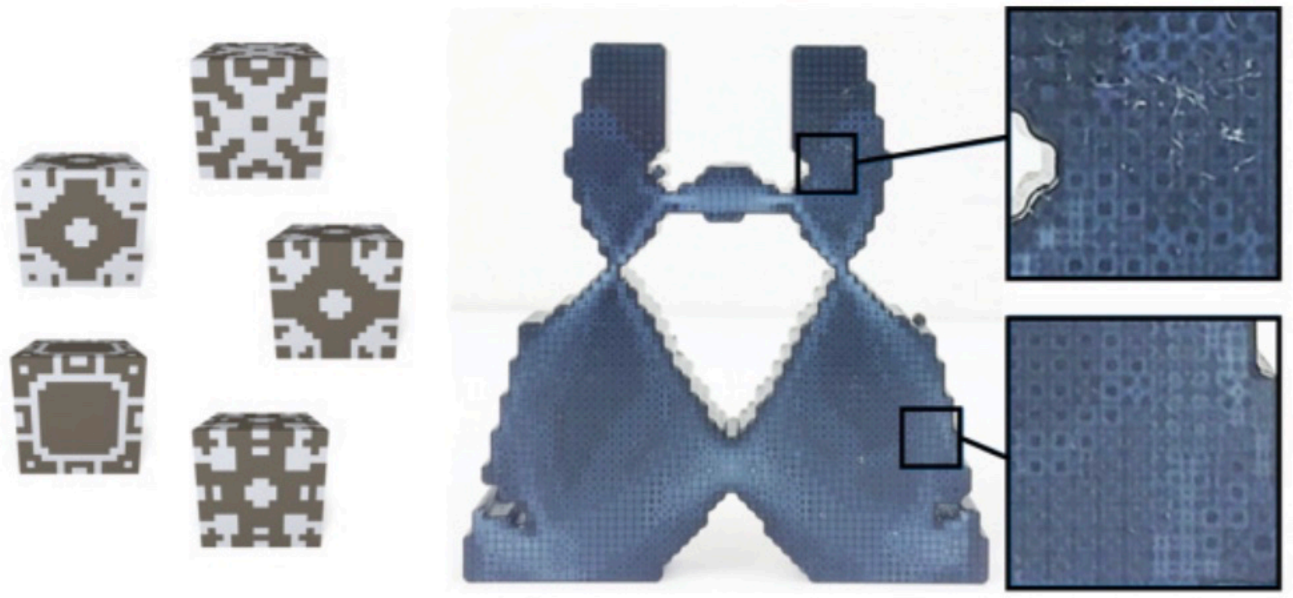


Bistable Auxetic Surface Structures
[Chen et al. 2021]

FlexMaps
[Malomo et al. 2018]

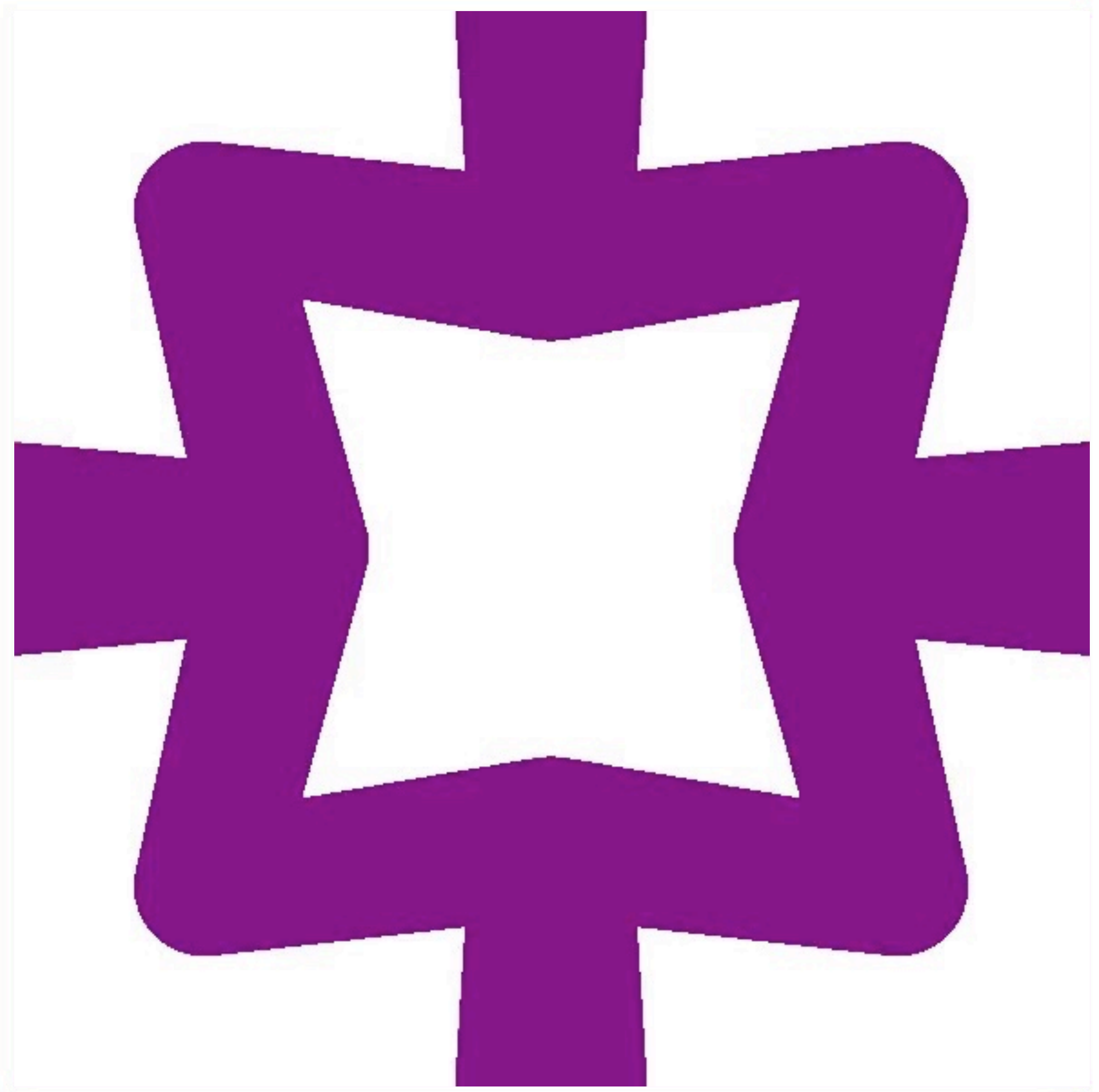


Desired Deformation Material Design
[Bickel et al. 2010]

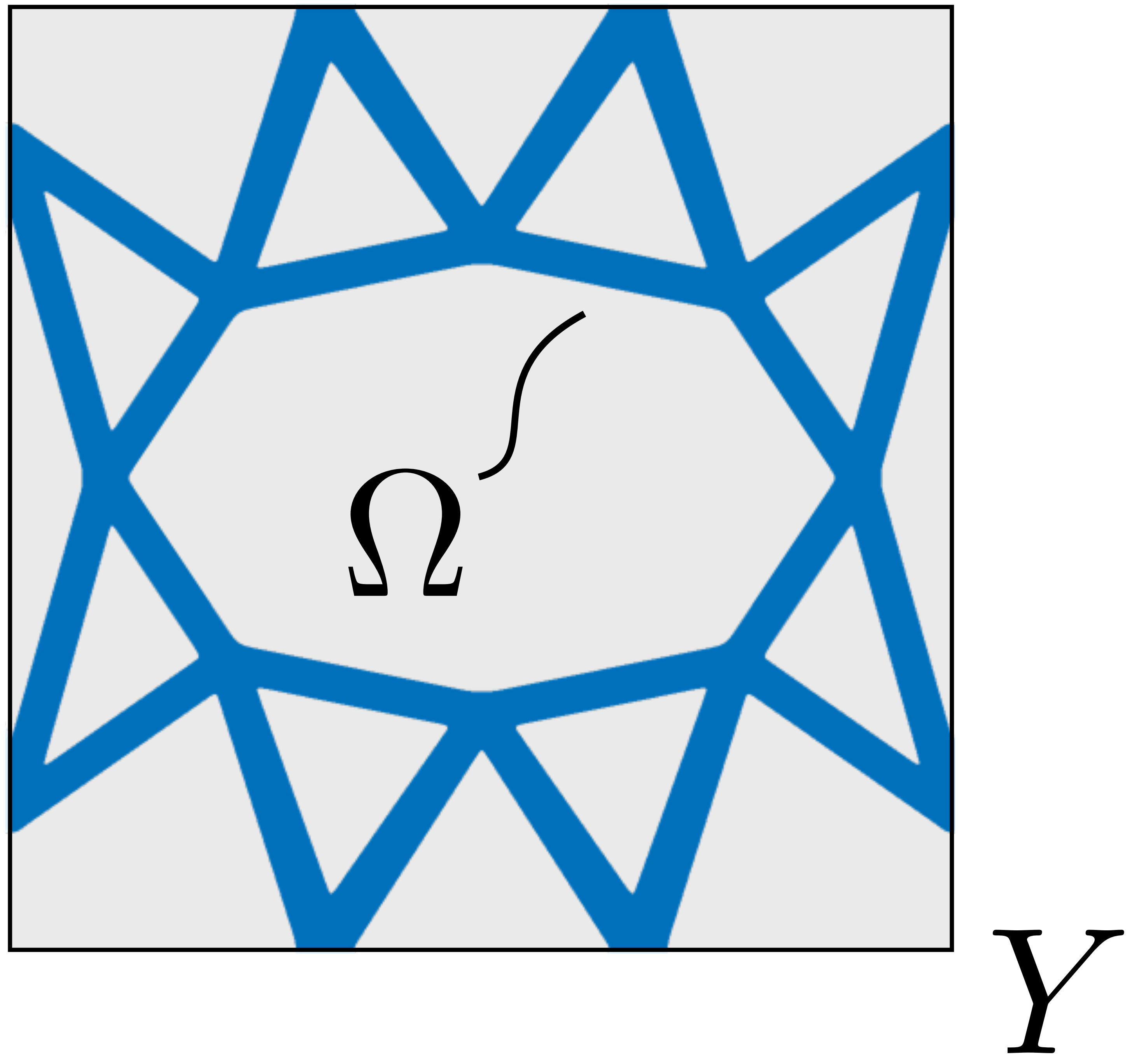


Two-Scale Topology Optimization
[Zhu et al. 2017]

HOMOGENIZATION



HOMOGENIZATION



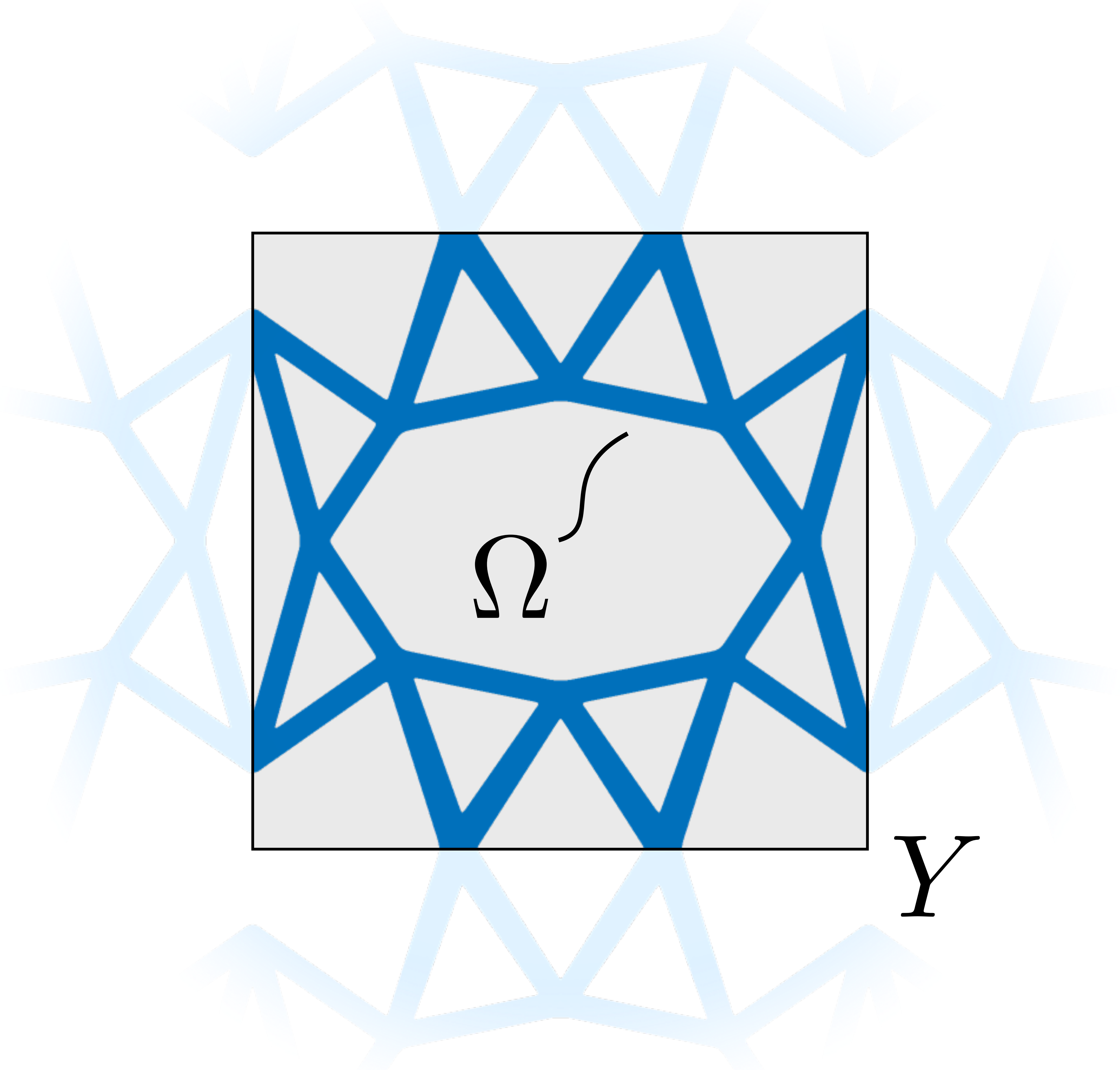
Deformation Function:

$$\Phi(\mathbf{X}) = \overline{F}\mathbf{X} + \omega(\mathbf{X})$$

macro deformation

micro deformation

HOMOGENIZATION



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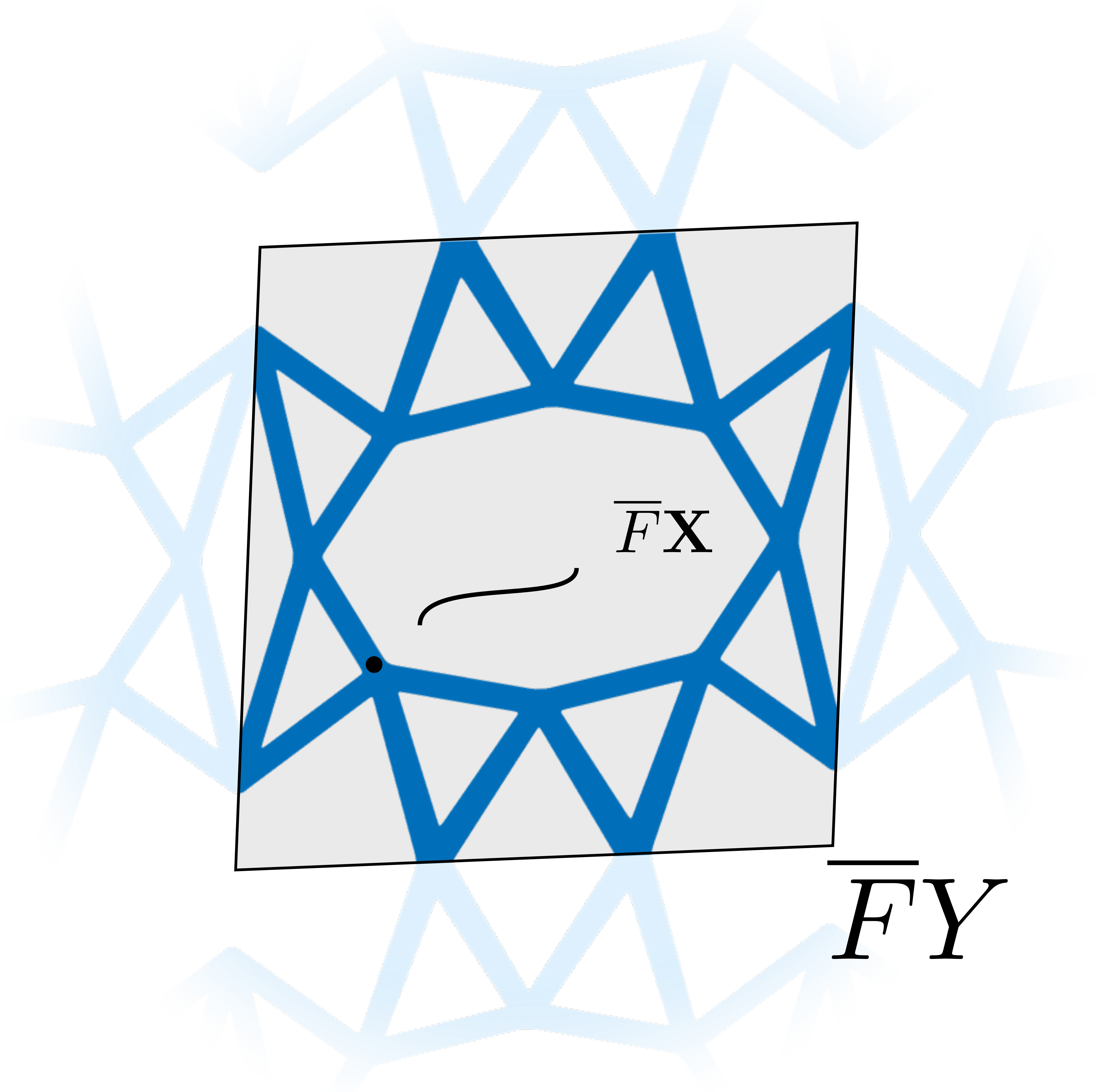
macro
deformation

HOMOGENIZATION



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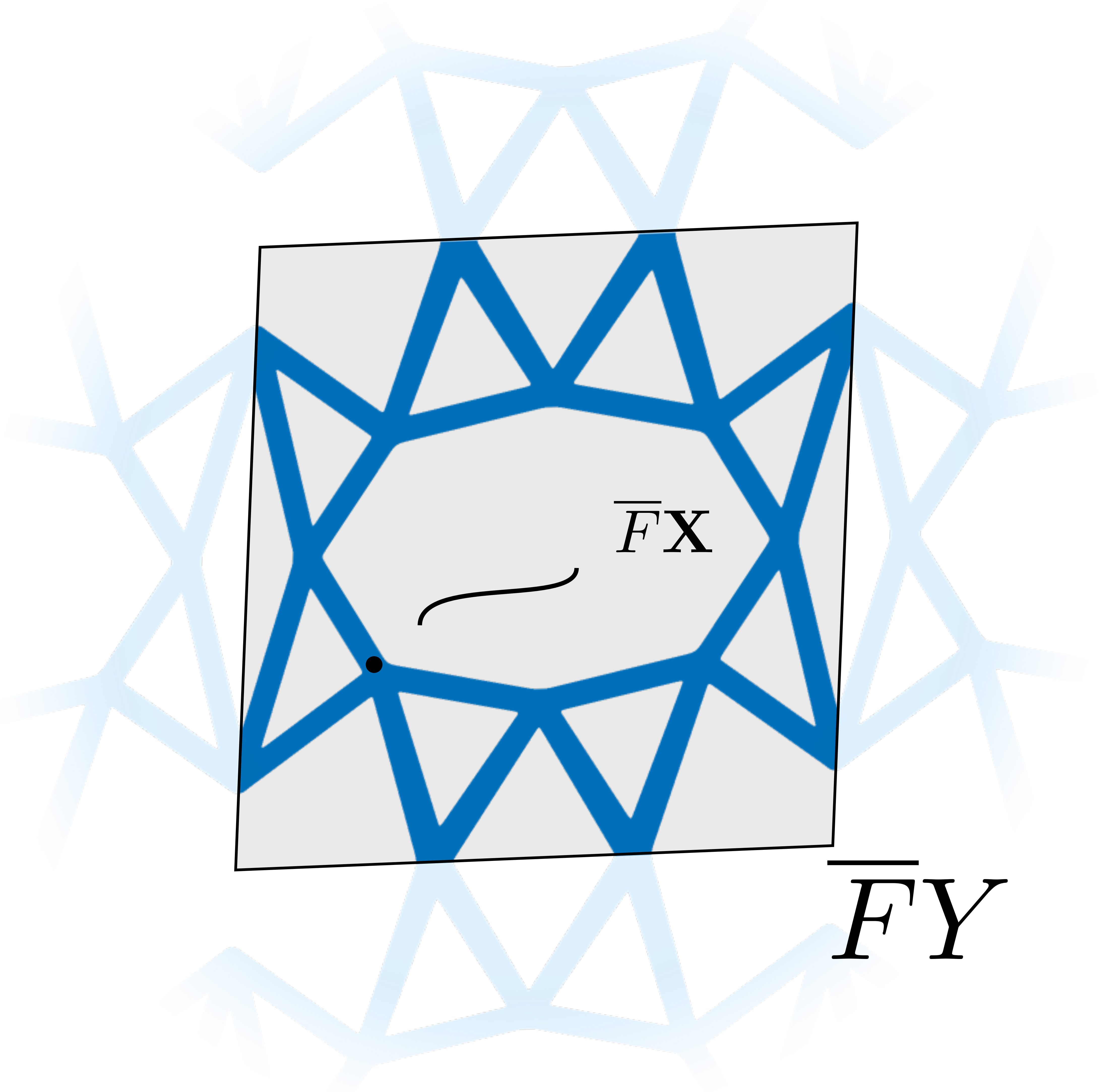
HOMOGENIZATION



$\overline{F}\mathbf{X}$ macro deformation

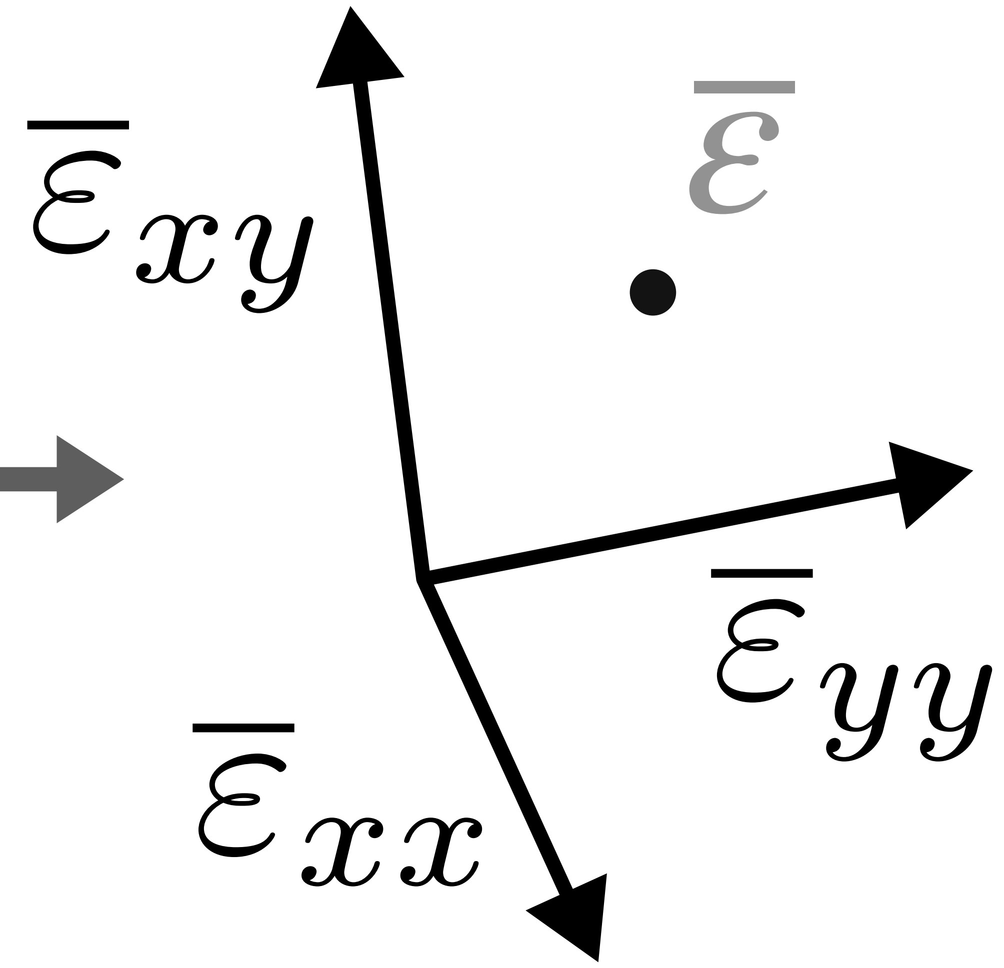
Symmetric $\overline{F} \in \mathbb{R}^{2\times2}$

$$\overline{F} = \overline{\varepsilon} + I$$

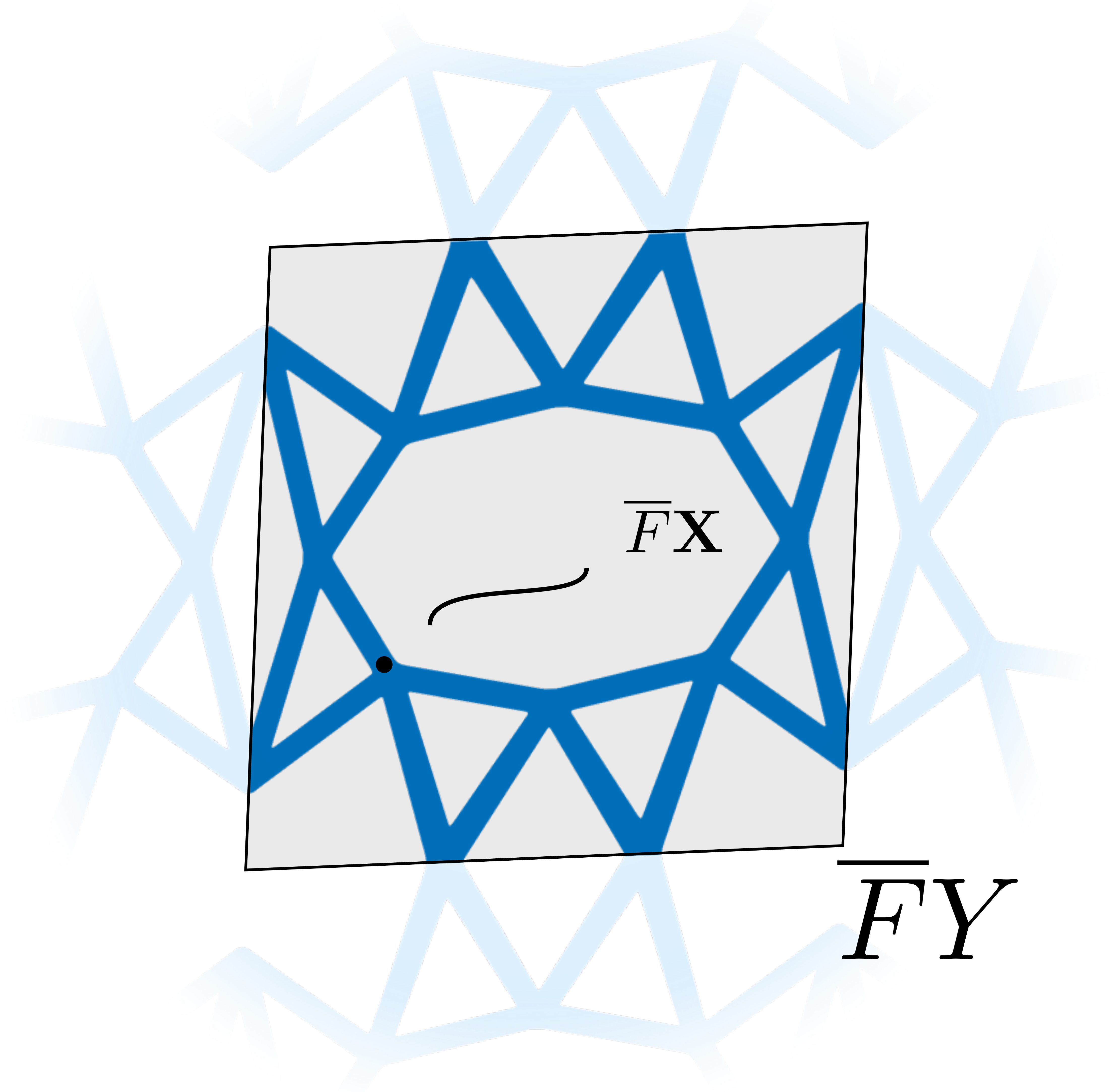
$$\begin{bmatrix} \overline{\varepsilon}_{xx} & \overline{\varepsilon}_{xy} \\ \overline{\varepsilon}_{xy} & \overline{\varepsilon}_{yy} \end{bmatrix}$$

Strain

(Biot)



HOMOGENIZATION



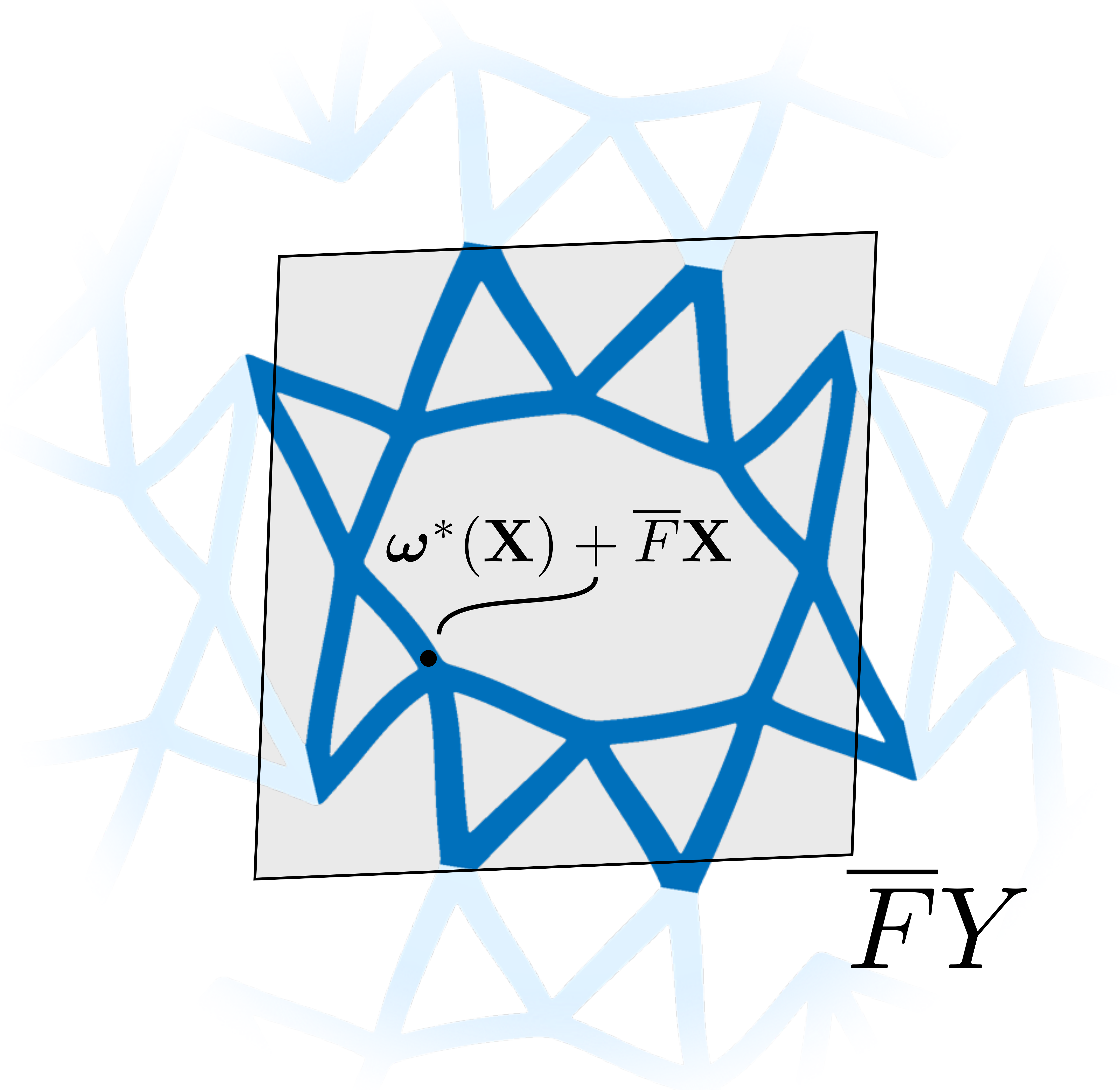
Deformation Function:

$$\Phi(\mathbf{X}) = \overline{F}\mathbf{X} + \boldsymbol{\omega}(\mathbf{X})$$

macro deformation

micro deformation

HOMOGENIZATION



Deformation Function:

Fluctuation Displacement Field

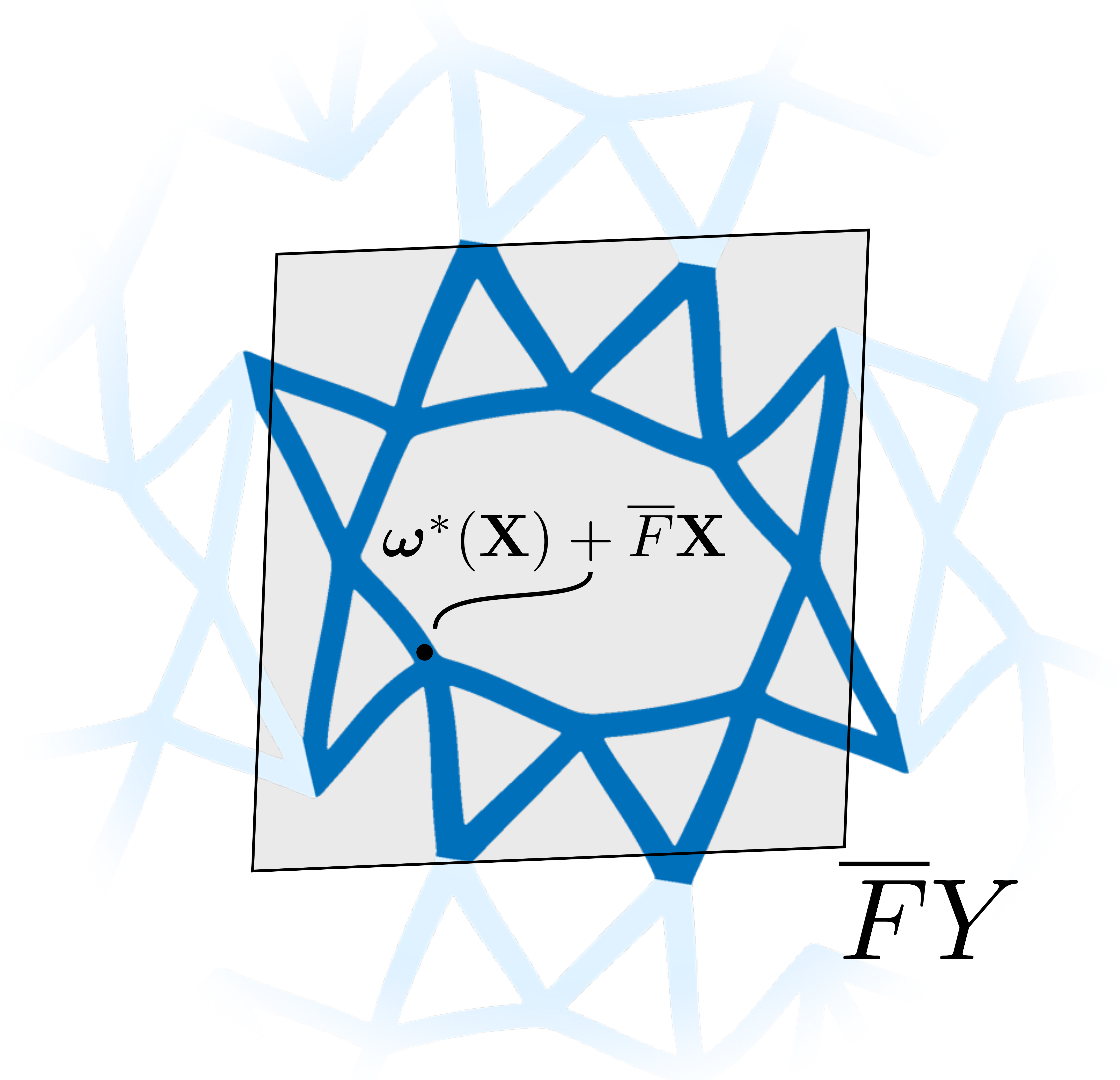
$$\Phi(\mathbf{X}) = \overline{F}\mathbf{X} + \boldsymbol{\omega}(\mathbf{X})$$

$\boldsymbol{\omega}$ *periodic*

macro deformation

micro deformation

HOMOGENIZATION



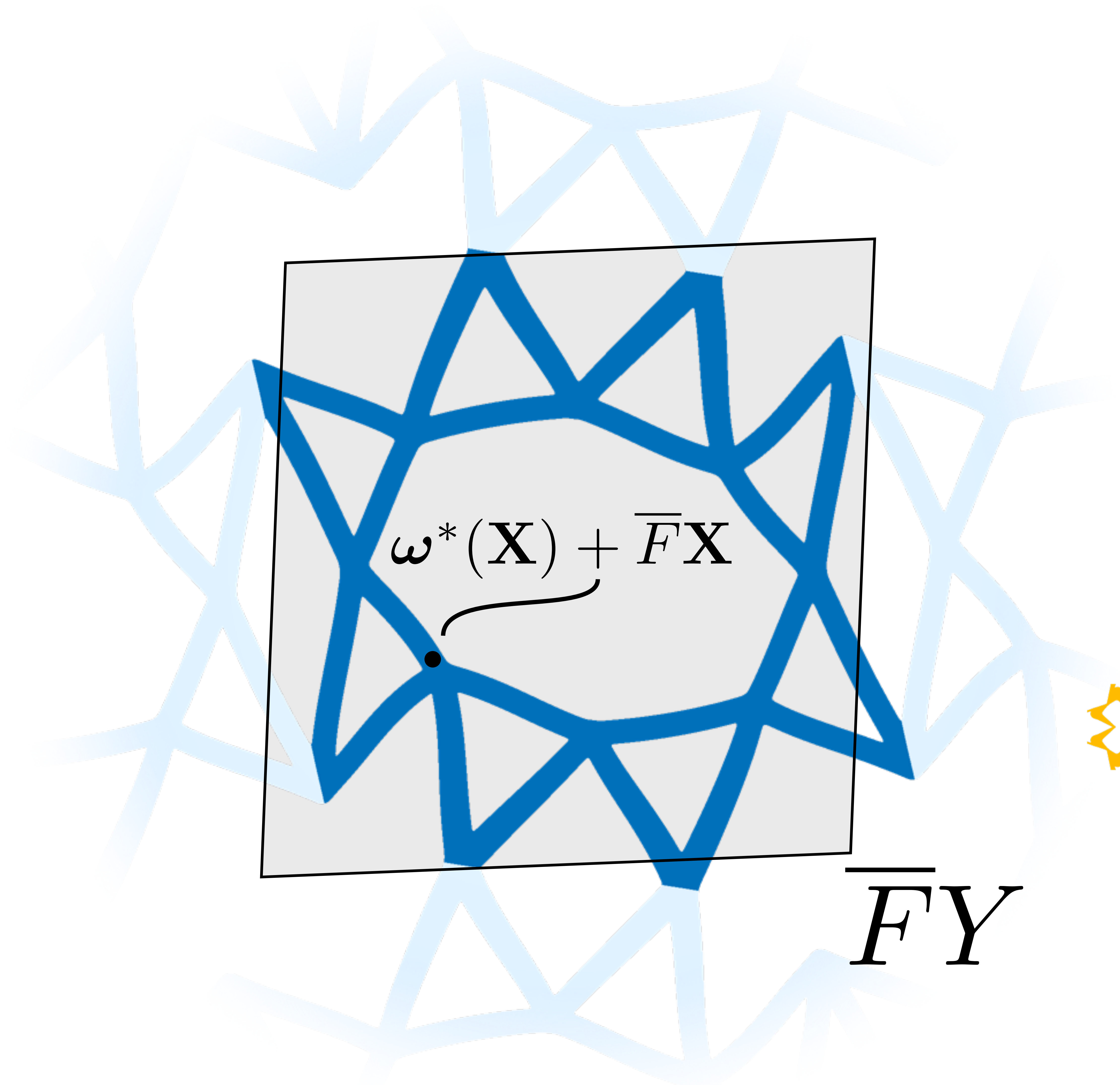
Deformation Function:

$$\Phi(\mathbf{X}) = \overline{F}\mathbf{X} + \boldsymbol{\omega}(\mathbf{X})$$

$$\min_{\omega\ periodic} \frac{1}{|Y|}\int_\Omega \psi(\nabla\boldsymbol{\omega} + \overline{F})\, d\mathbf{X}$$

’s elastic energy density

HOMOGENIZATION



Deformation Function:

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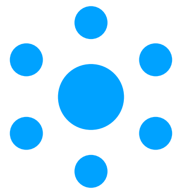
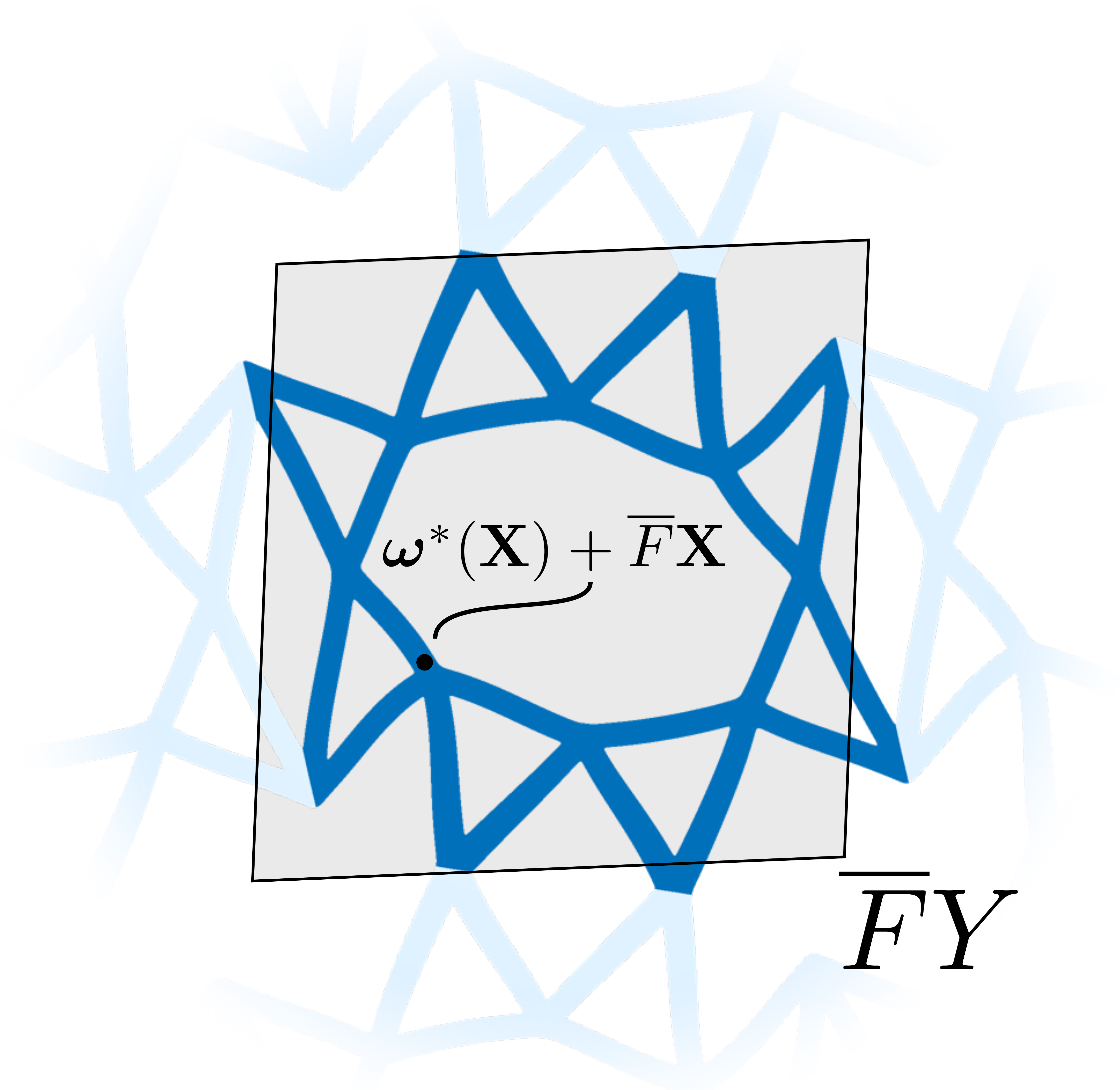
 *’s elastic energy density*  *’s elastic energy density*

Homogenized energy density function

HOMOGENIZATION



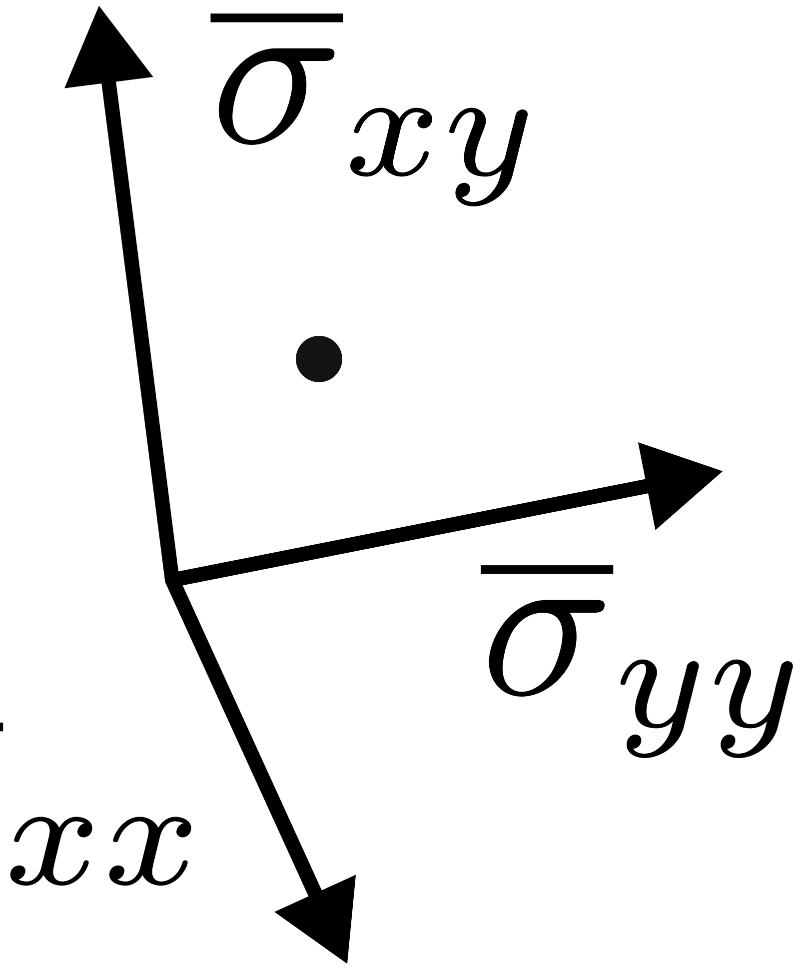
$$\overline{\psi}(\overline{F}) := \min_{\boldsymbol{\omega}\ periodic} \frac{1}{|Y|} \int_{\Omega} \psi(\nabla \boldsymbol{\omega} + \overline{F})\, d\mathbf{X}$$

$$\overline{\psi}'(\overline{F}) = \frac{1}{|Y|} \int_{\Omega} \psi'(\nabla \boldsymbol{\omega}^*(\mathbf{X};\overline{F}) + \overline{F})\, d\mathbf{X}$$

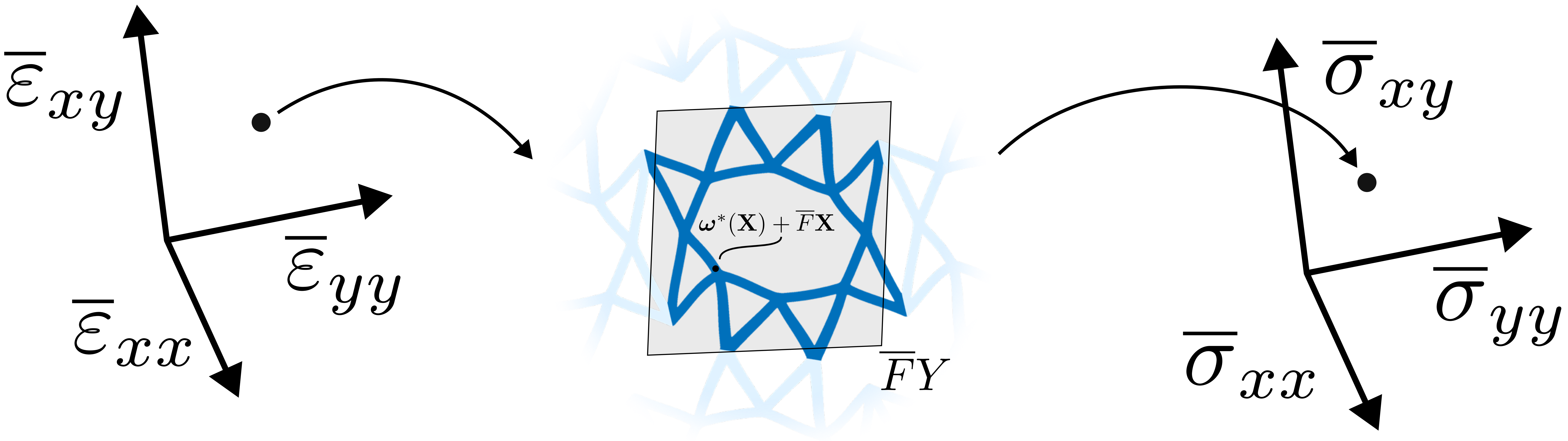
$$= \overline{\sigma}(\overline{F})$$

$$\begin{bmatrix} \overline{\sigma}_{xx} & \overline{\sigma}_{xy} \\ \overline{\sigma}_{xy} & \overline{\sigma}_{yy} \end{bmatrix}$$

Stress



HOMOGENIZATION



Step 1: Apply macro strain

Step 2: Solve displacement field

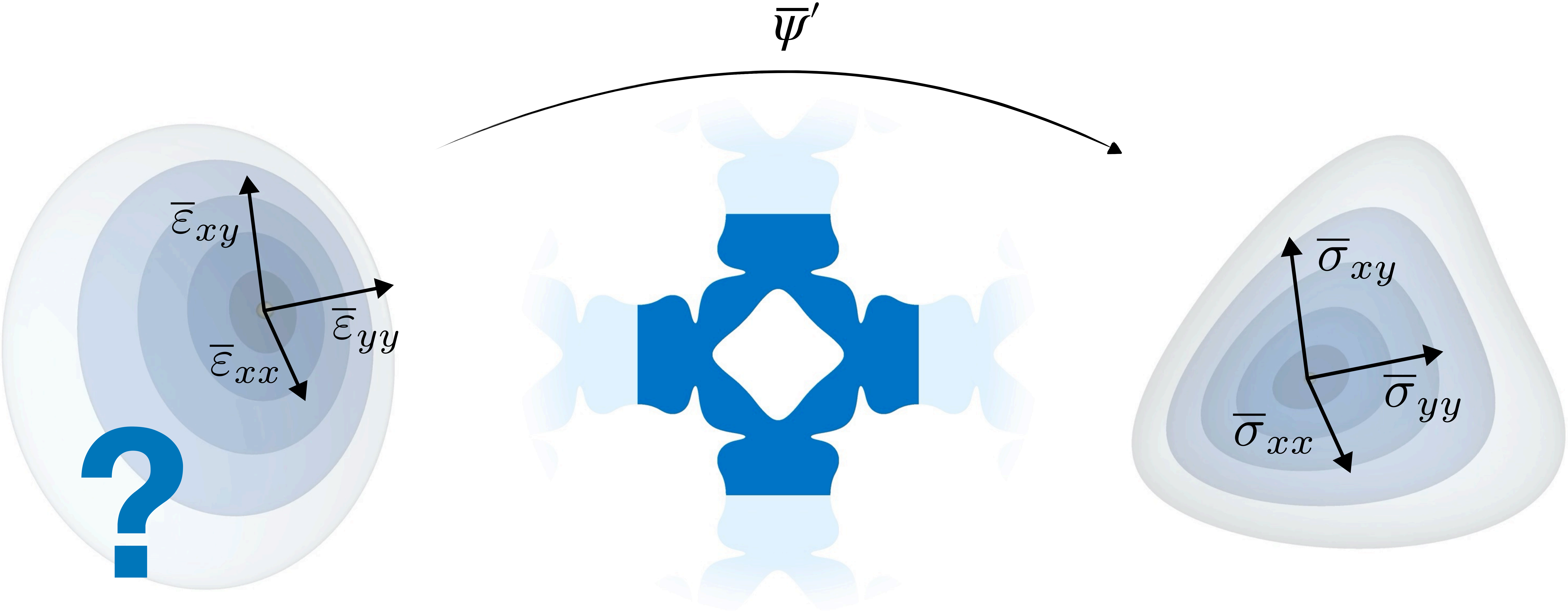
Step 3: Get response stress

Macro Deformation

$$\overline{\psi}(\overline{F}) := \min_{\boldsymbol{\omega}\, periodic} \frac{1}{|Y|} \int_{\Omega} \psi(\nabla \boldsymbol{\omega} + \overline{F})\, d\mathbf{X}$$

Response Force

HOMOGENIZATION



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Response Force

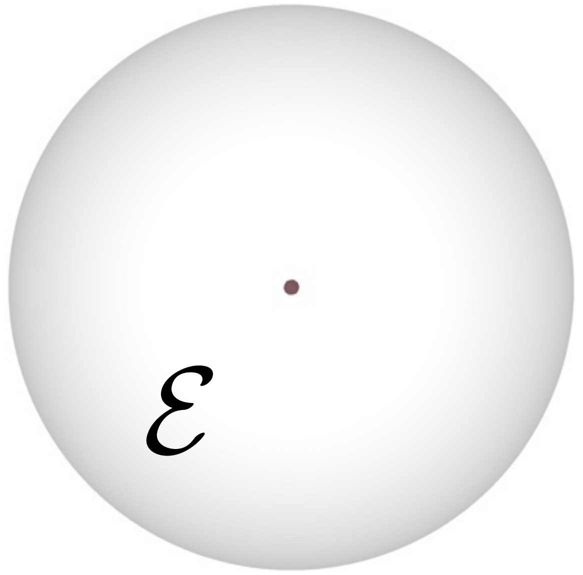
HOMOGENIZATION

Past Works

Infinitely small deformation

$$\overline{\psi}(\overline{F}) := \min_{\boldsymbol{\omega}\ periodic} \frac{1}{|Y|}\int_{\Omega} \psi(\nabla\boldsymbol{\omega} + \overline{F})\, d\mathbf{X}$$

ψ Linear Elasticity



$\mathcal{E}$

$\{origin\}$

[Neves et al. 2000] …

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HOMOGENIZATION

Past Works

Infinitely small deformation

$$\overline{\psi}(\overline{F}) := \min_{\omega\, periodic} \frac{1}{|Y|} \int_{\Omega} \psi(\nabla \boldsymbol{\omega} + \overline{F})\, d\mathbf{X}$$

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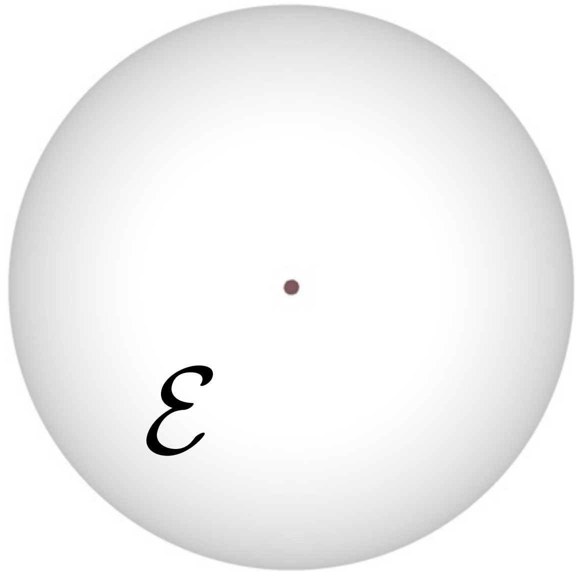
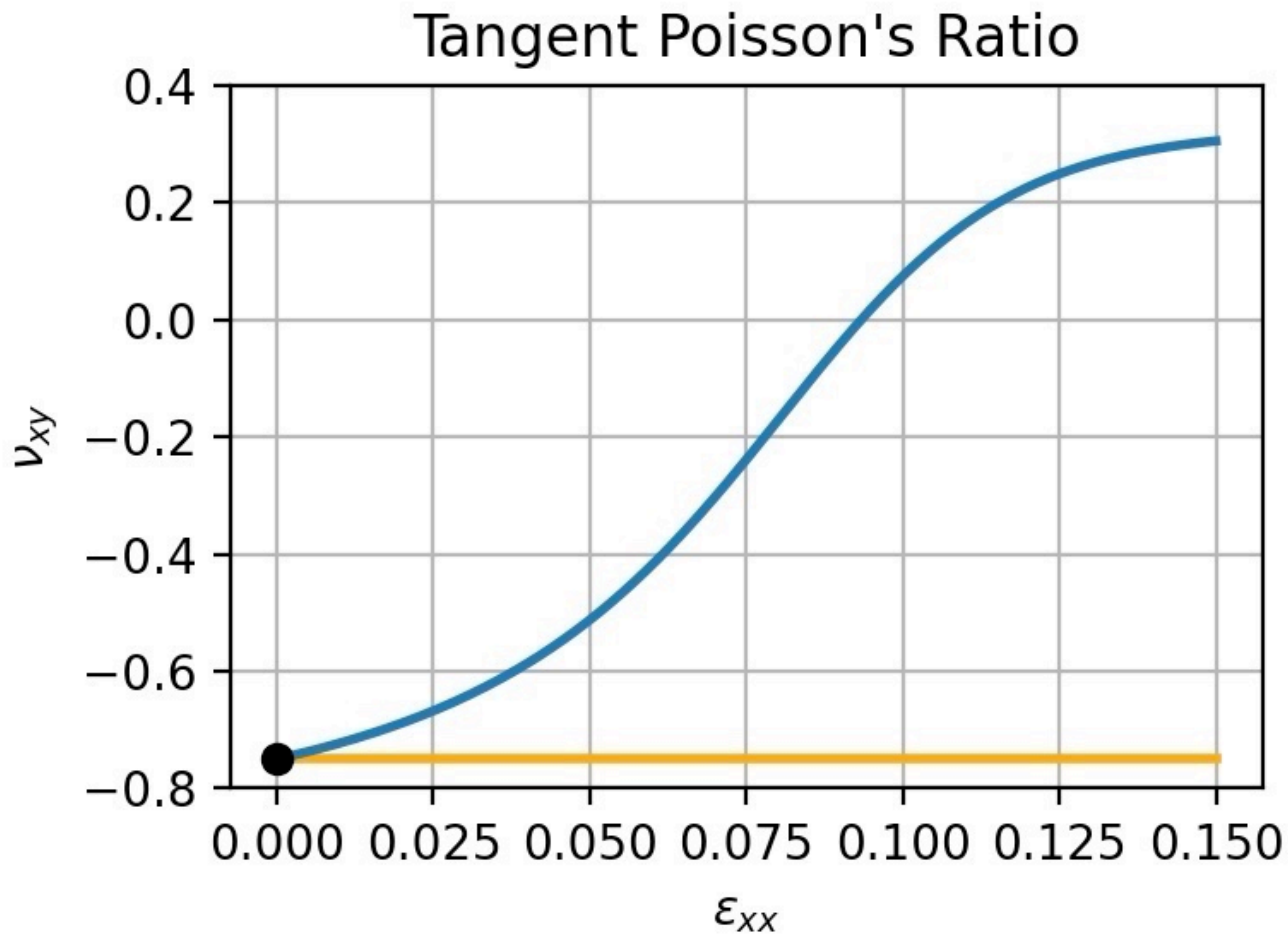


$\mathscr{E}$

$\{origin\}$



[Neves et al. 2000]

HOMOGENIZATION

Past Works

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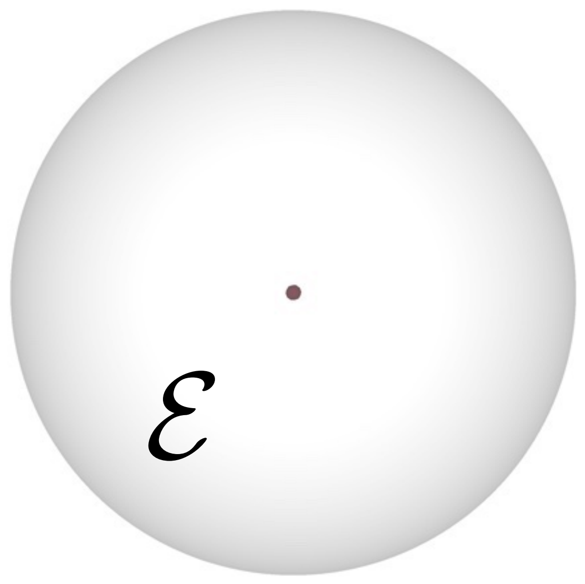


$\mathscr{E}$

$\{origin\}$

ψ **Nonlinear** Elasticity Model

- Corotated
- Saint Venant-Kirchhoff
- Neo-Hookean
- …

Flexible

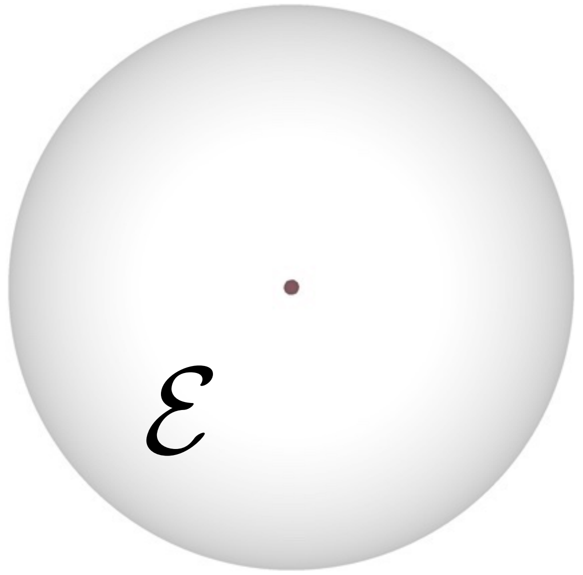
[Neves et al. 2000]

HOMOGENIZATION

Past Works

Flexible

Infinitely small deformation

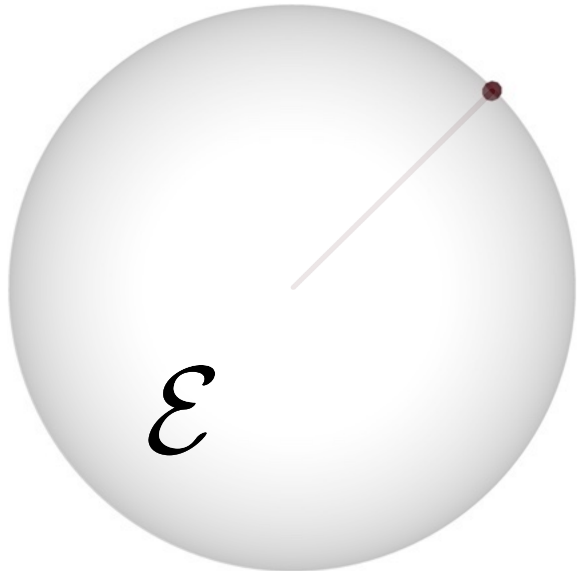


$\mathscr{E}$

$\{origin\}$

[Neves et al. 2000]

A few sampled biaxial strains

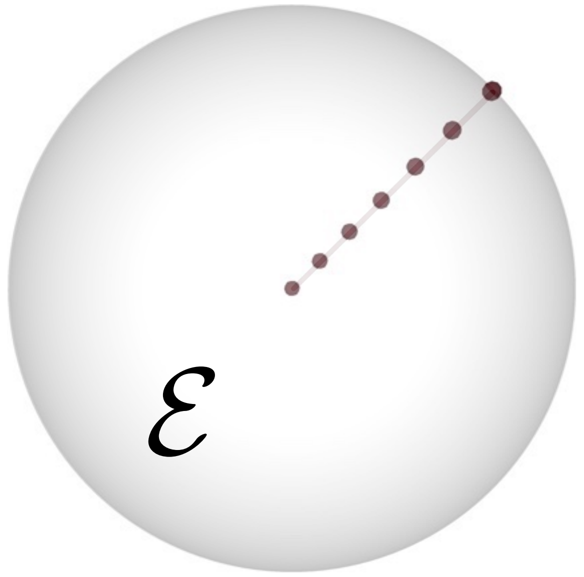


$\mathscr{E}$

$\{points\}$

[Behrou et al. 2021]

Along uniaxial stretch path

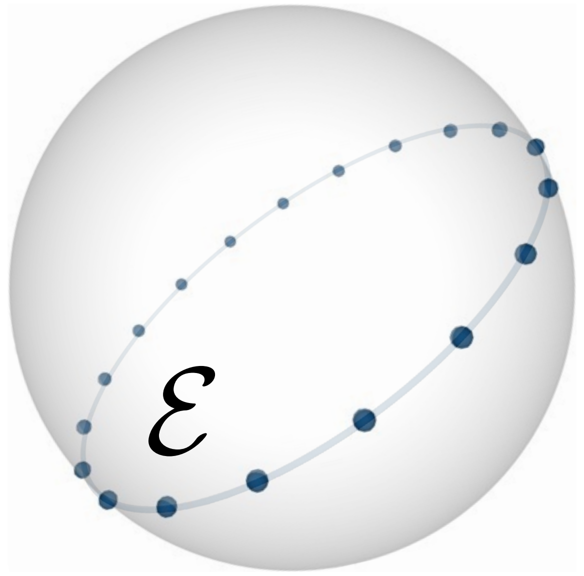


$\mathscr{E}$

$\{a\ line\}$

[Clausen et al. 2015]

Trajectories through strain space



$\mathscr{E}$

$\{a\ circle\}$

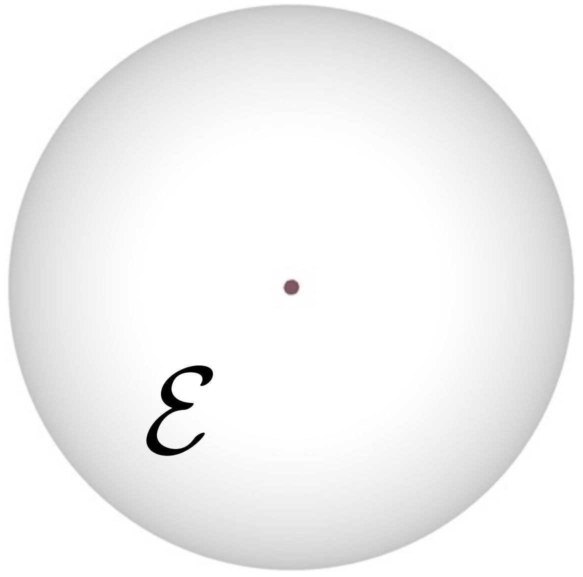
[Schumacher et al. 2018]

HOMOGENIZATION

Past Works

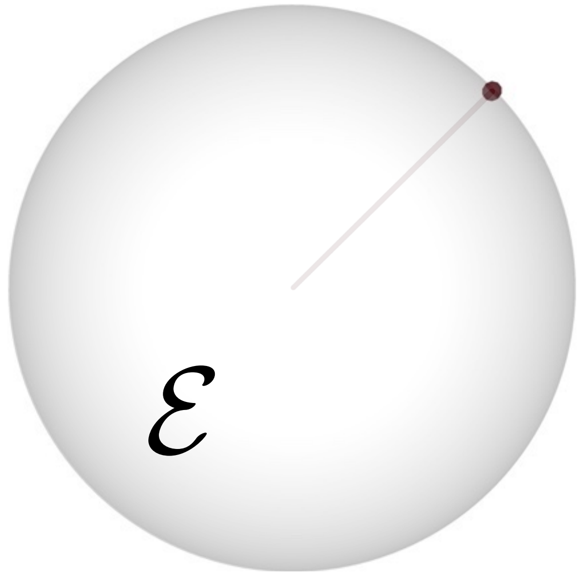
Flexible

Infinitely small deformation | *A few sampled biaxial strain* | *Along uniaxial stretch path* | *Trajectories through strain space*



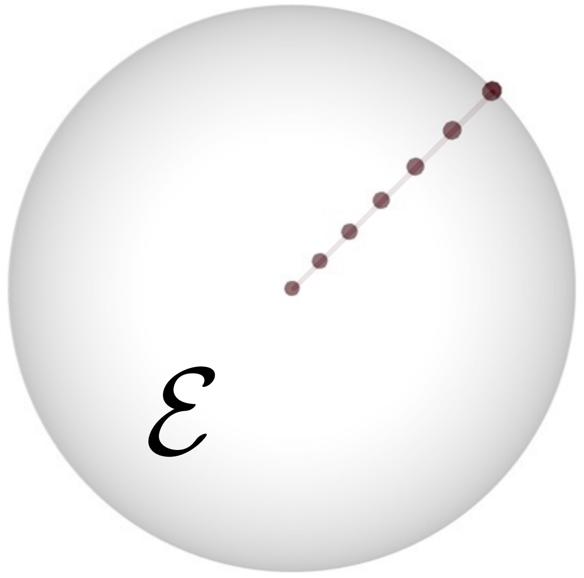
$$\mathcal{E}$$
$$\{origin\}$$

[Neves et al. 2000]



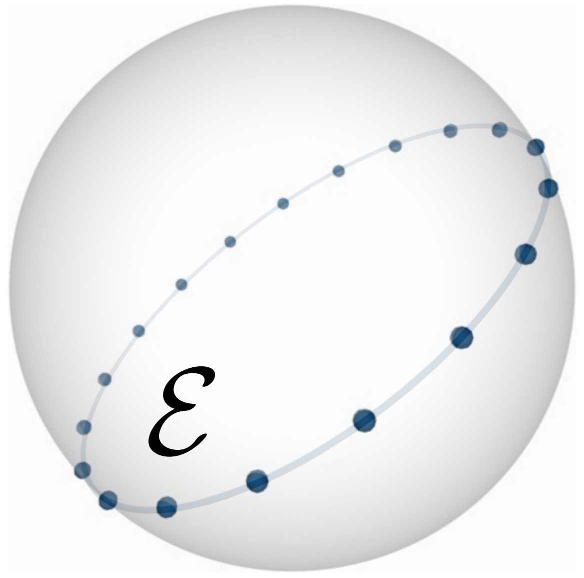
$$\mathcal{E}$$
$$\{points\}$$

[Behrou et al. 2021]



$$\mathcal{E}$$
$$\{a\ line\}$$

[Clausen et al. 2015]

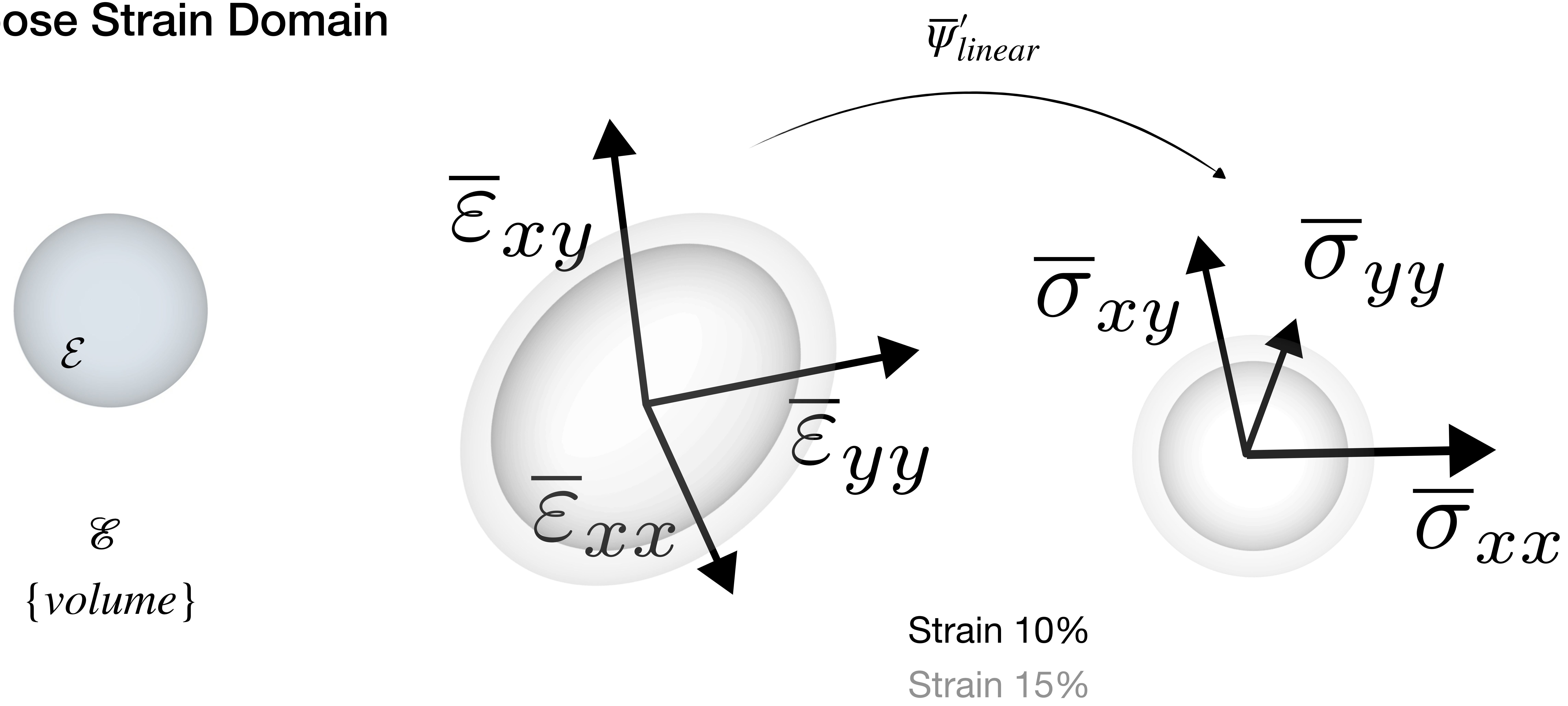


$$\mathcal{E}$$
$$\{a\ circle\}$$

[Schumacher et al. 2018]

HOMOGENIZATION

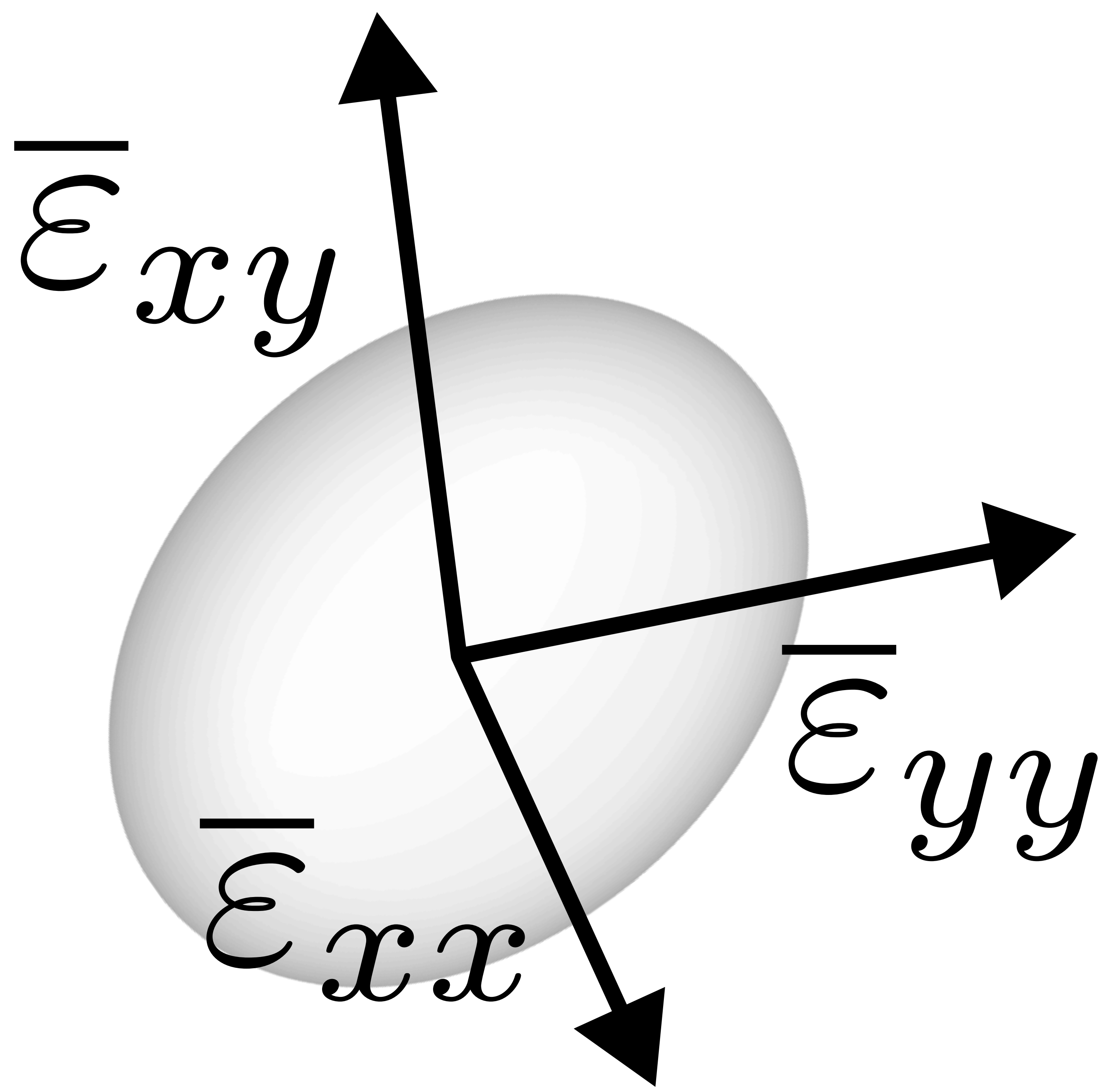
Choose Strain Domain



HOMOGENIZATION

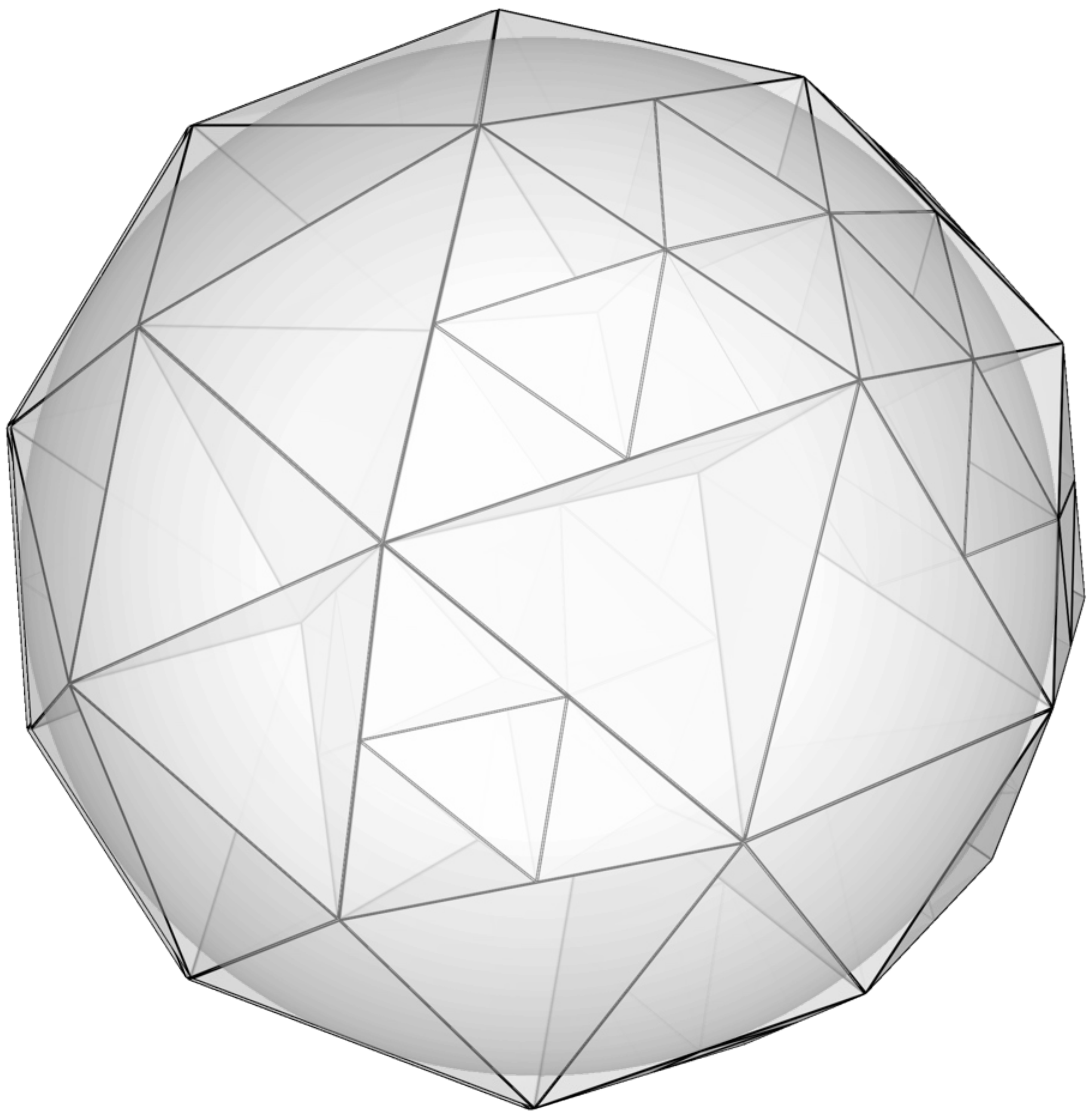
Interpolation

Choose Strain Domain



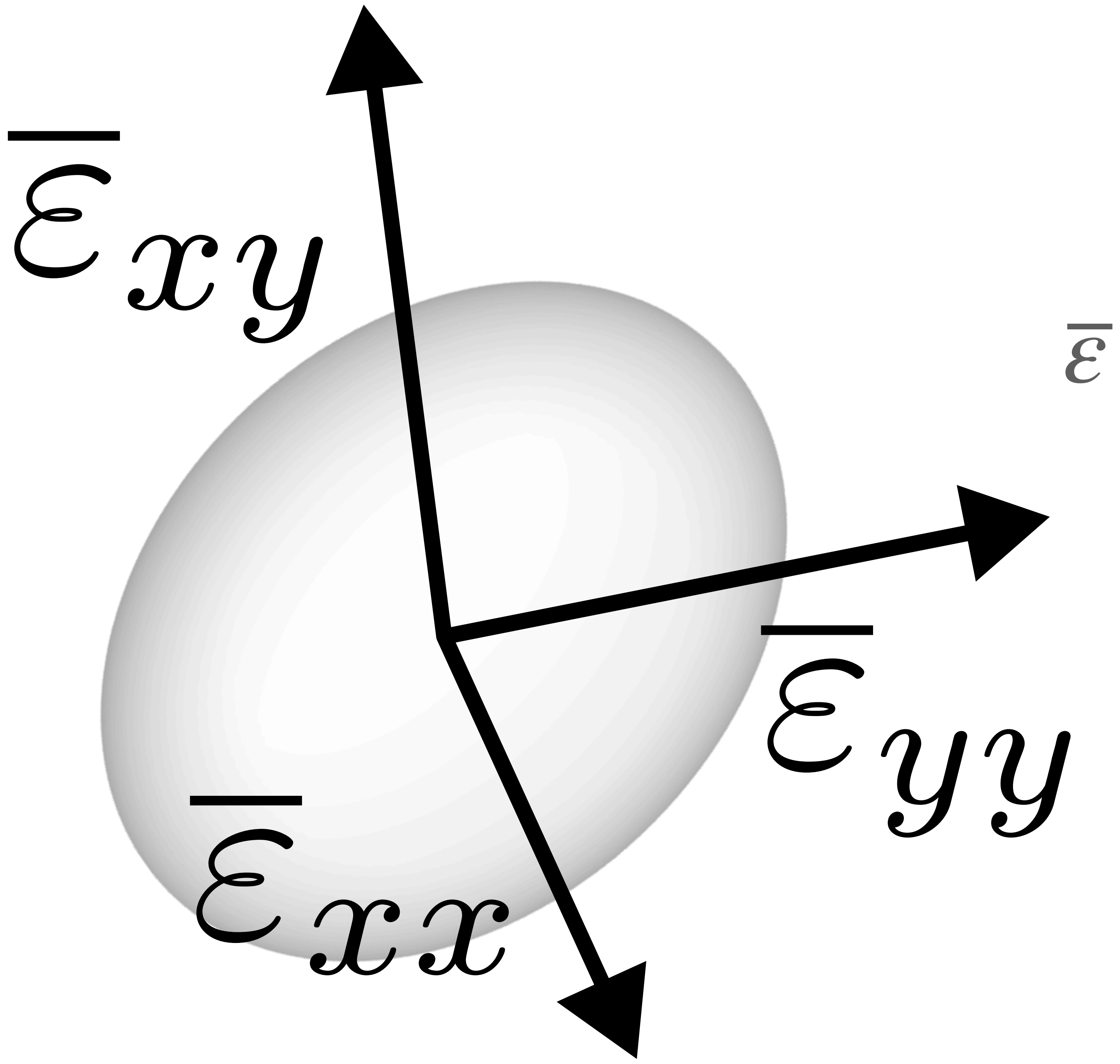
$\mathscr{E}$ Strain 10%

Adaptive Subdivision



HOMOGENIZATION

Interpolation

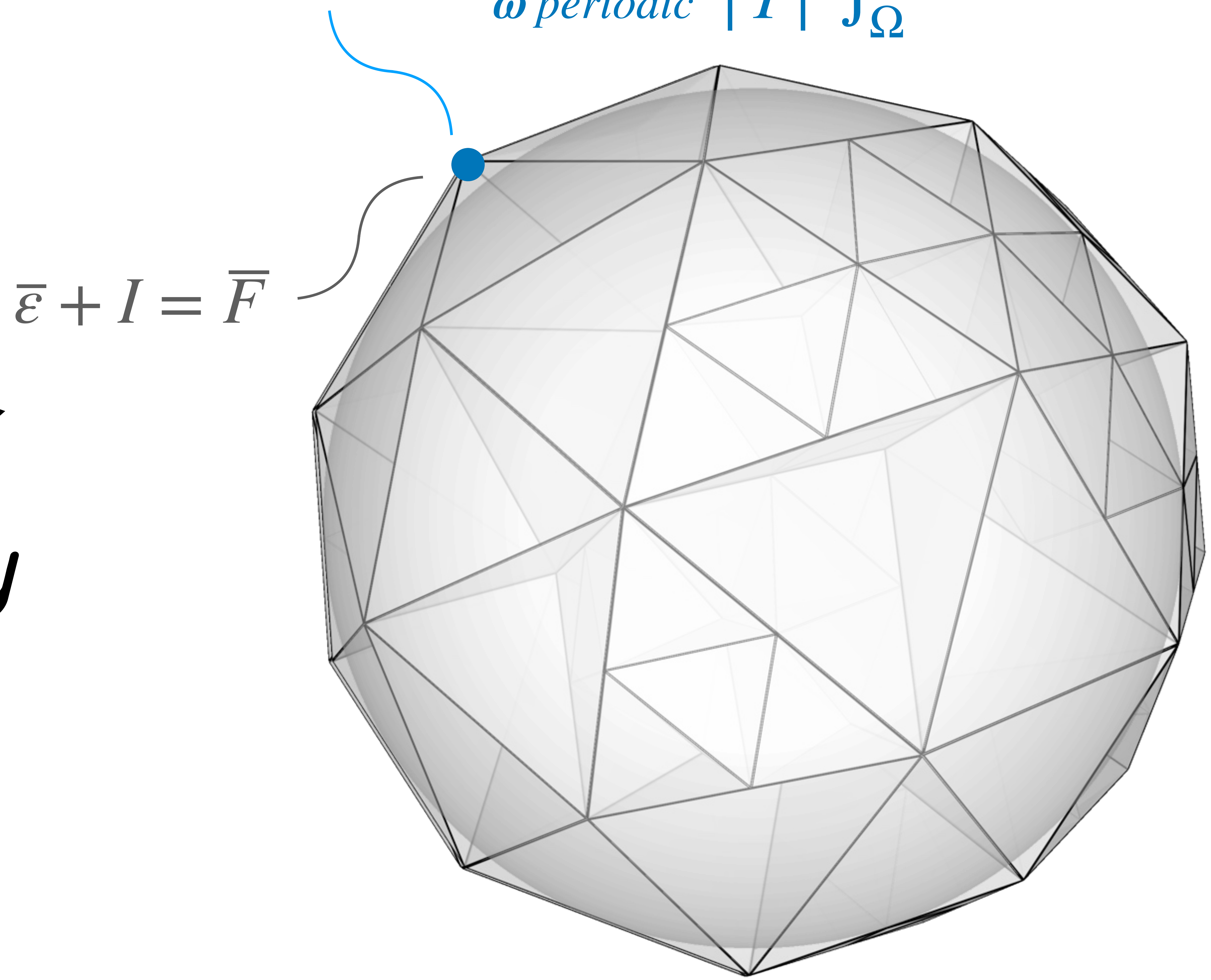


$\mathscr{E}$

Strain 10%

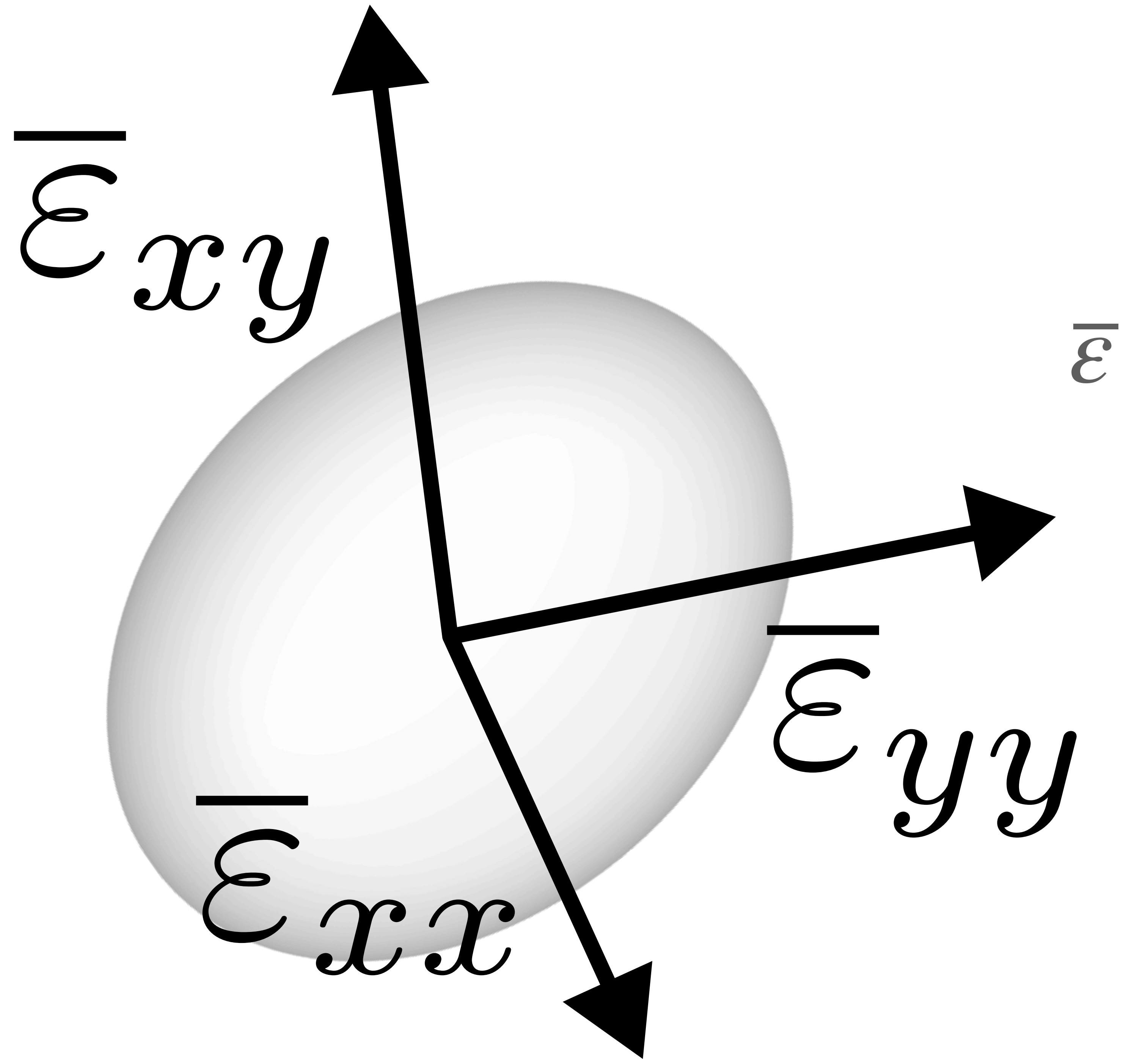
Interpolate displacement field ω^*

$$\boldsymbol{\omega}^* = \arg \min_{\boldsymbol{\omega}\, periodic} \frac{1}{|Y|} \int_{\Omega} \psi(\nabla \boldsymbol{\omega} + \overline{F})\, d\mathbf{X}$$



HOMOGENIZATION

Interpolation



$\mathscr{E}$

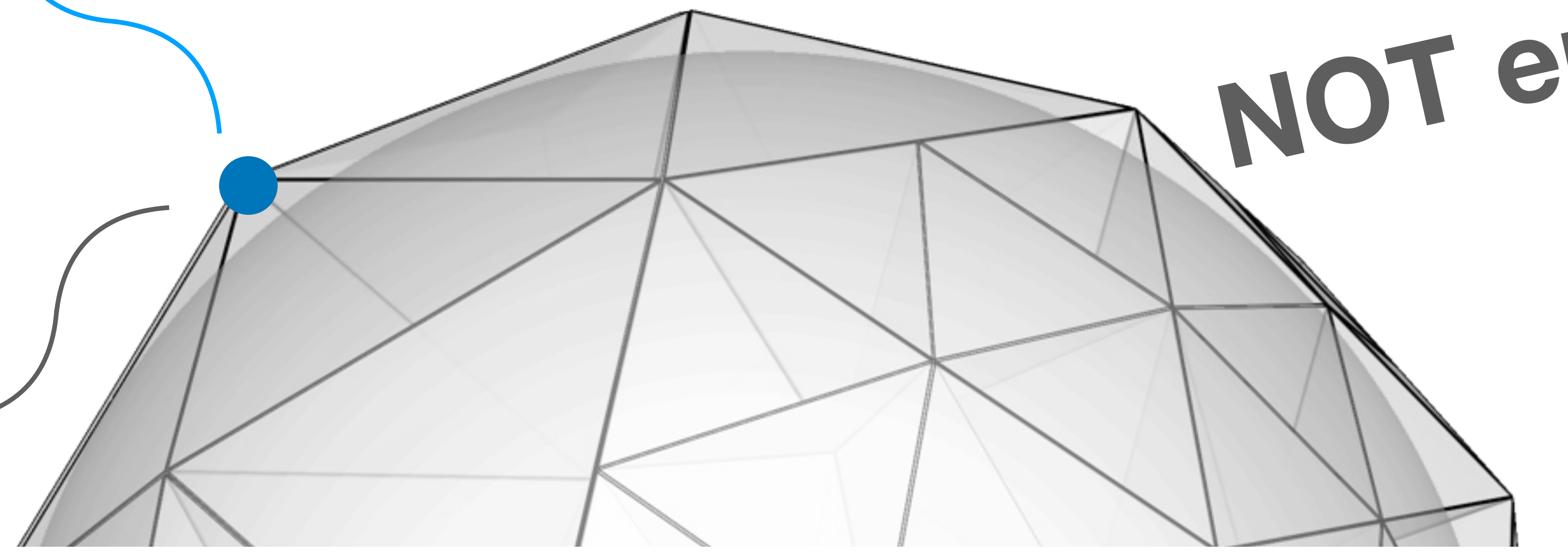
Strain 10%

Interpolate displacement field ω^*

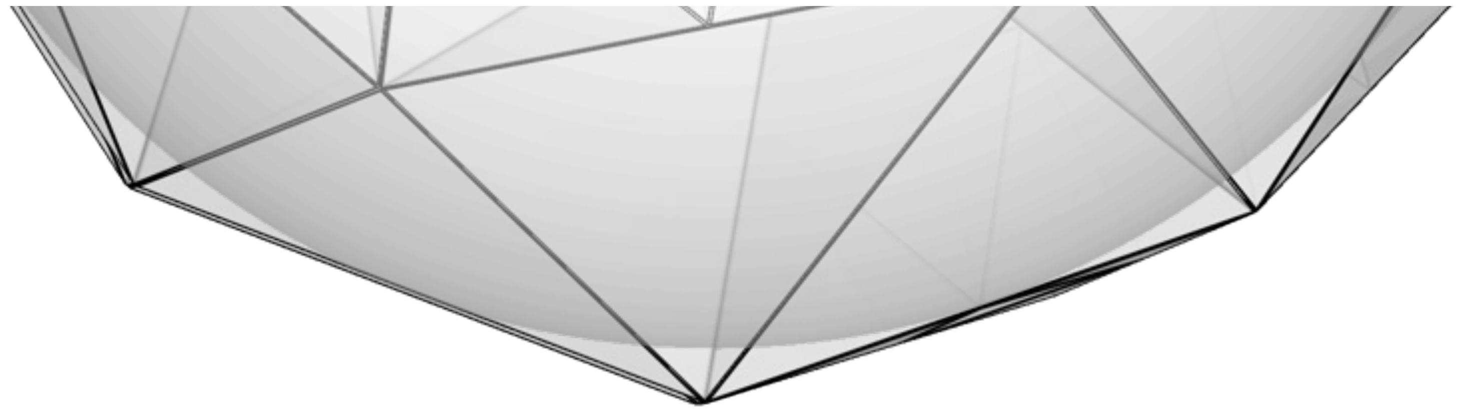
$$\omega^* = \arg \min_{\omega\, periodic} \frac{1}{|Y|} \int_\Omega \psi(\nabla \omega + \overline{F})\, d\mathbf{X}$$

NOT energy $\overline{\Psi}$!

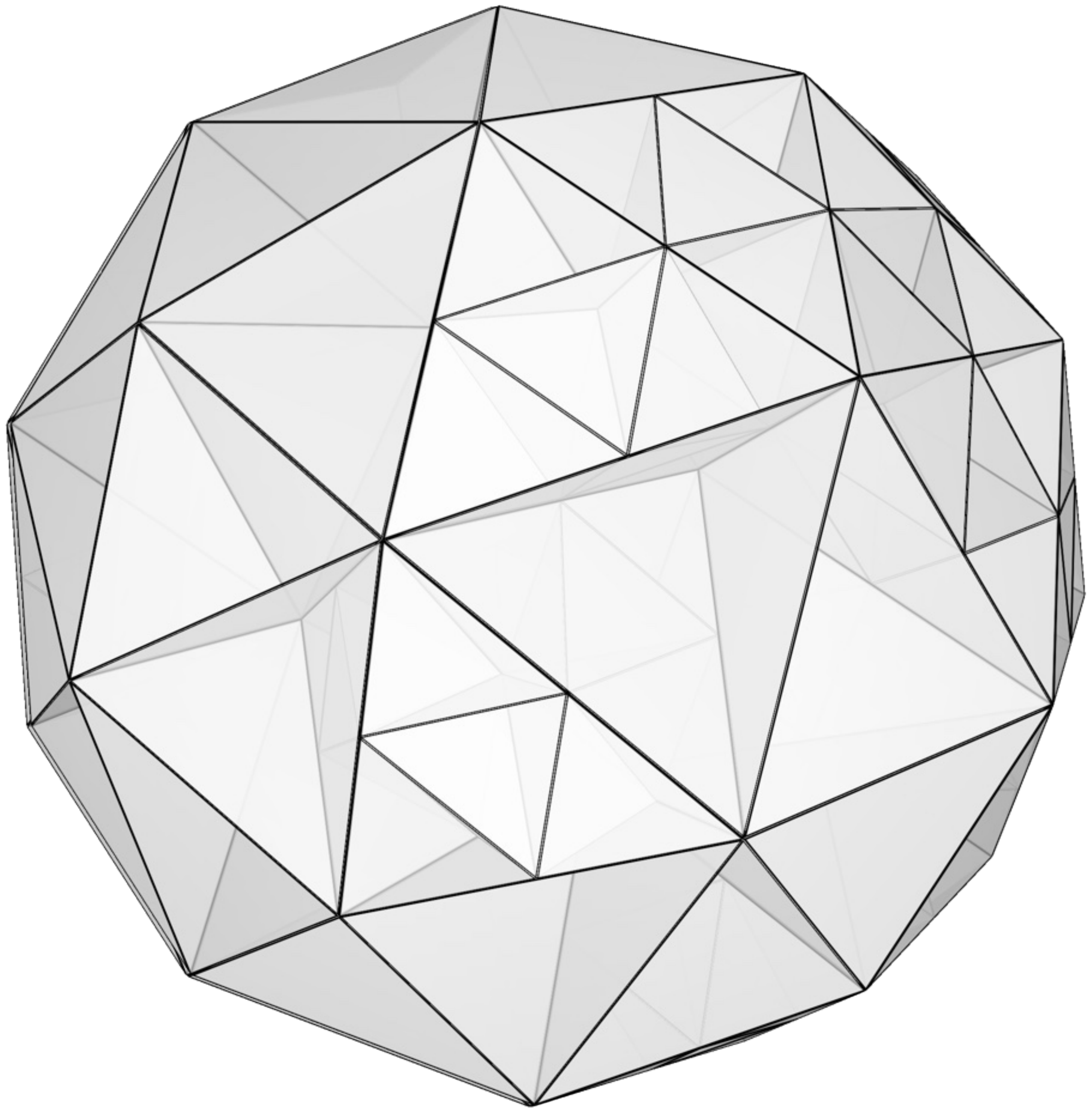
$\overline{\varepsilon} + I = \overline{F}$



1. Efficient to evaluate the accuracy (no need for ground truth)
2. Accelerates nonlinear solves (provides high-quality initialization)



HOMOGENIZATION
Interpolation



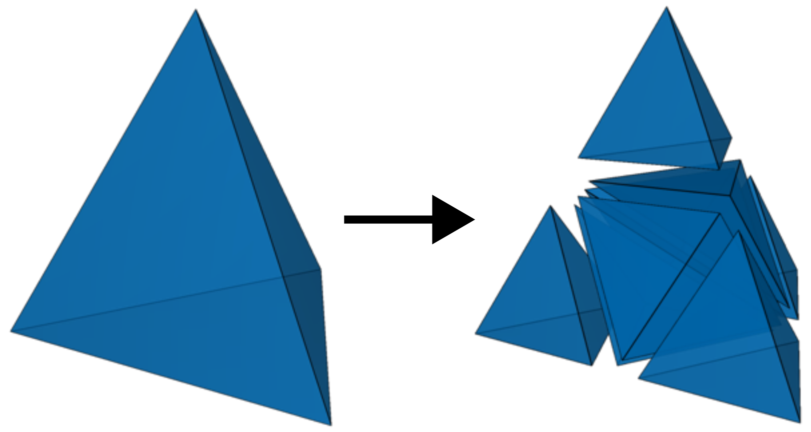
Interpolate displacement field ω^*

$$\boldsymbol{\omega}^* = \arg \min_{\boldsymbol{\omega}\, periodic} \frac{1}{|Y|} \int_{\Omega} \psi(\nabla \boldsymbol{\omega} + \overline{F})\, d\mathbf{X}$$

Linear Interpolation

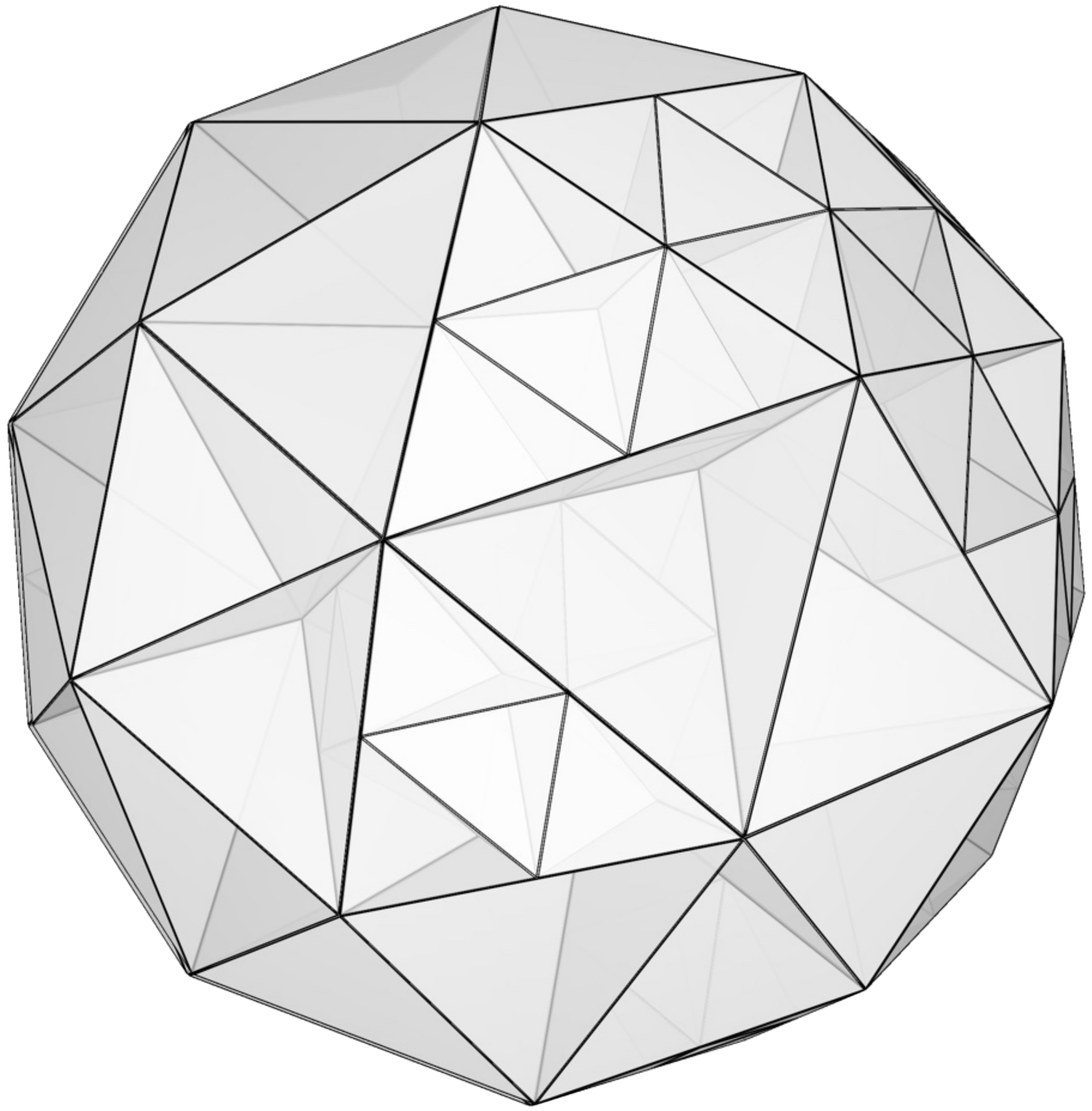
Evaluate at every center of tet

Check for the residual only



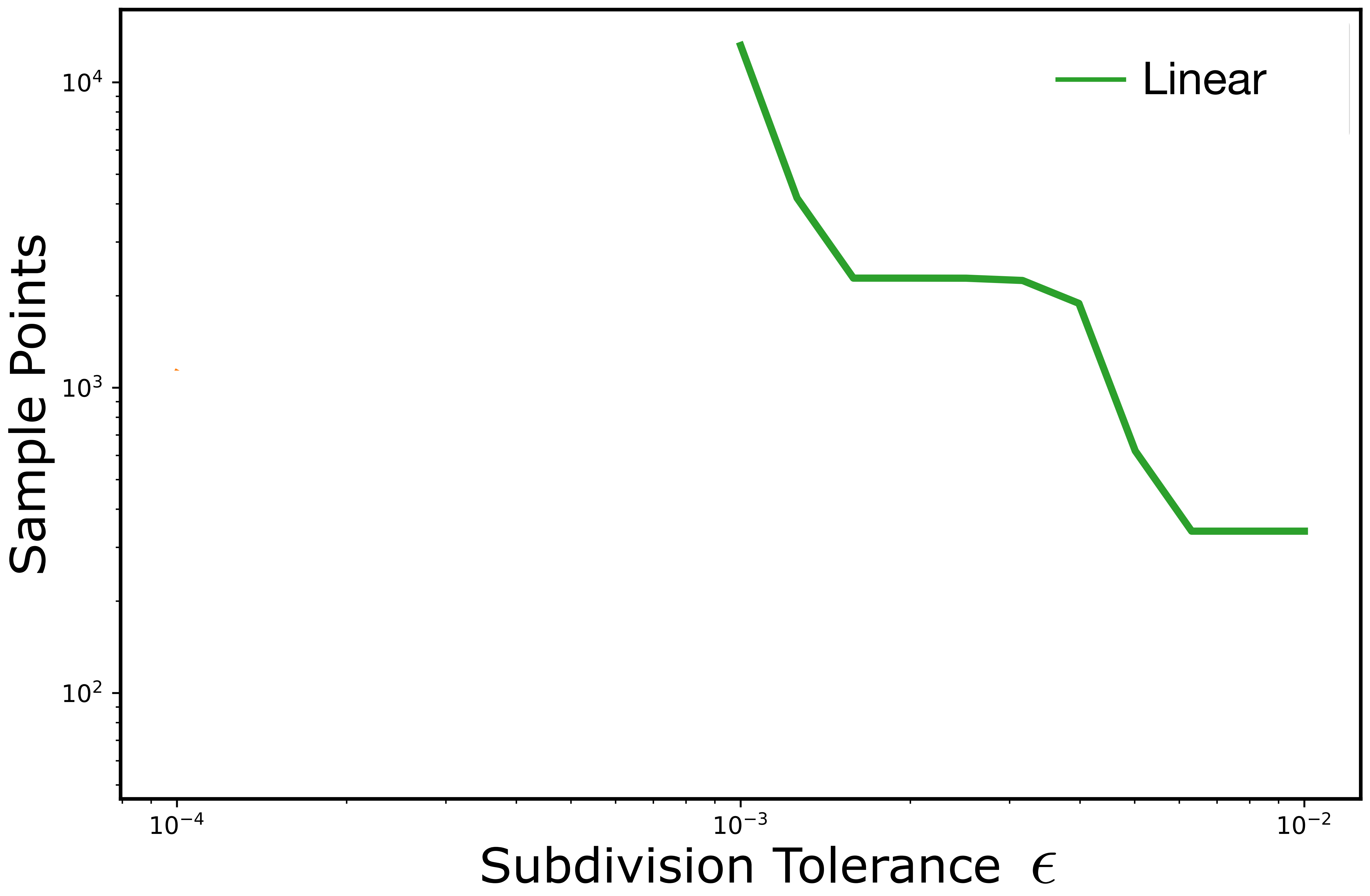
HOMOGENIZATION

Interpolation

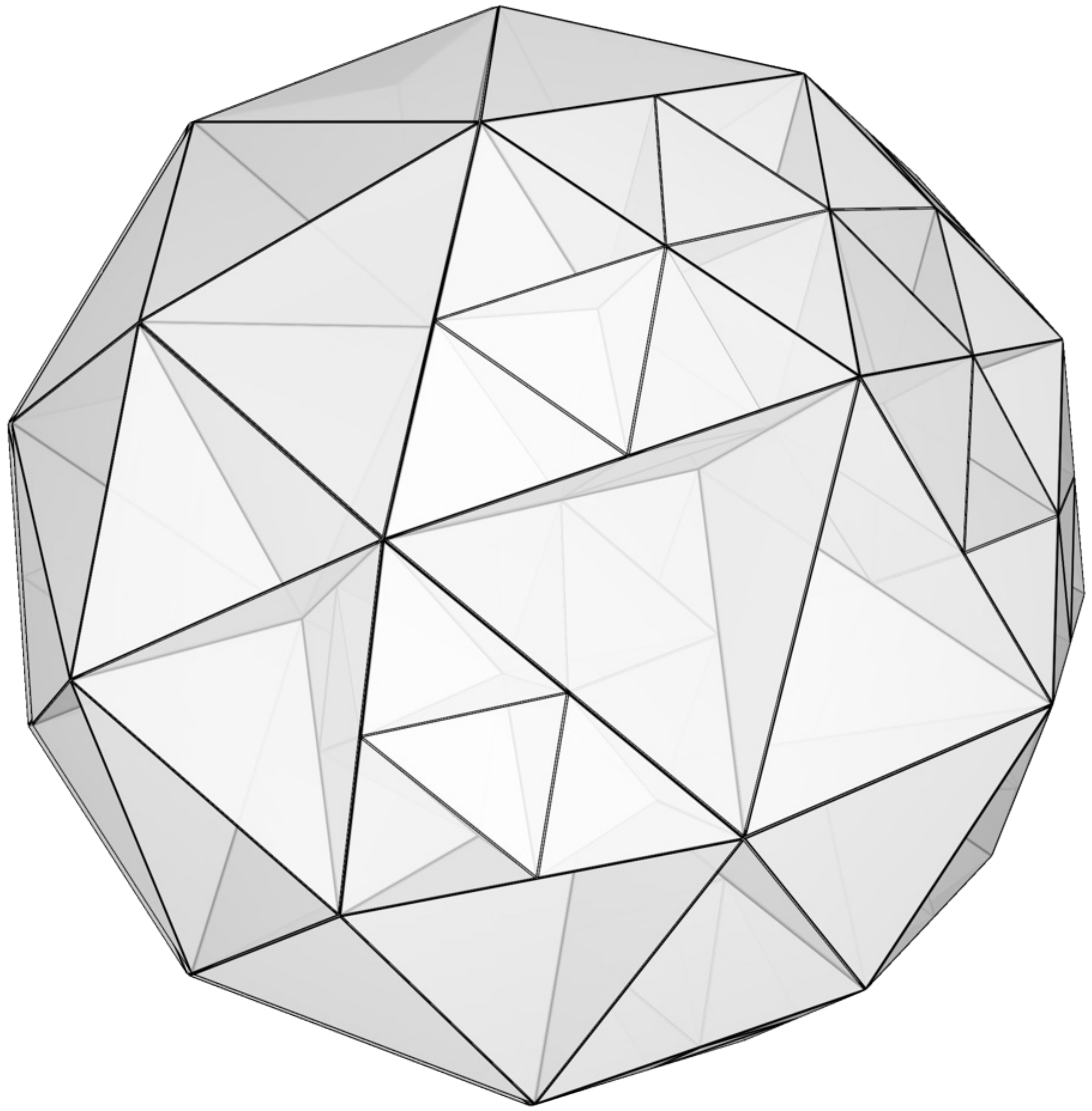


Interpolate displacement field ω^*

$$\boldsymbol{\omega}^* = \arg \min_{\boldsymbol{\omega}\, periodic} \frac{1}{|Y|} \int_{\Omega} \psi(\nabla \boldsymbol{\omega} + \overline{F})\, d\mathbf{X}$$



HOMOGENIZATION
Interpolation



Interpolate displacement field ω^*

$$\boldsymbol{\omega}^* = \arg \min_{\boldsymbol{\omega}\, periodic} \frac{1}{|Y|} \int_{\Omega} \psi(\nabla \boldsymbol{\omega} + \overline{F})\, d\mathbf{X}$$

Powell-Sabin Interpolation

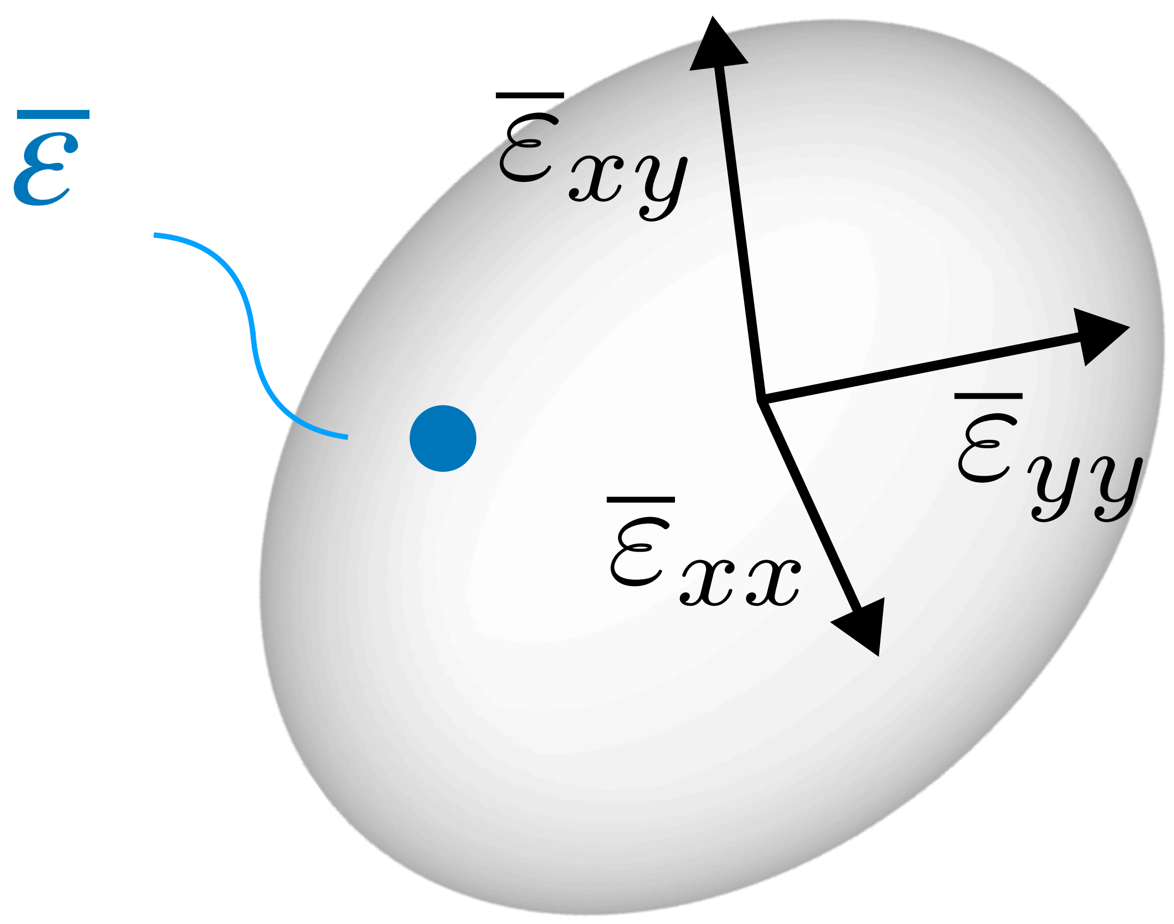
C^1

$$\left(\omega^*, \quad \frac{\partial \omega^*}{\partial \overline{F}}\right)$$

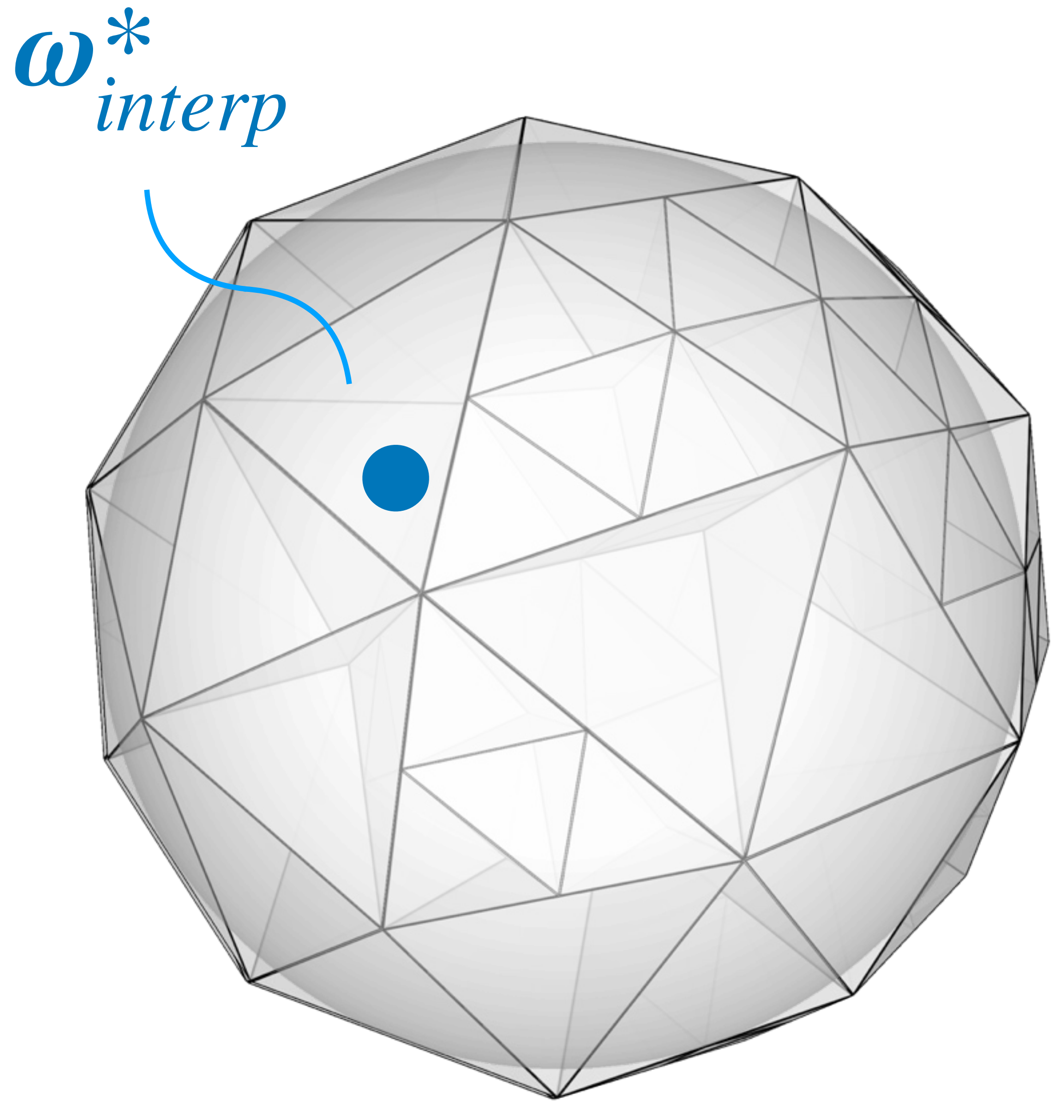
derivative with respect to the macroscopic strain

HOMOGENIZATION

Interpolation



Adaptive Subdivision

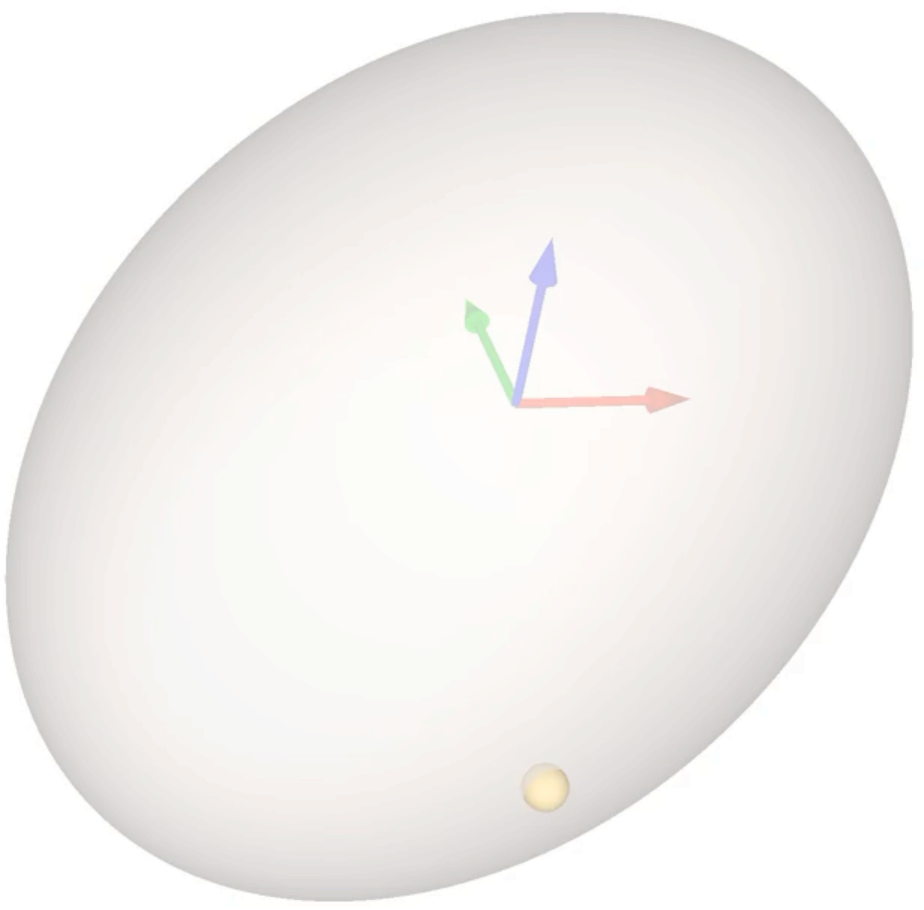


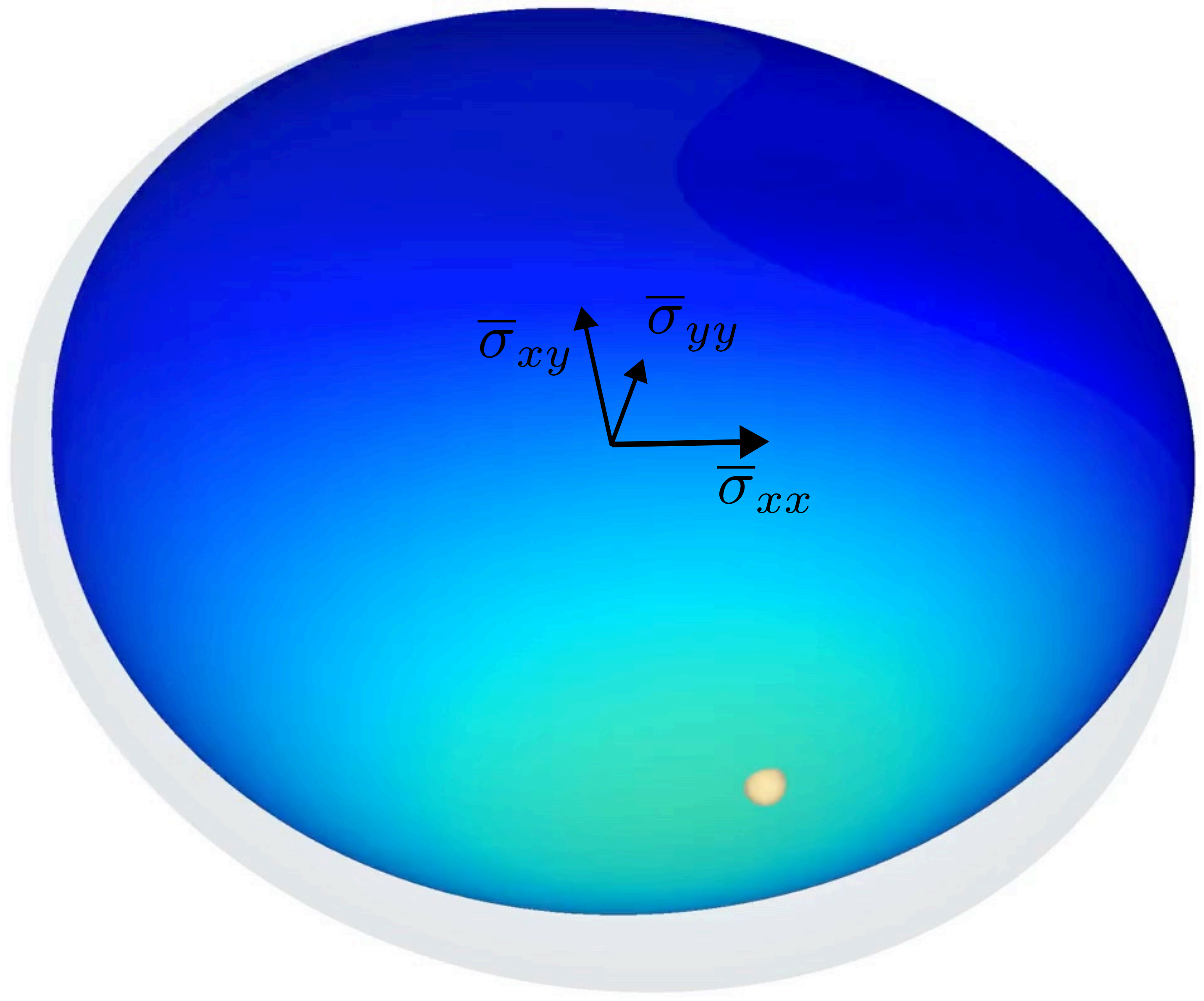
$\mathcal{E}$

Strain 10%

HOMOGENIZATION

Interpolation





$\mathcal{E}$

Strain 10%

Stress Domain

DESIGN PROBLEM

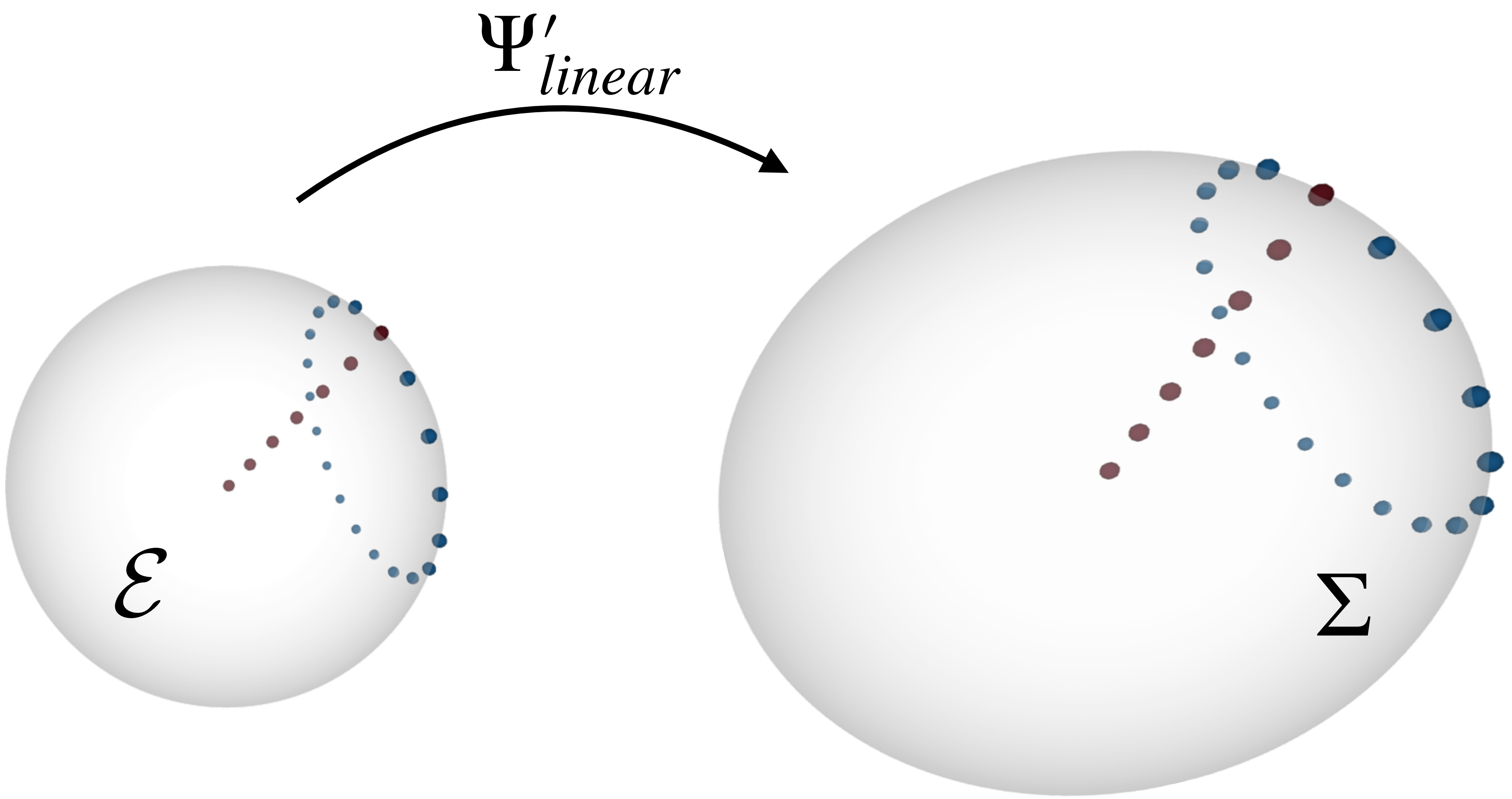
$$\overline{\psi} \rightarrow \overline{\Psi}_{tgt}$$

$\overline{\Psi}_{tgt}$ Linear Elasticity

$$\Psi_{linear}(\overline{F}) = \frac{1}{2}\,\overline{\varepsilon} : C : \overline{\varepsilon}$$

$$\Psi'_{linear}(\overline{F}) = \overline{\sigma} = C : \overline{\varepsilon}$$

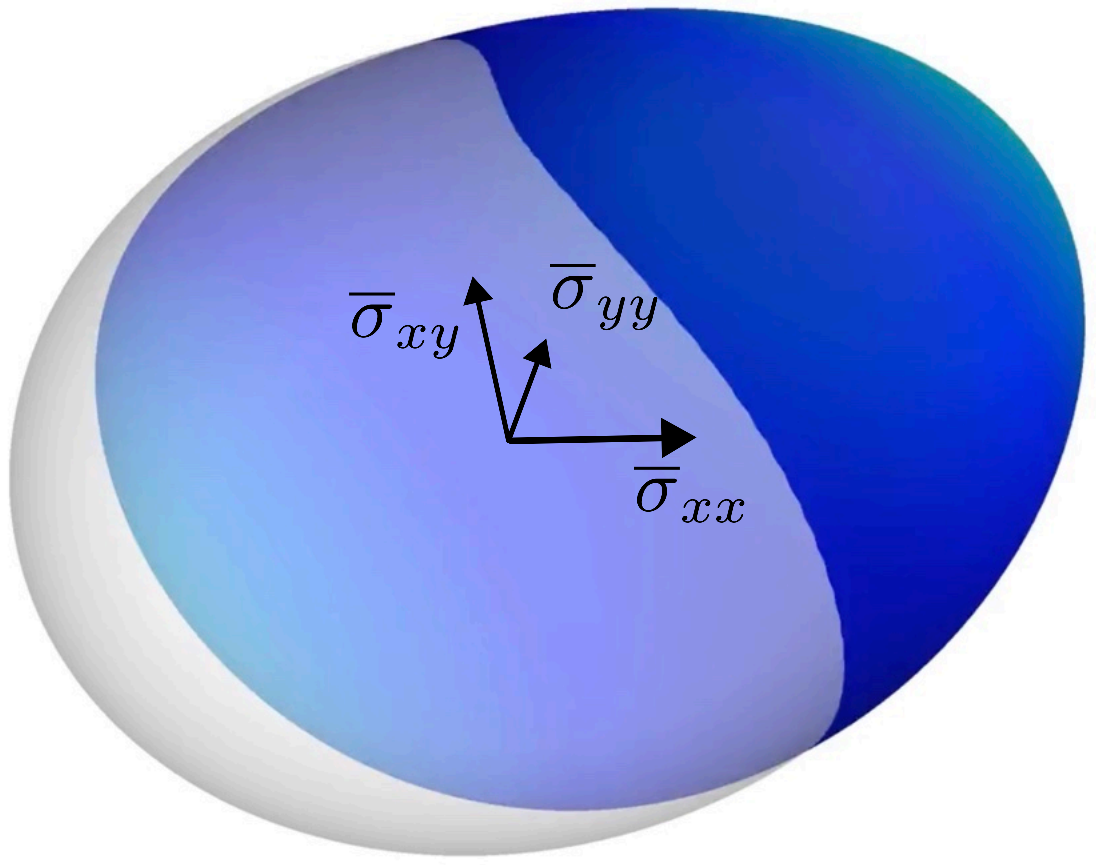
fourth-order elasticity tensor



DESIGN PROBLEM

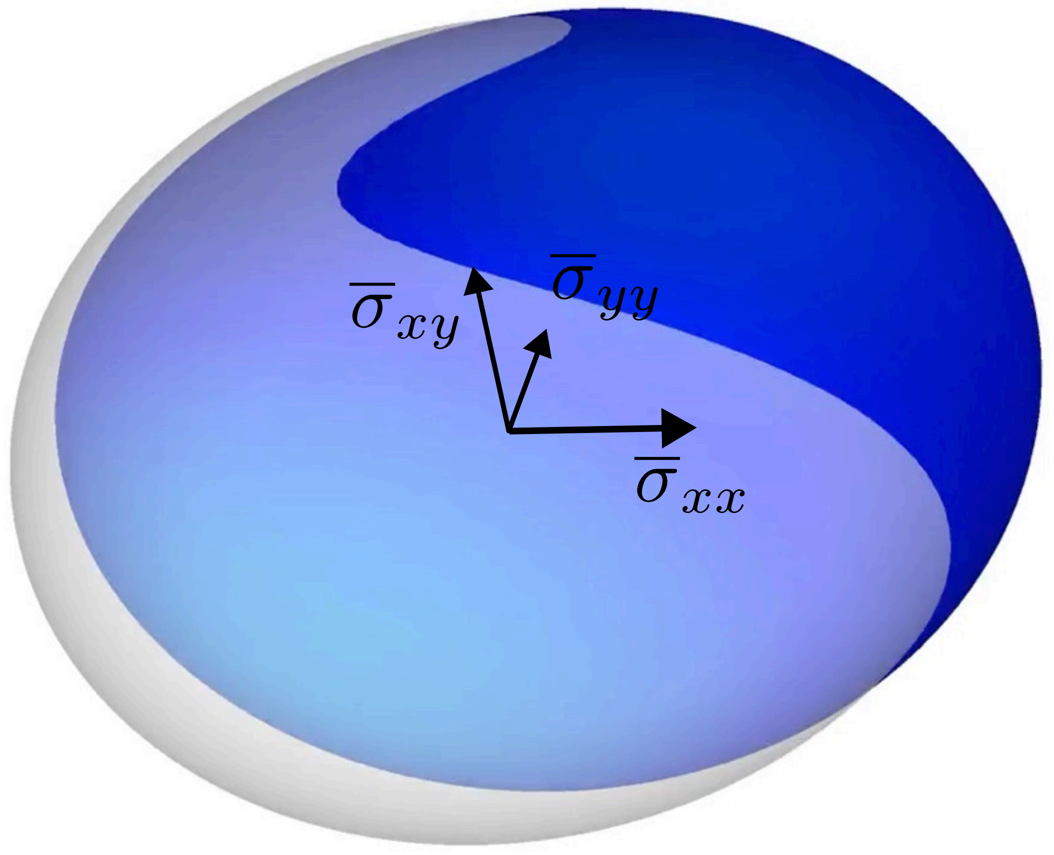
$$\overline{\psi} \longrightarrow \overline{\psi}_{tgt}$$

Change geometry Ω



DESIGN PROBLEM

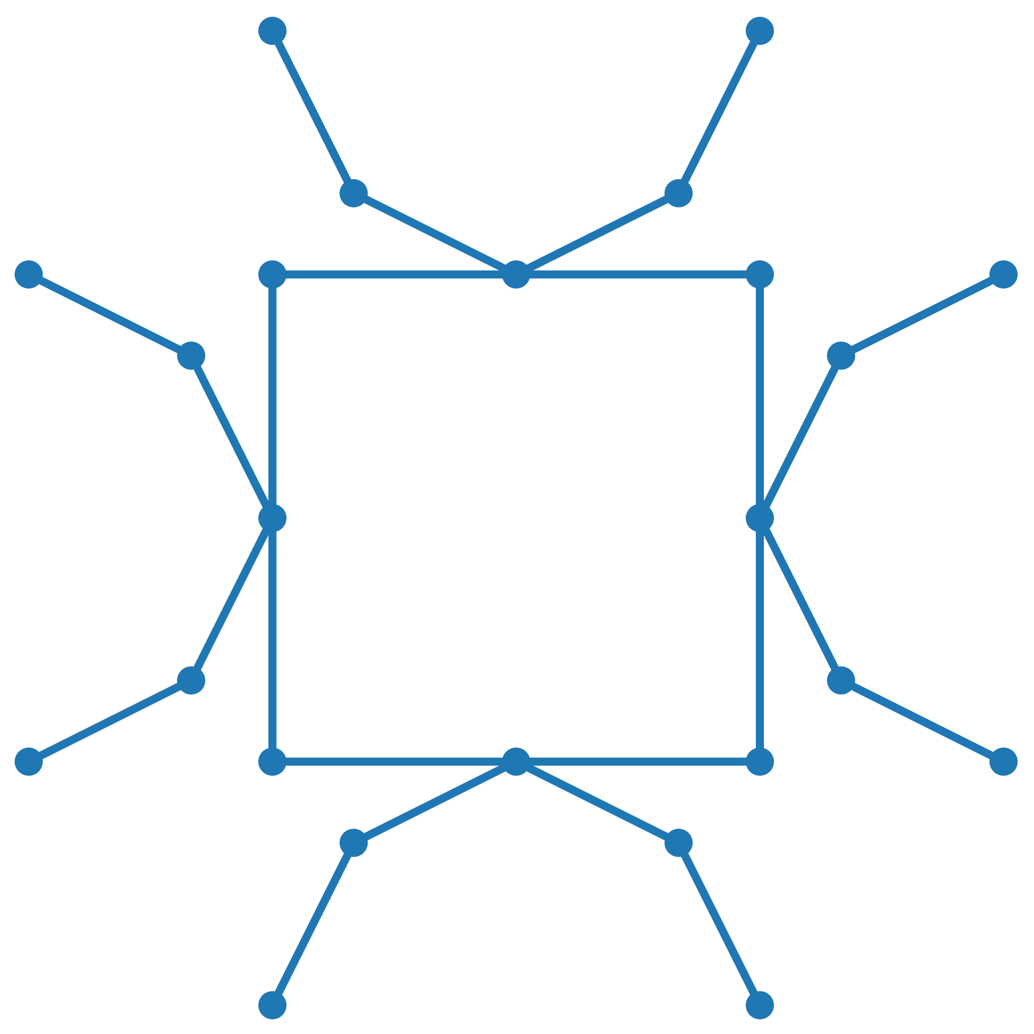
$$\min_{\Omega} \frac{1}{|\mathscr{F}|}\int_{\mathscr{F}} \mathbf{w}_e \frac{\left(\overline{\psi} - \overline{\psi}_{tgt}\right)^2}{\overline{\psi}_{tgt}^2} + \mathbf{w}_\sigma \frac{\left\| \overline{\psi}' - \overline{\psi}'_{tgt} \right\|^2}{\left\| \overline{\psi}'_{tgt} \right\|^2} d\overline{F}$$



DESIGN PROBLEM

Shape Derivative

$$\min_{\Omega} \frac{1}{|\mathscr{F}|} \int_{\mathscr{F}} \mathbf{w}_e \left(\overline{\psi} - \overline{\psi}_{tgt}\right)^2_{rel} + \mathbf{w}_\sigma \left\| \overline{\psi}' - \overline{\psi}'_{tgt} \right\|^2_{rel} d\overline{F}$$



Topology

$\Omega(p)$

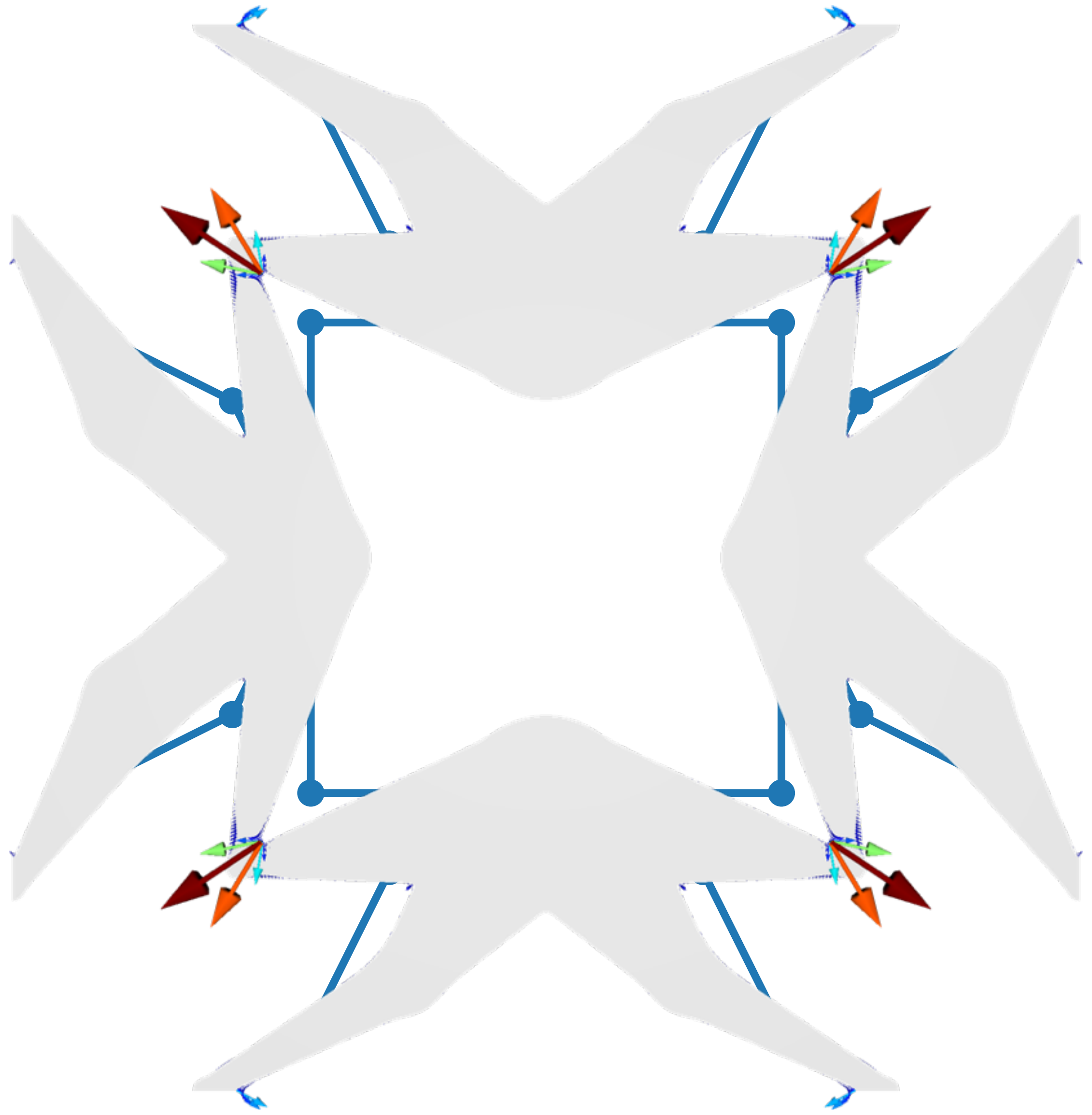
$$p \in \mathbb{R}^{20\sim25}$$

DESIGN PROBLEM

Shape Derivative

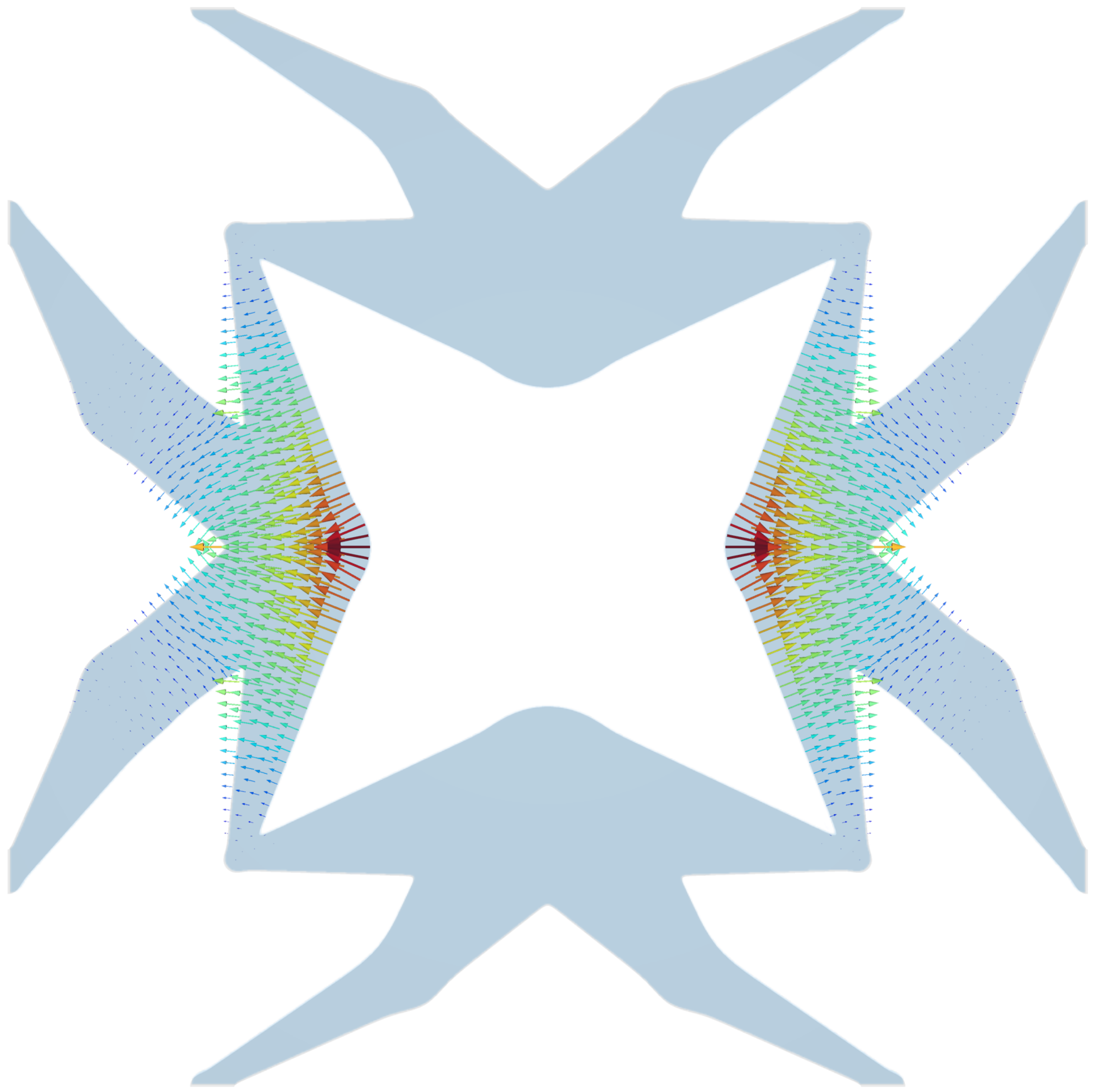
$$\min_{\Omega} \frac{1}{|\mathscr{F}|} \int_{\mathscr{F}} \mathbf{w}_e \left(\overline{\psi} - \overline{\psi}_{tgt}\right)^2_{rel} + \mathbf{w}_\sigma \left\| \overline{\psi}' - \overline{\psi}'_{tgt} \right\|^2_{rel} \, d\overline{F}$$

Derivative of fitting objective
with respect to mesh node positions



$\frac{\partial \overline{\psi}(\overline{F})}{\partial \Omega}$ Topology

Derivative of mesh node positions
with respect to shape parameter



$\mathbf{v} = \partial \Omega(p) / \partial p$ $\Omega(p)$

Gauss–Newton algorithm

analytical gradient

$$\left\langle \frac{\partial \overline{\psi}(\overline{F})}{\partial \mathbf{X}}, \mathbf{v} \right\rangle = \int_{\Omega} G_{\overline{\psi}} : \nabla \mathbf{v} \, d\mathbf{X}$$

$$\left\langle \frac{\partial \overline{\sigma}_{ij}(\overline{F})}{\partial \mathbf{X}}, \mathbf{v} \right\rangle = \int_{\Omega} G_{\overline{\sigma}_{ij}} : \nabla \mathbf{v} \, d\mathbf{X}$$

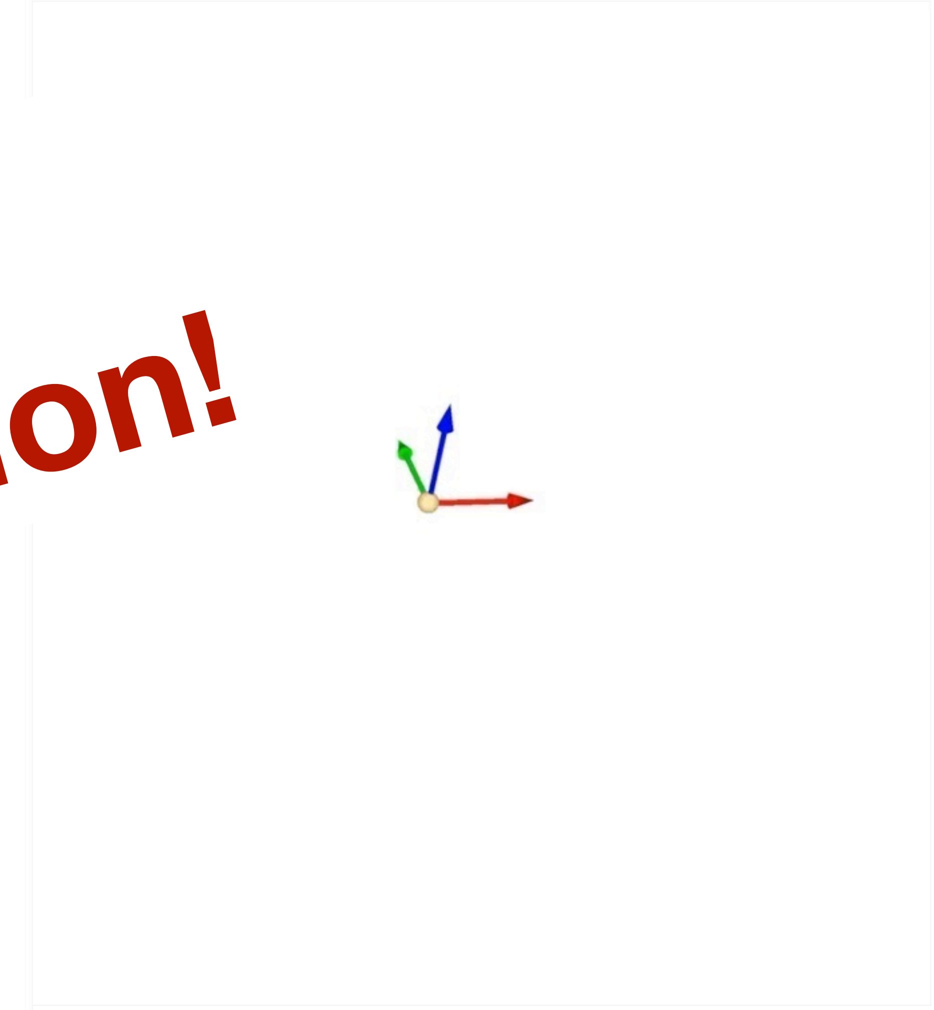
DESIGN PROBLEM

Collision Removal

$$\min_{\Omega} \frac{1}{|\mathcal{F}|} \int_{\mathcal{F}} \mathbf{w}_e \left(\overline{\psi} - \overline{\psi}_{tgt} \right)^2_{rel} + \mathbf{w}_\sigma \left\| \overline{\psi}' - \overline{\psi}'_{tgt} \right\|^2_{rel} d\overline{F}$$

Avoid collisions!

No contact simulation!



Strain Domain

Red for high collision area

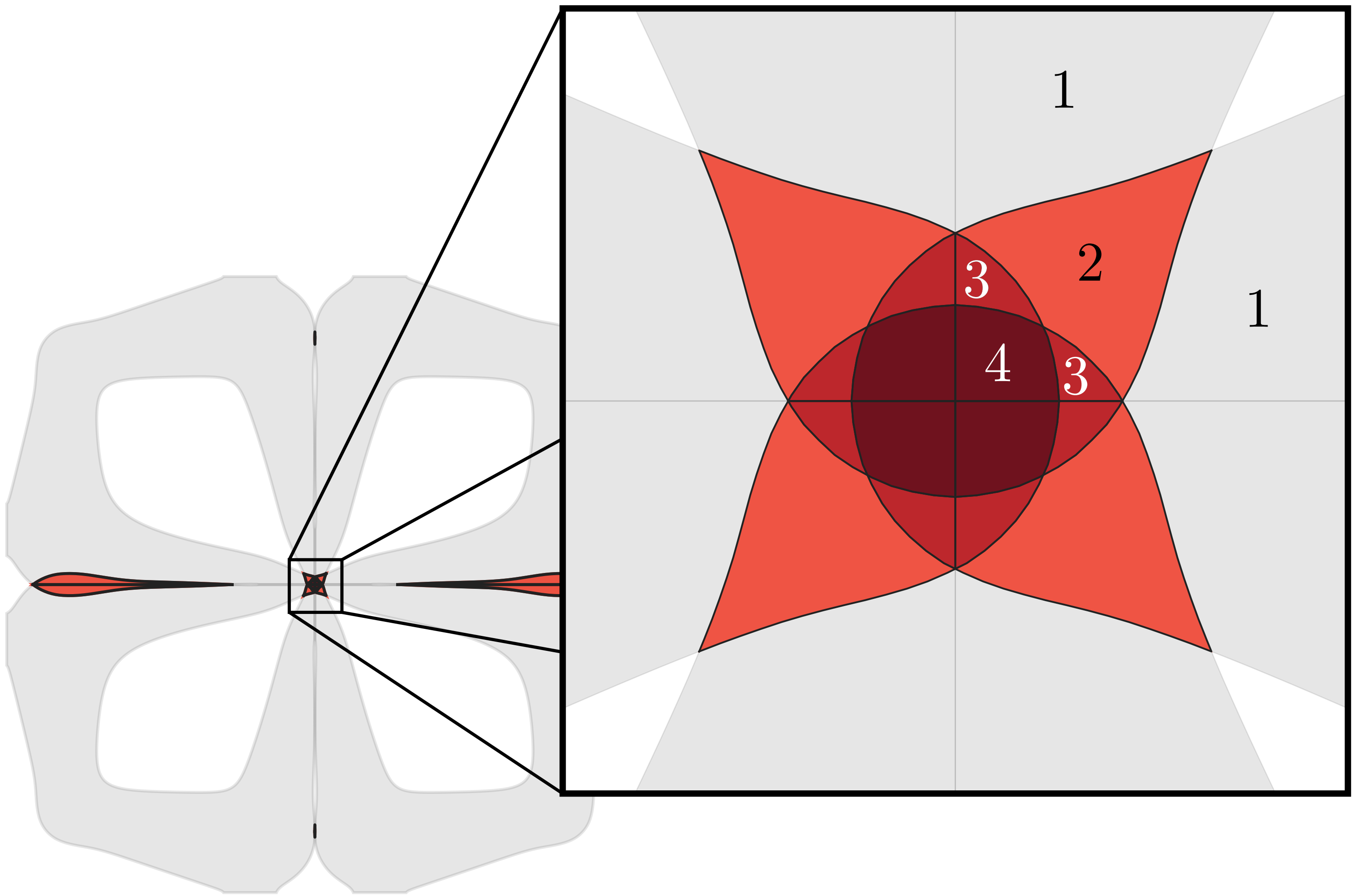
Stress Domain

Blue for low relative error

DESIGN PROBLEM

Collision Removal

$$\min_{\Omega} \frac{1}{|\mathscr{F}|} \int_{\mathscr{F}} \mathbf{w}_e \left(\overline{\psi} - \overline{\psi}_{tgt}\right)^2_{rel} + \mathbf{w}_\sigma \left\| \overline{\psi}' - \overline{\psi}'_{tgt} \right\|^2_{rel} + \mathbf{w_c} \left[\mathbf{A}(\boldsymbol{\omega}^*(\overline{\mathbf{F}}) + \overline{\mathbf{F}}\mathbf{X}; \boldsymbol{\Omega})\right]^{\mathbf{2}} d\overline{F}$$

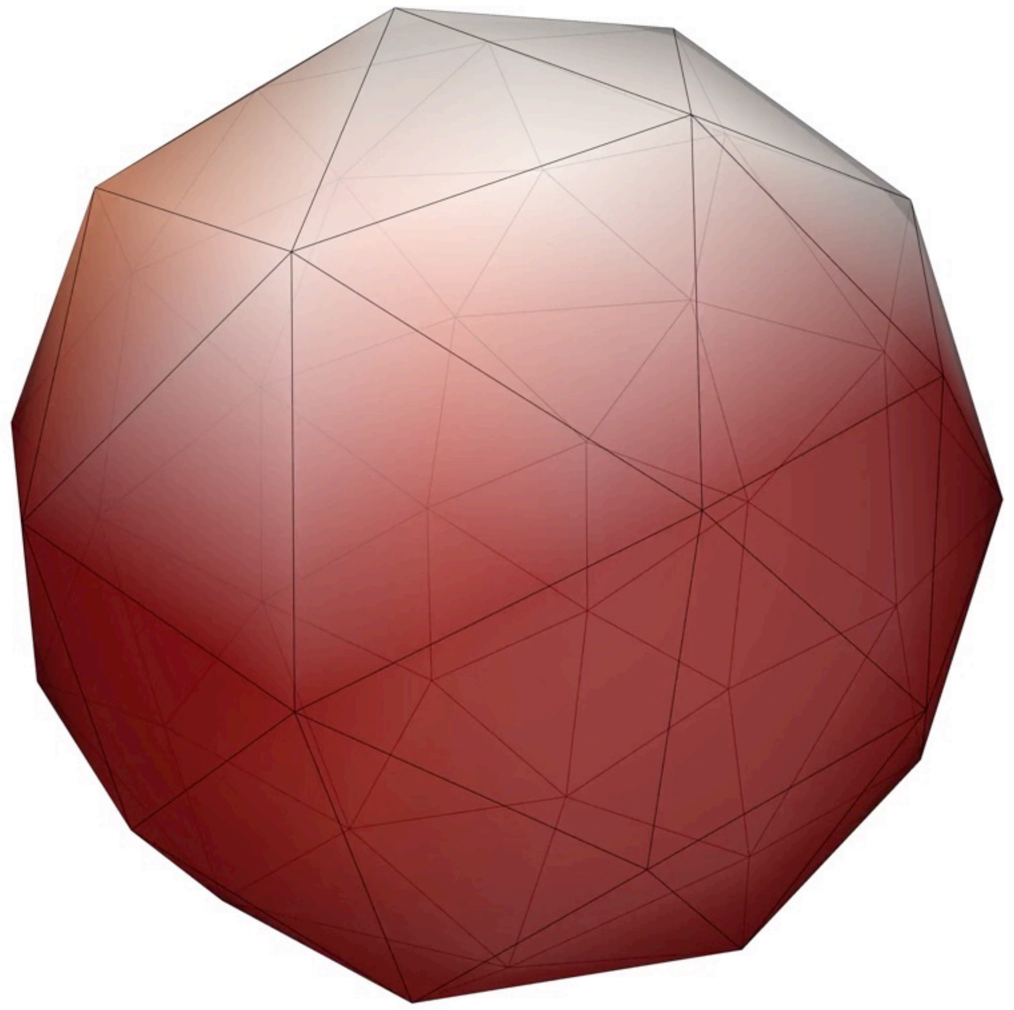


$$A(\Phi;\Omega) = \int_{\Phi(\Omega)} (\text{wind}(\mathbf{x}, \Phi(\partial\Omega)) - 1)_+ \, d\mathbf{x}$$

DESIGN PROBLEM

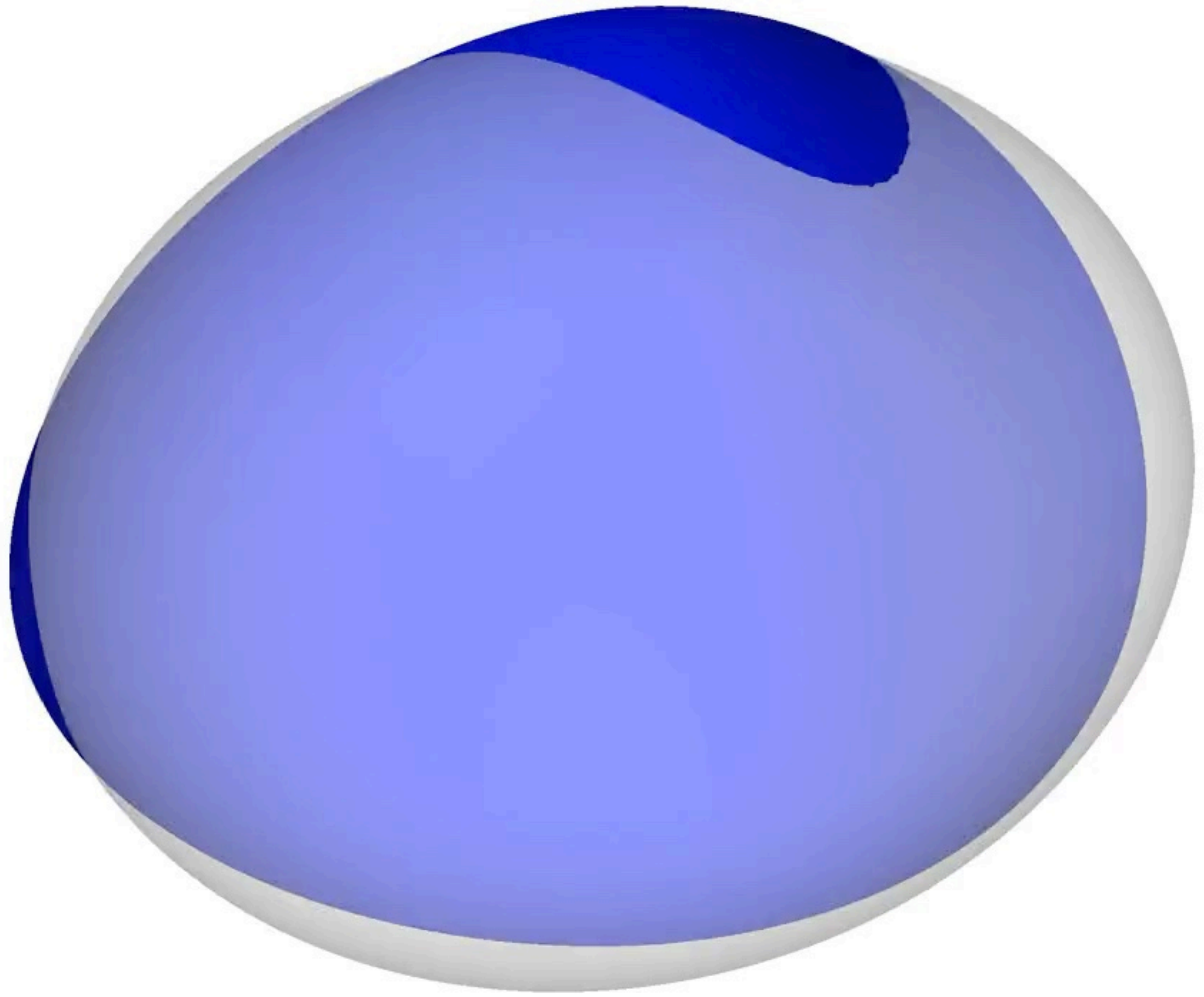
Collision Removal

$$\min_{\Omega} \frac{1}{|\mathcal{F}|}\int_{\mathcal{F}} \mathbf{w}_e \left(\overline{\psi}-\overline{\psi}_{tgt}\right)^2_{rel} + \mathbf{w}_\sigma \left\| \overline{\psi}' - \overline{\psi}'_{tgt}\right\|^2_{rel} + \mathbf{w_c}\left[\mathbf{A}(\omega^*(\overline{\mathbf{F}}) + \overline{\mathbf{F}}\mathbf{X};\Omega)\right]^2 d\overline{F}$$



Discrete Strain Domain

Red for high collision area

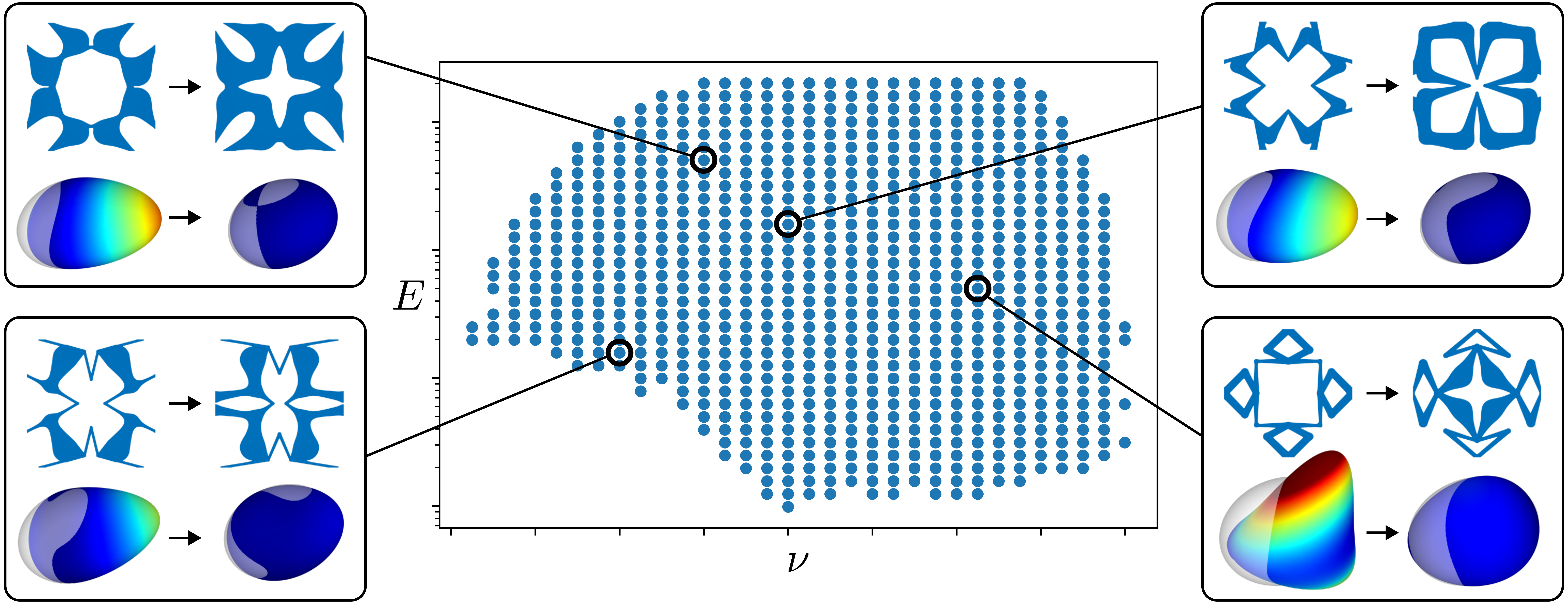


Stress Domain

Blue for low relative error

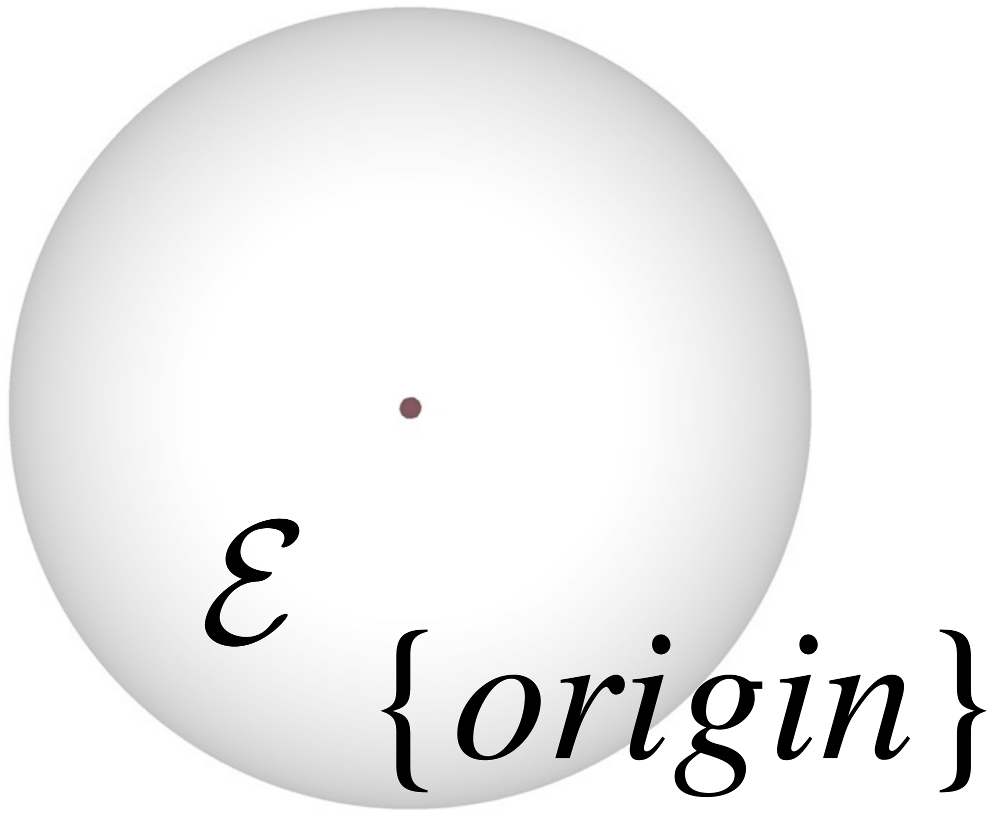
LARGE-SCALE VALIDATION

Isotropic Linear Material Design



LARGE-SCALE VALIDATION

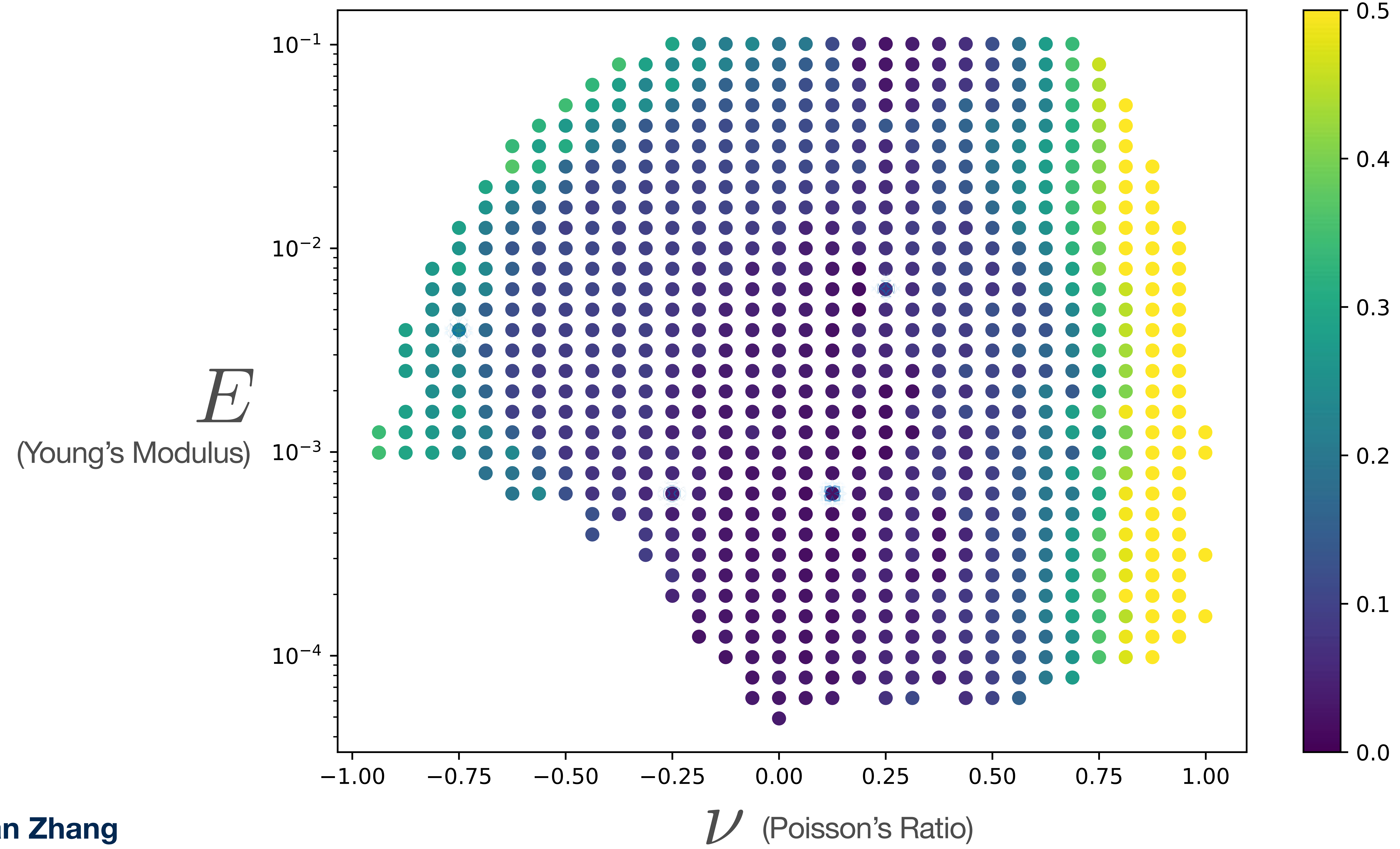
Isotropic Linear Material Design



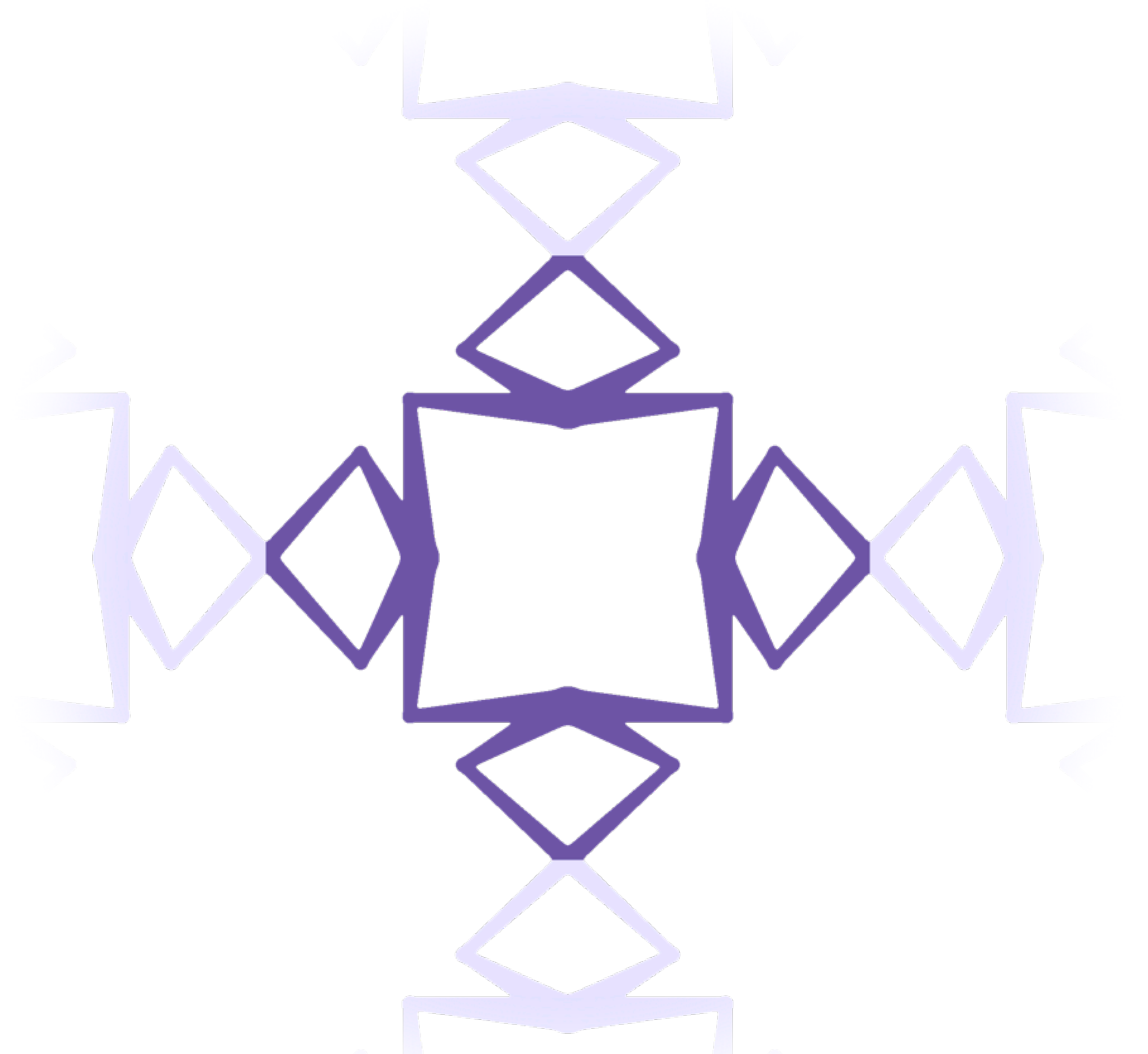
LARGE-SCALE VALIDATION

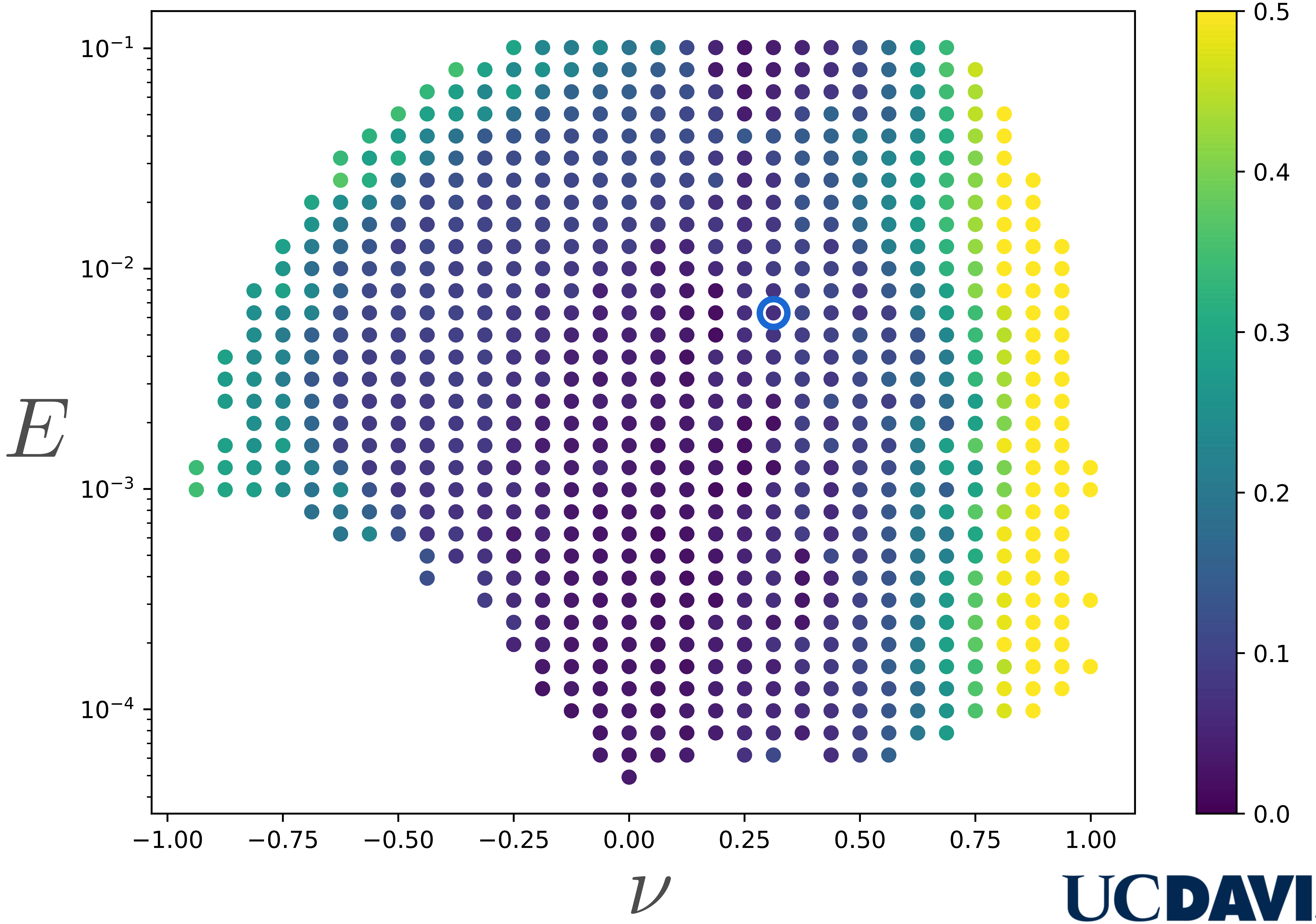
Isotropic Linear Material Design

LARGE-SCALE VALIDATION



RESULTS



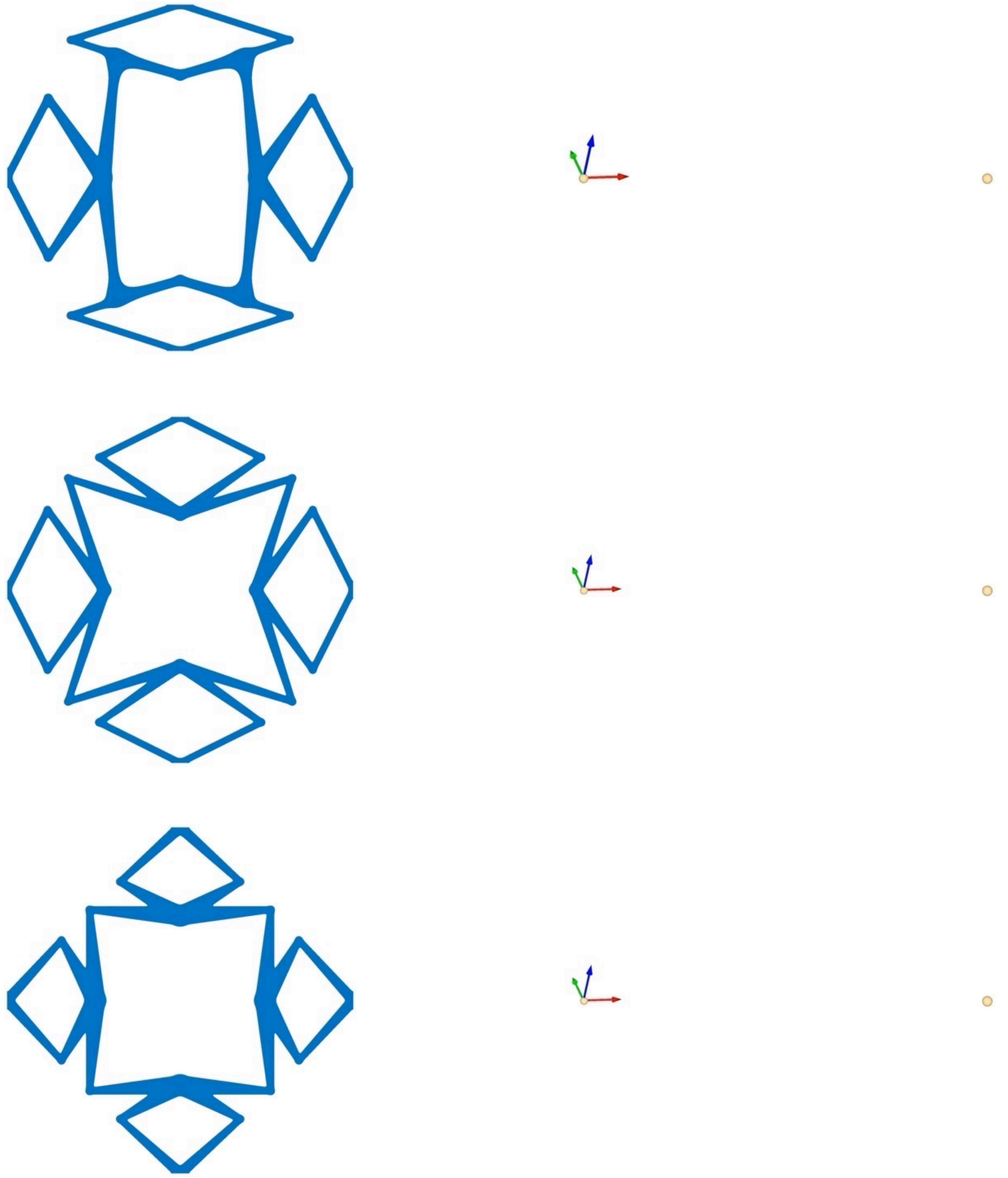


Zhan Zhang

UC DAVIS

RESULTS

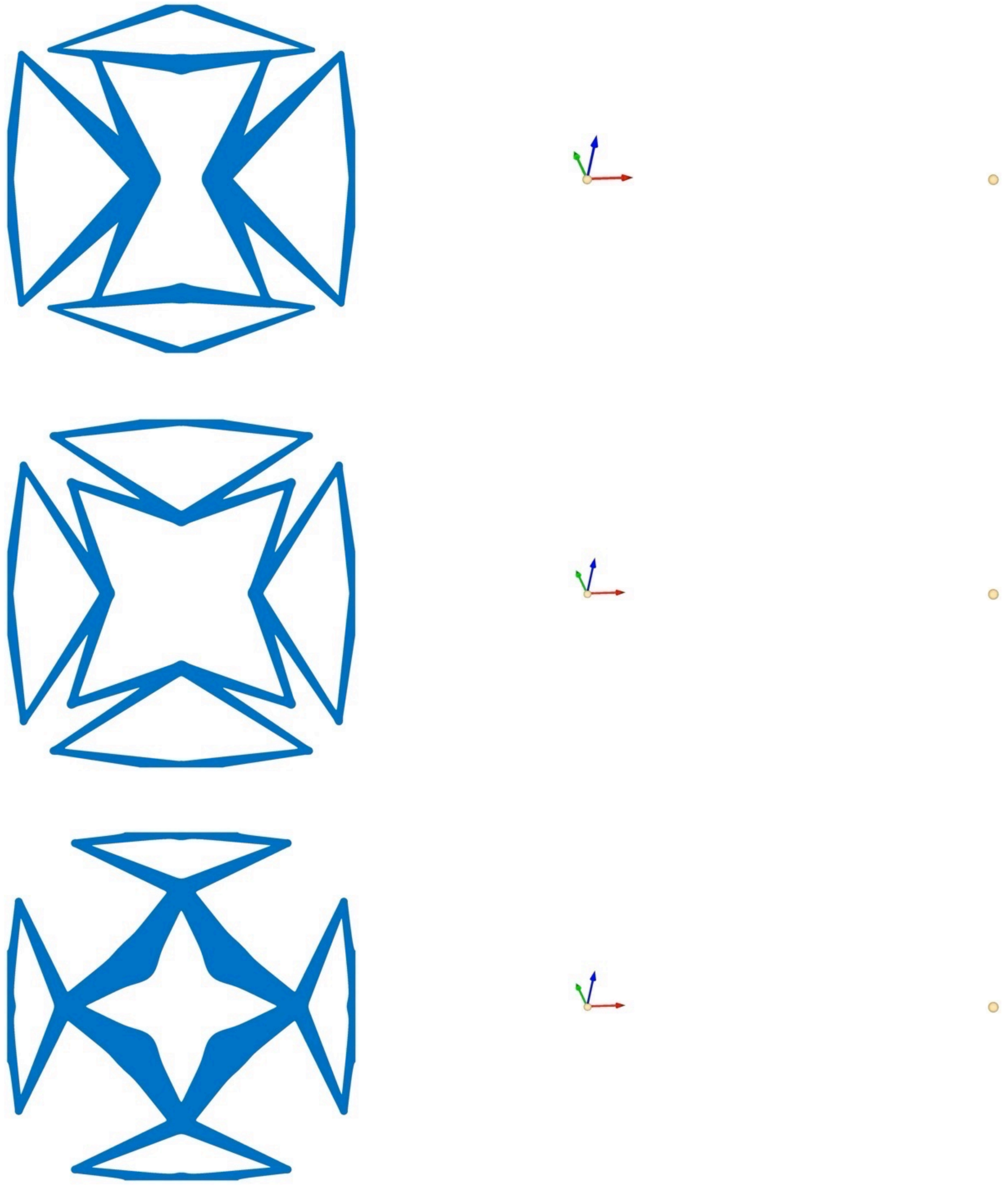
Different Poisson's Ratio



-0.25

0.00

0.25



Old

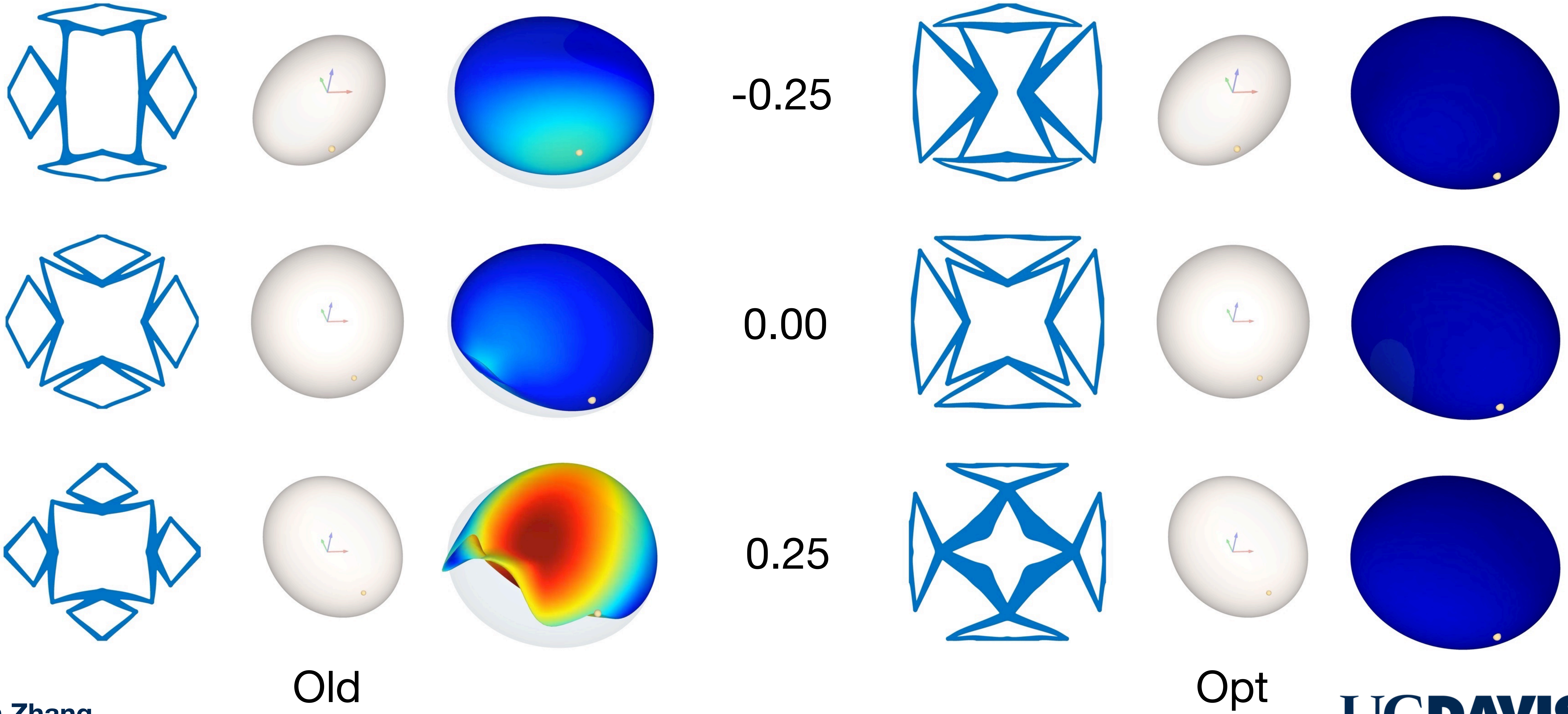
Opt

Zhan Zhang

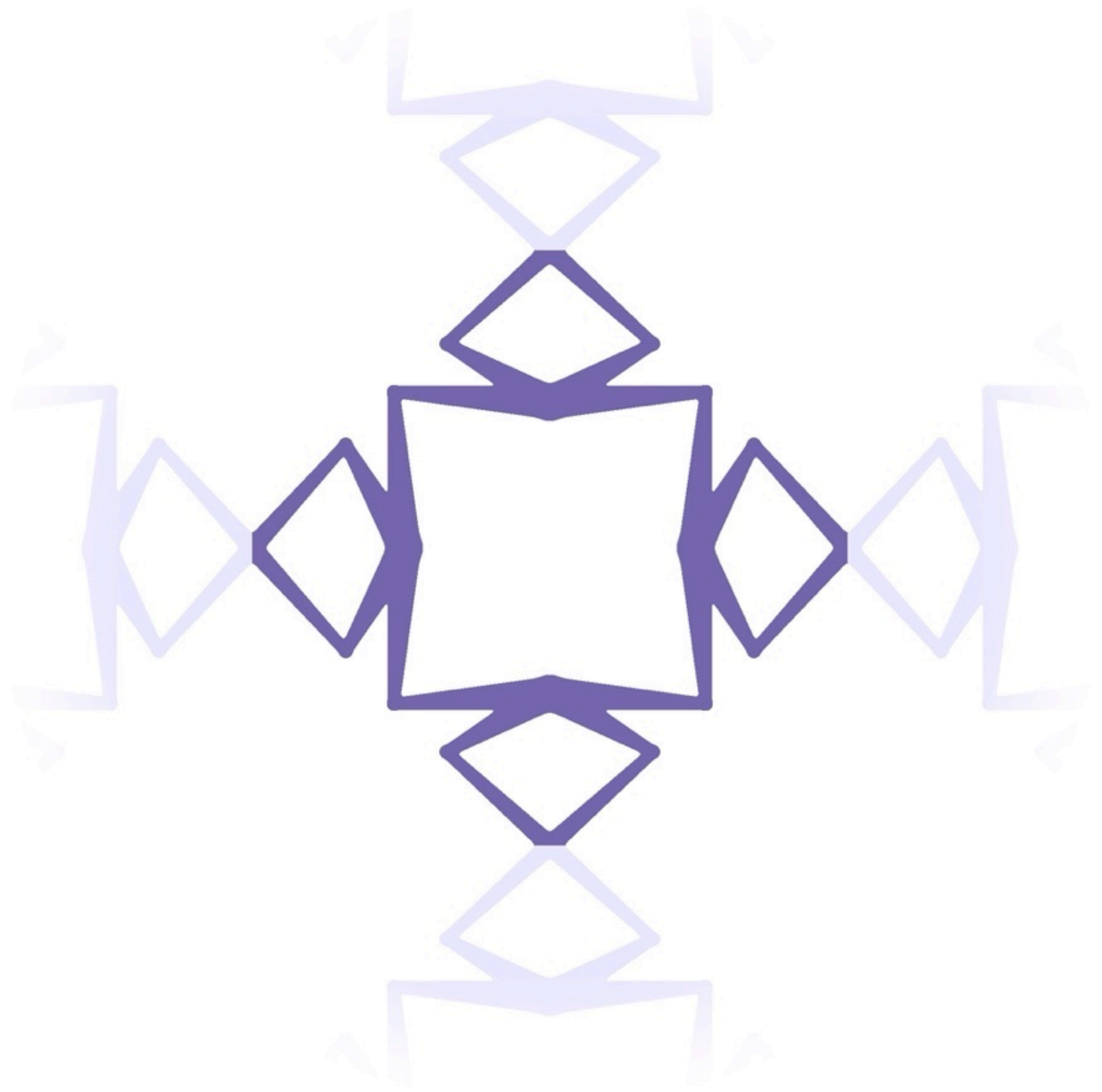
UCDAVIS

RESULTS

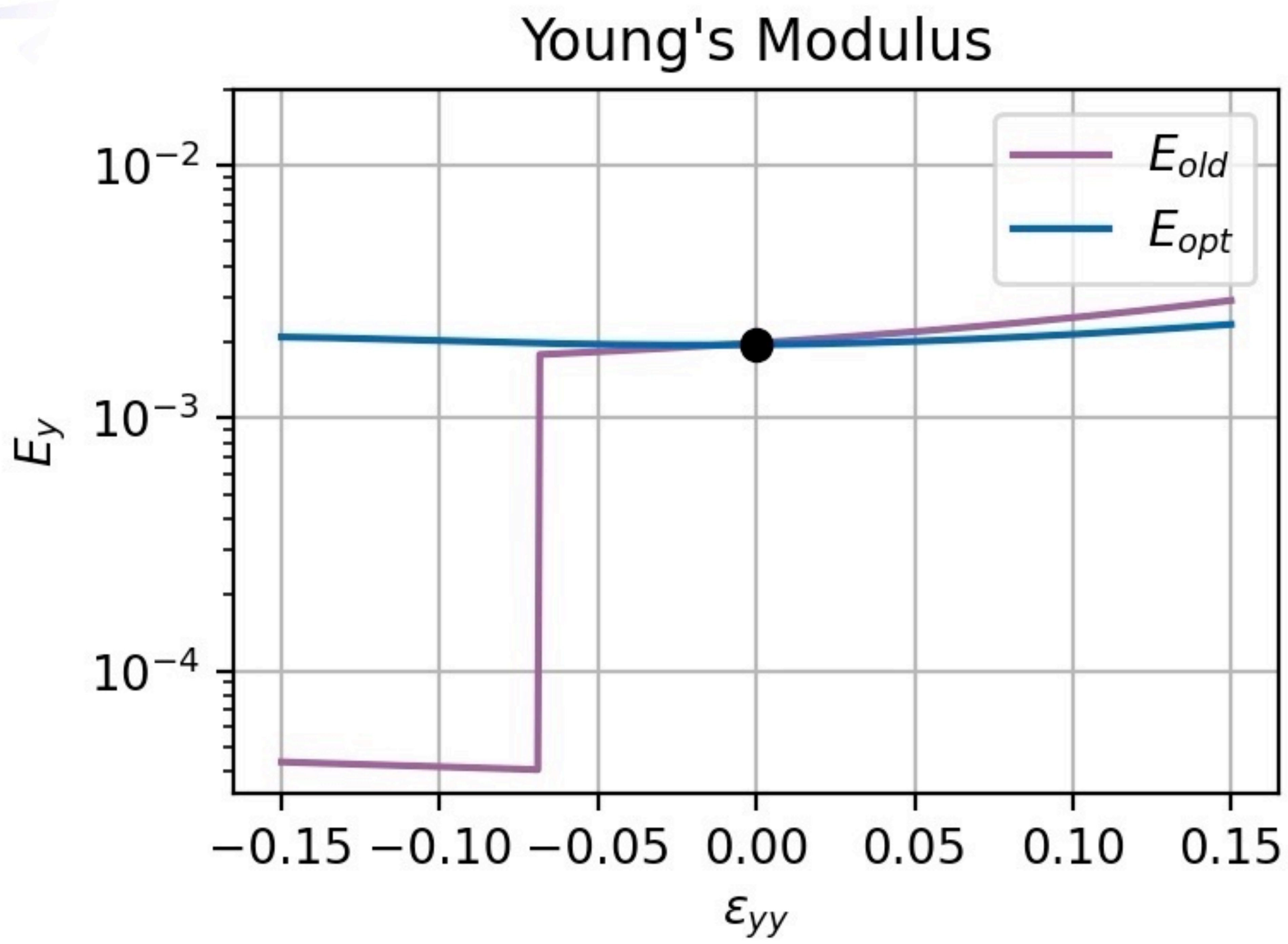
Different Poisson's Ratio

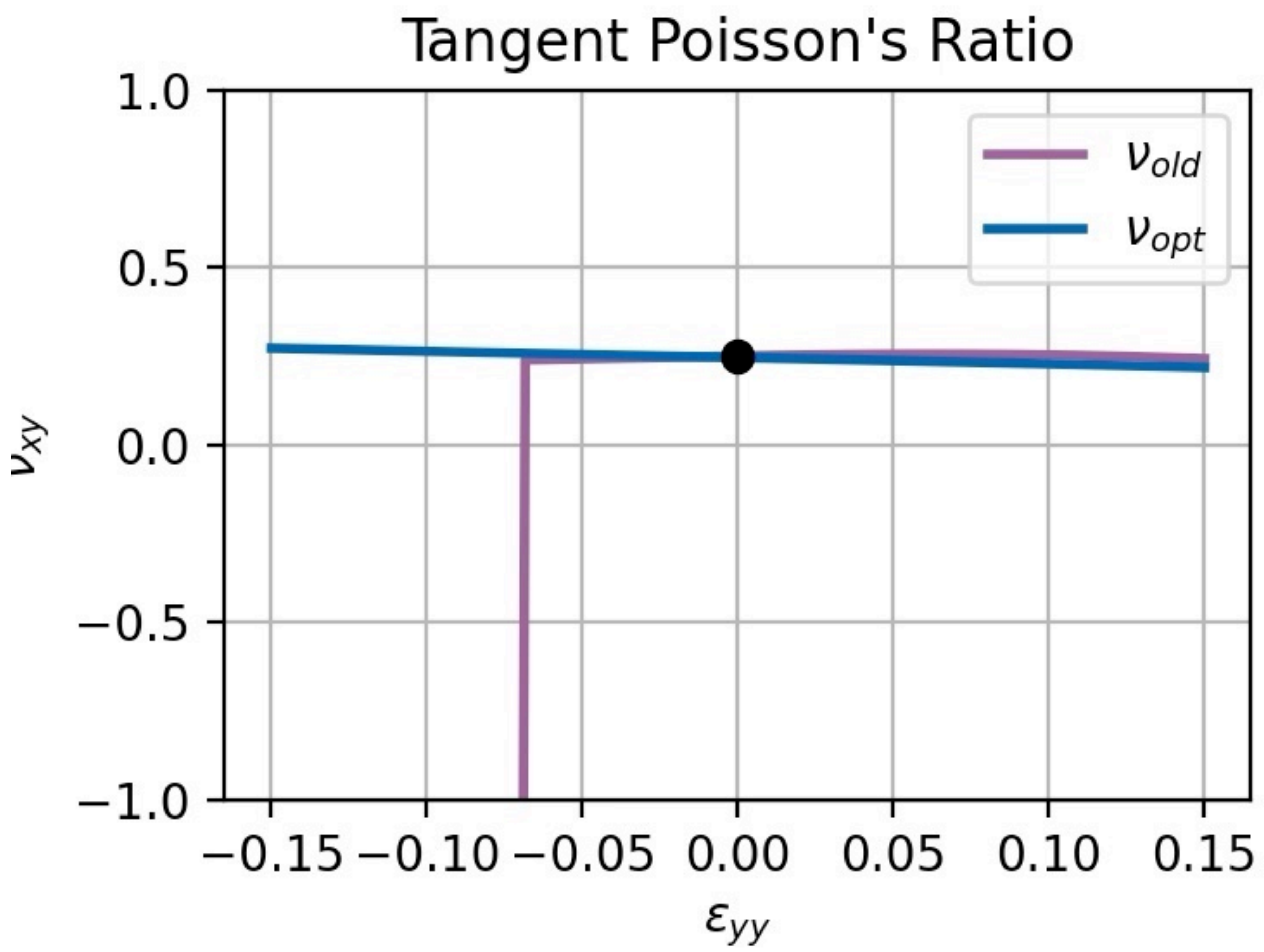


RESULTS



Young’s Modulus & Poisson’s Ratio



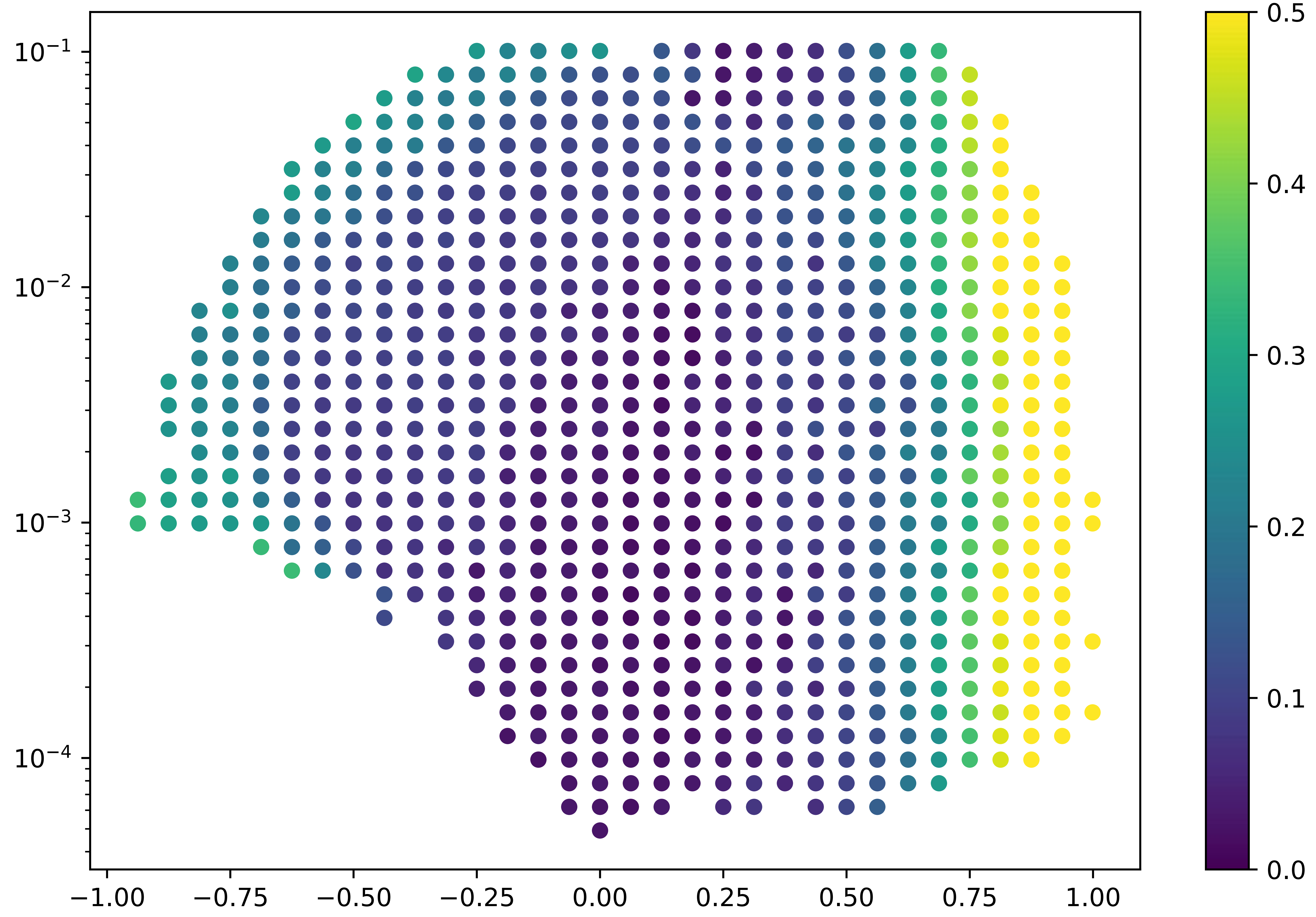


RESULTS

Max Relative Error

$$\text{eval}_{opt} = \max_{\bar{F} \in \mathscr{F}} \left\| \bar{\psi}'(\bar{F}) - \bar{\psi}'_{tgt}(\bar{F}) \right\|_{rel}$$

Ignoring Collisions

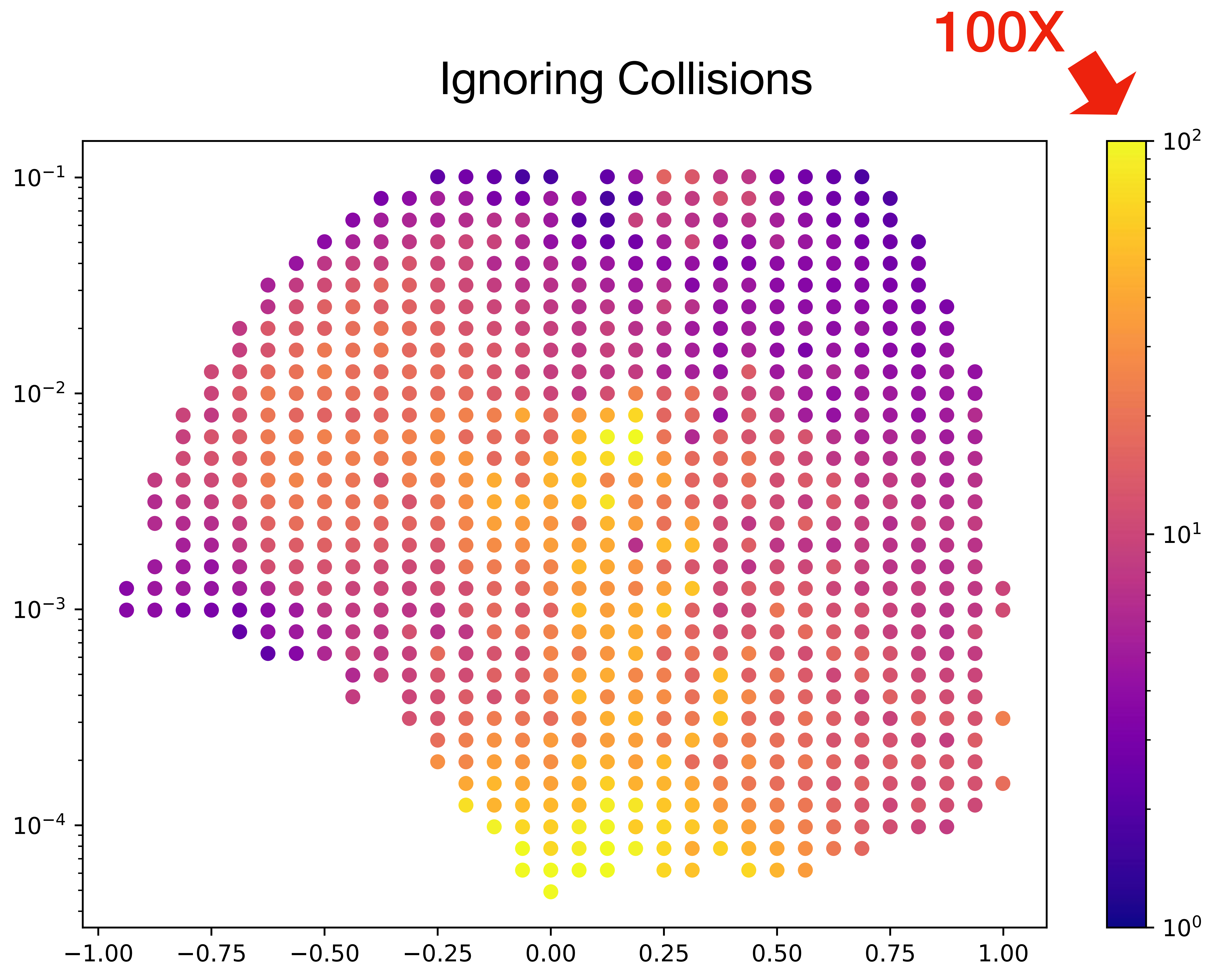


Results for 10% Strain

RESULTS

Improvement

$$\frac{eval_{old}}{eval_{opt}}$$



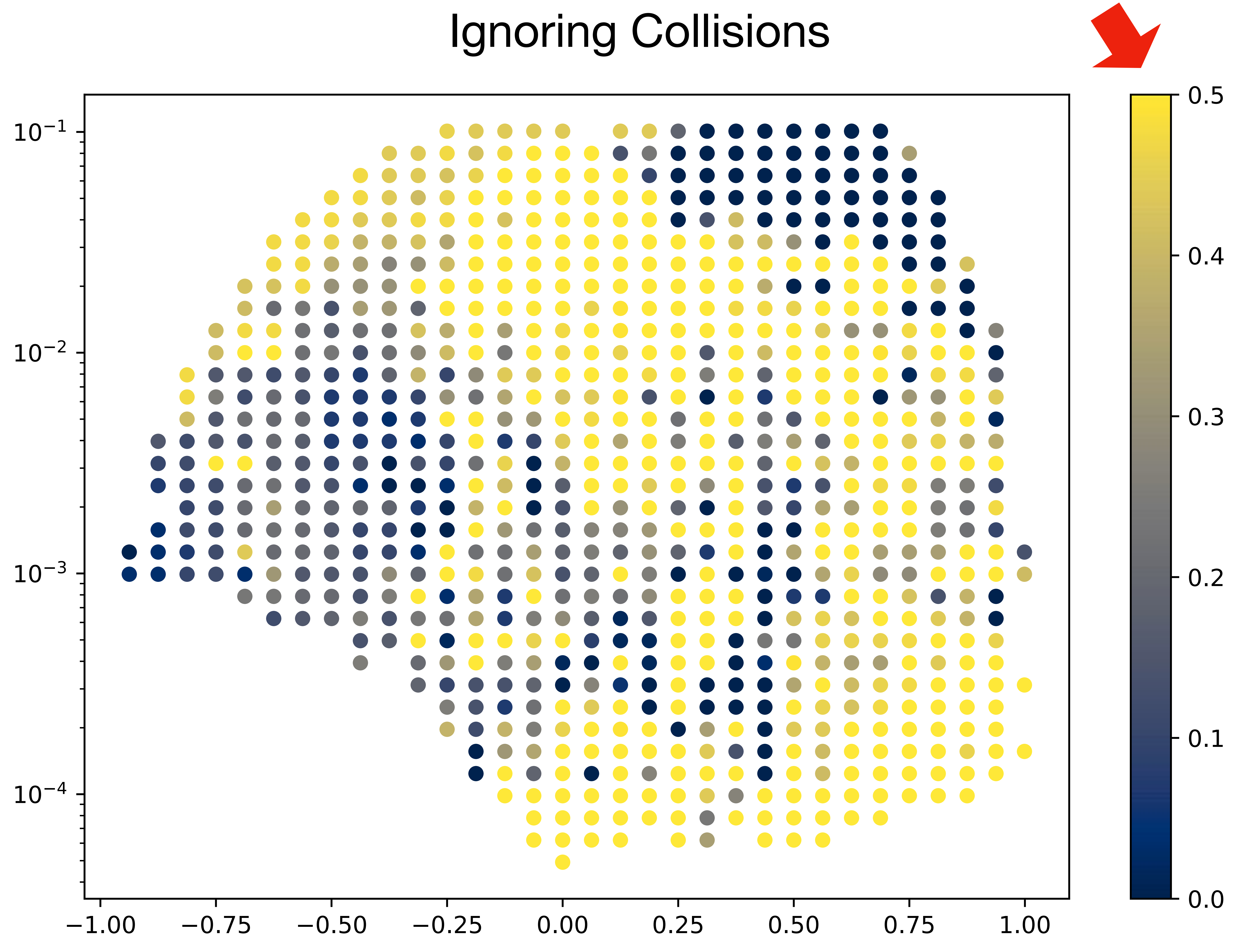
Results for 10% Strain

RESULTS

Collision Ratio

$$\frac{V_{collision}}{V_{domain}}$$

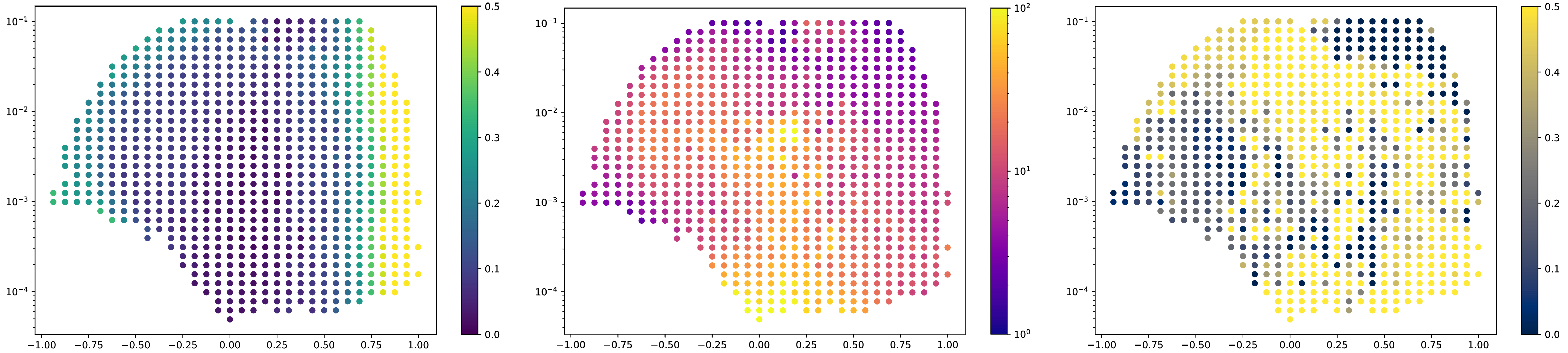
50% of strains have collision



Results for 10% Strain

RESULTS

Ignoring Collisions



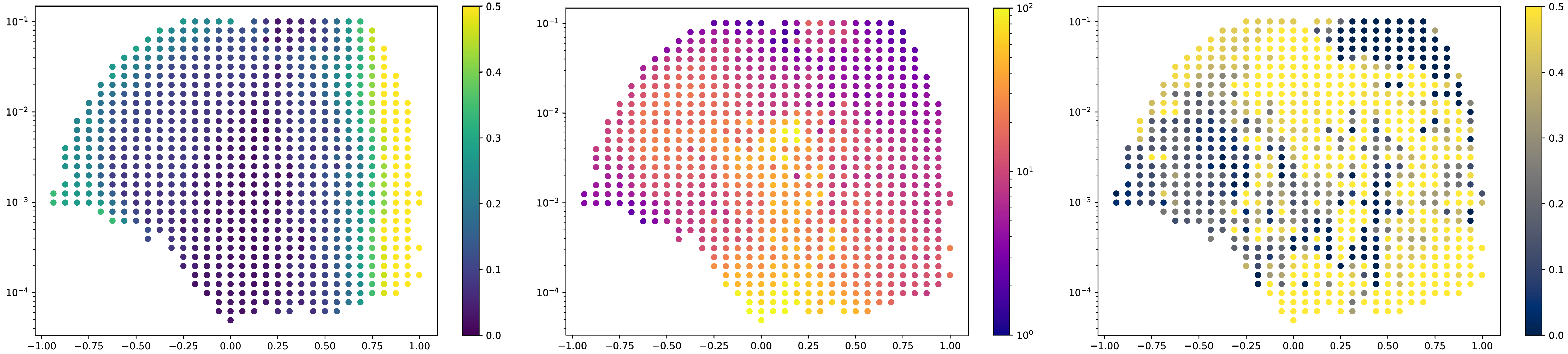
Relative Error

Improvement

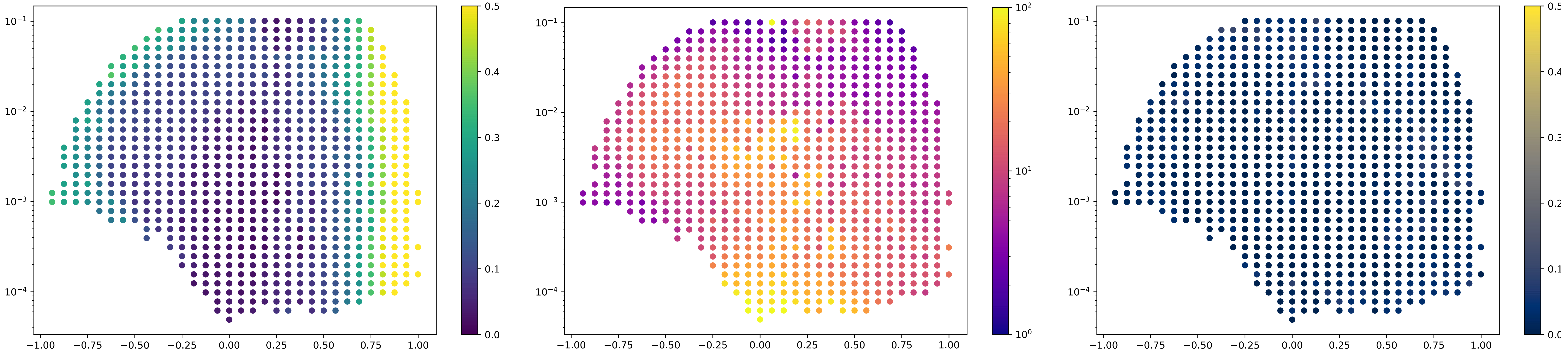
Collision Ratio

RESULTS

Ignoring Collisions



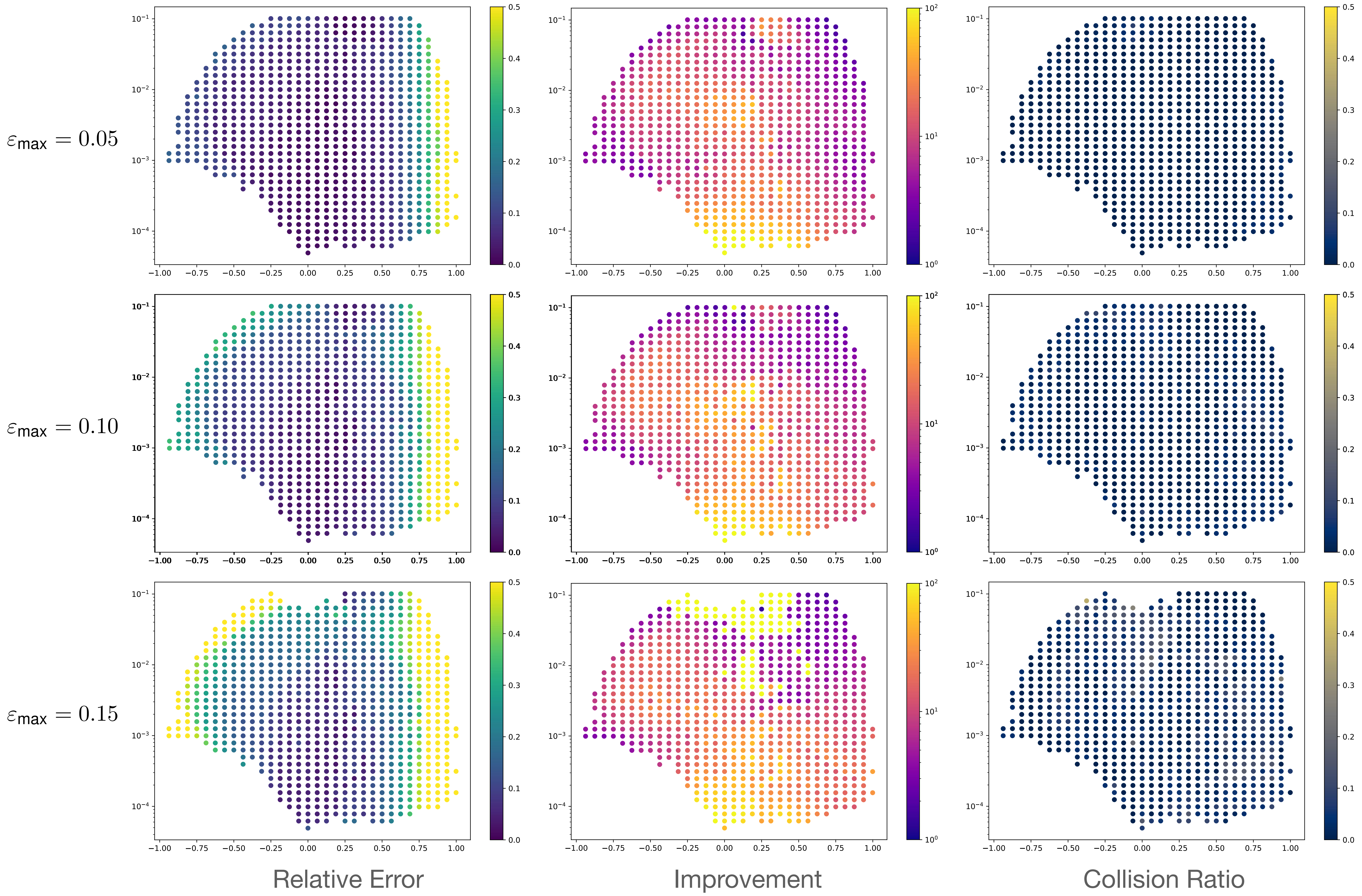
Removing Collisions



Relative Error | Improvement | Collision Ratio

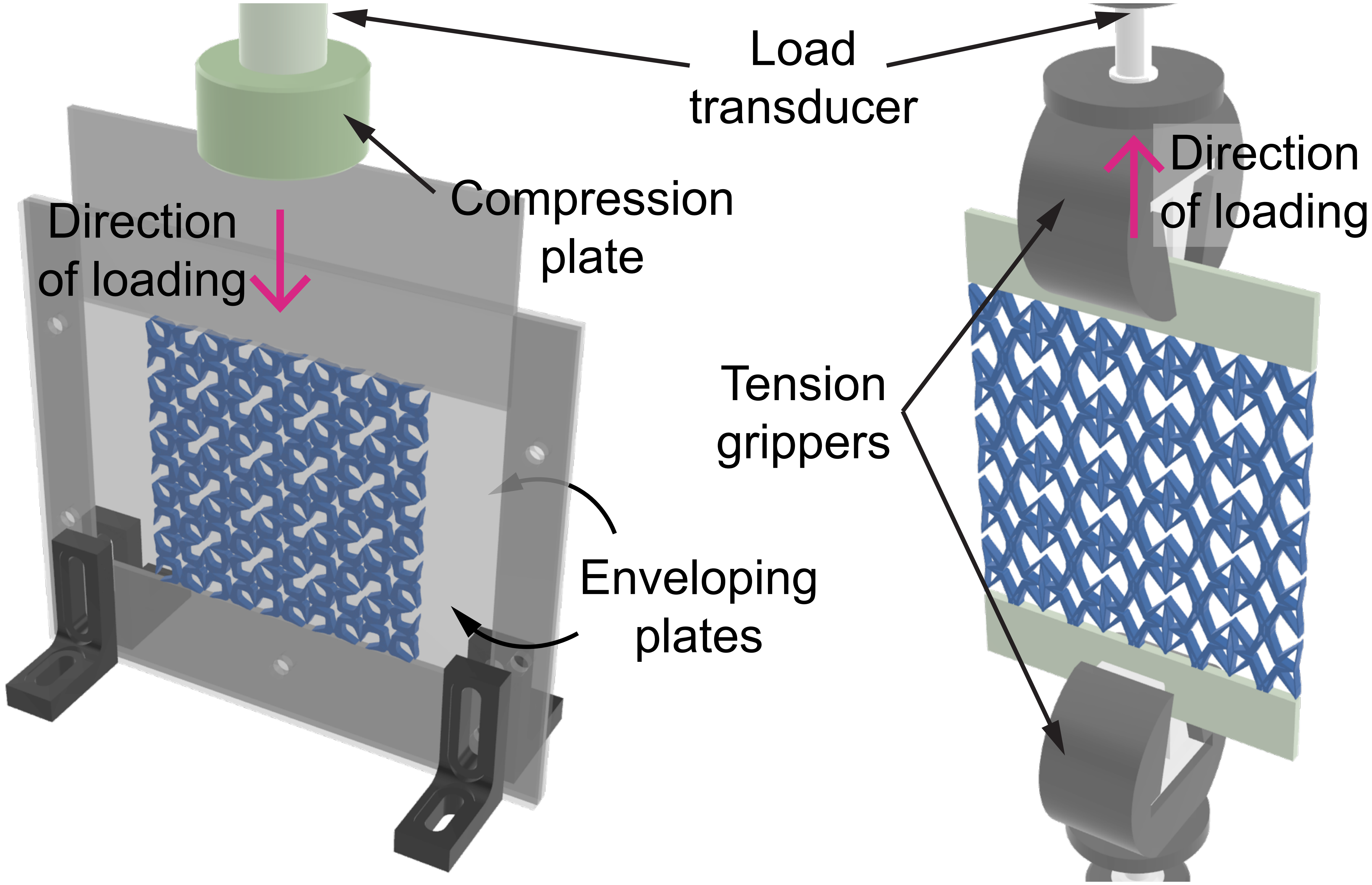
RESULTS

Removing collisions



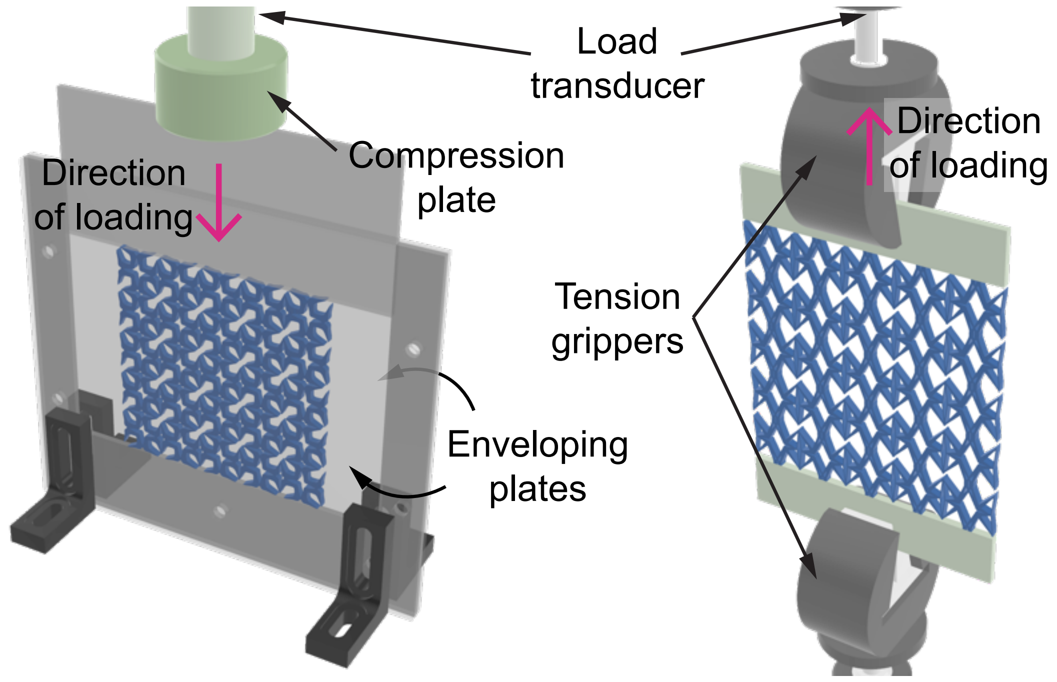
VALIDATIONS

Physical Tests



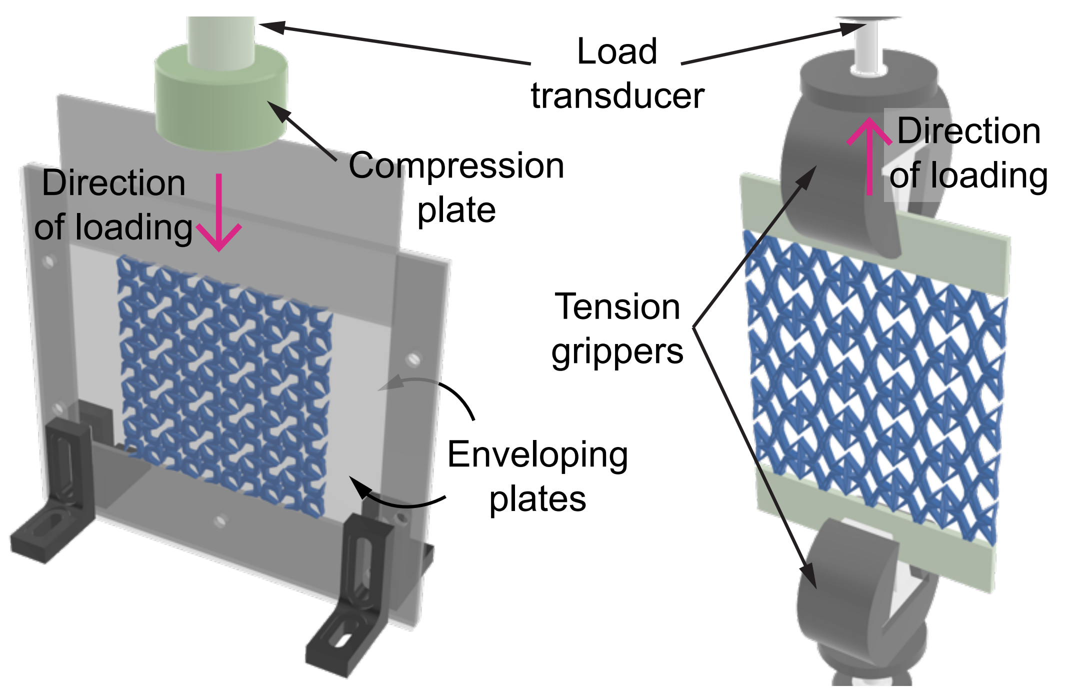
VALIDATIONS

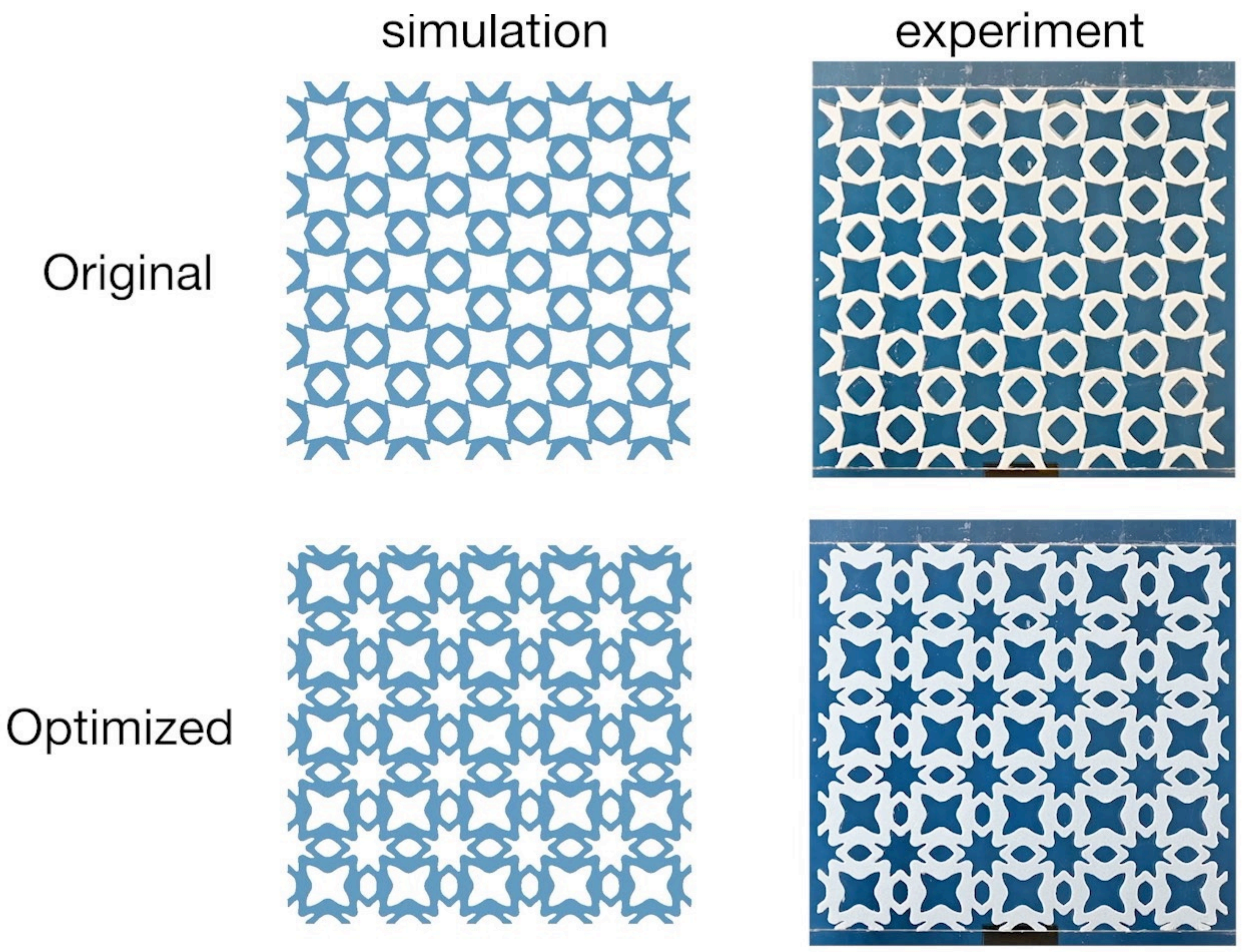
Physical Tests

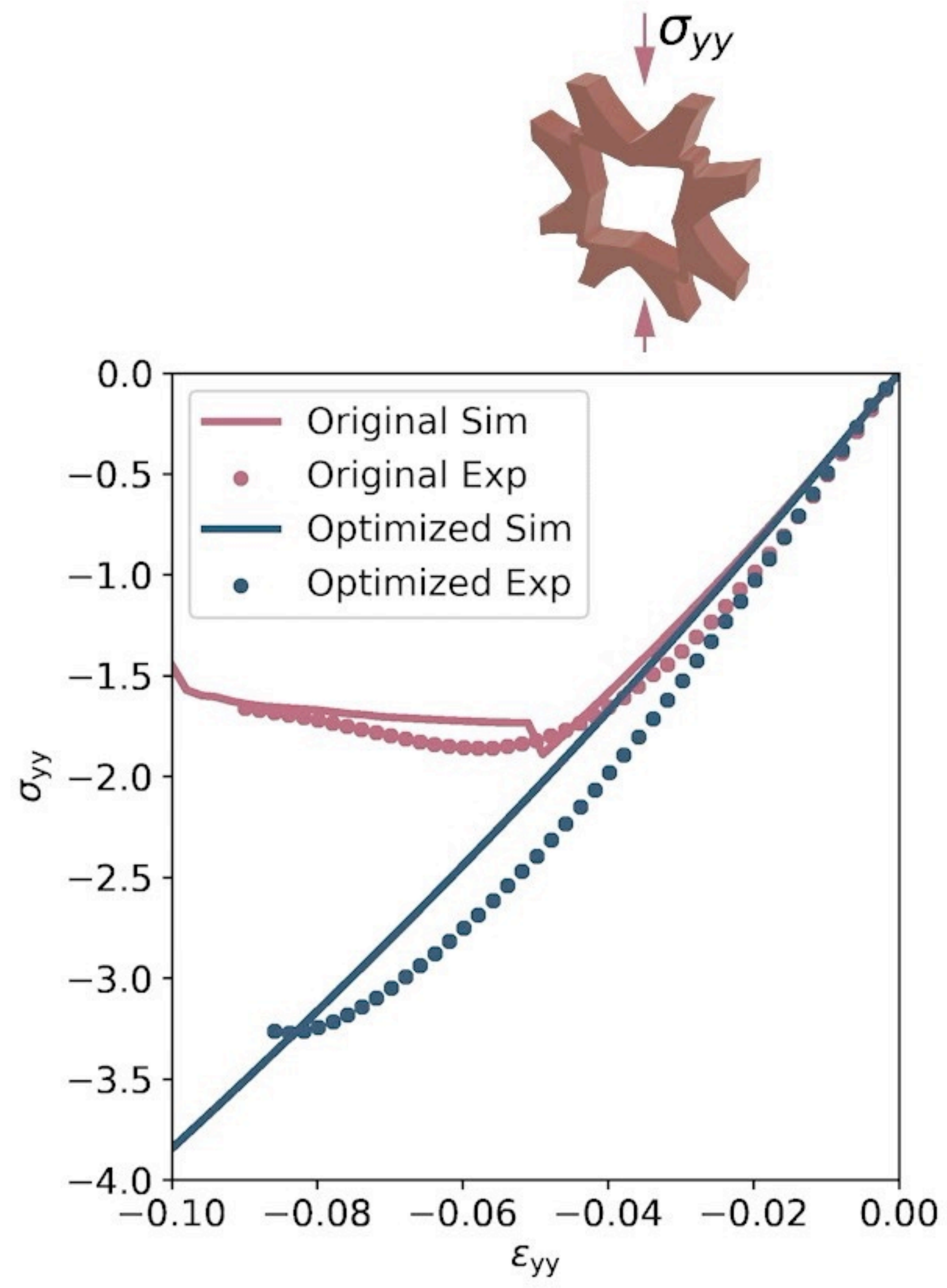


VALIDATIONS

Physical Tests





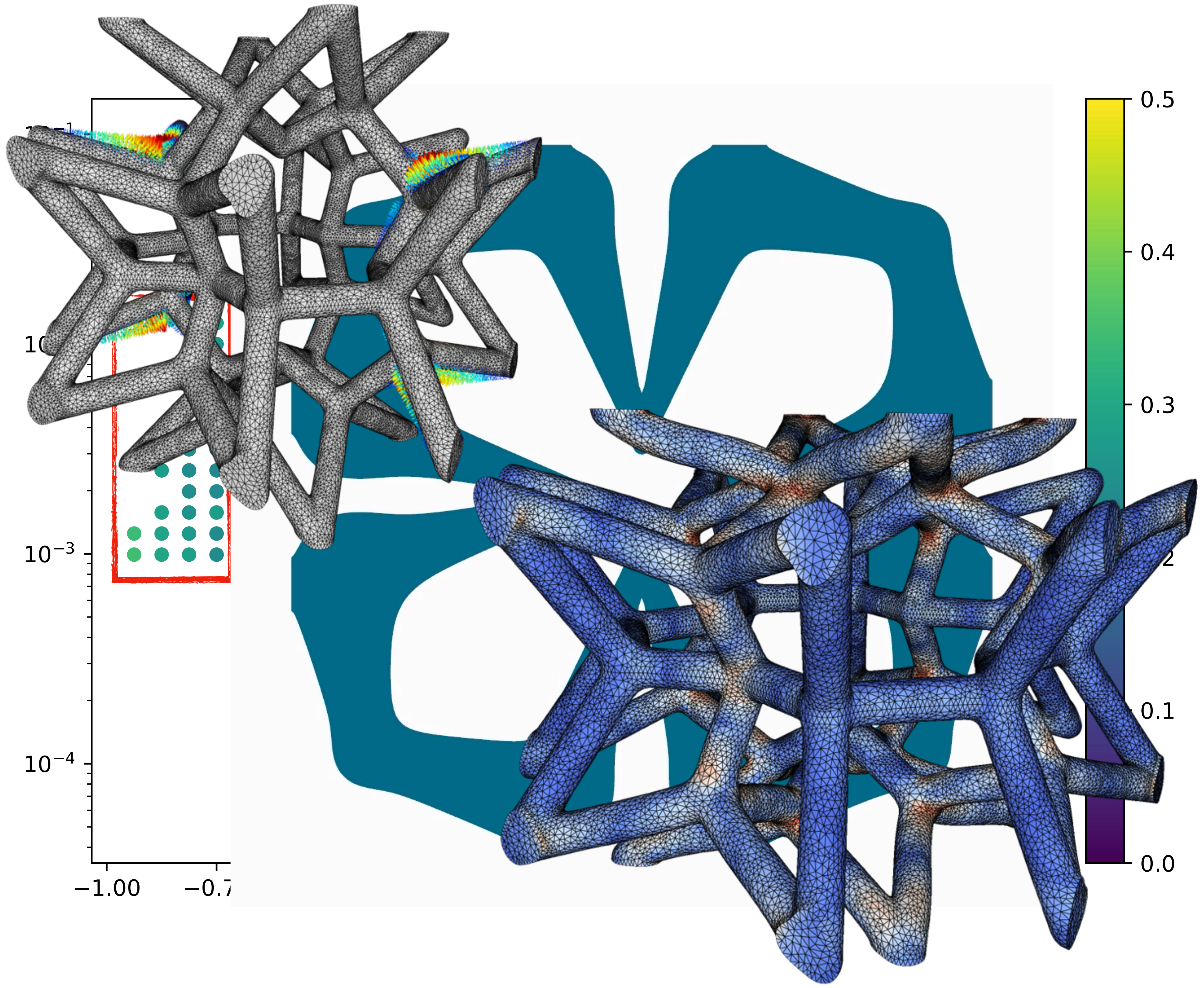


CONCLUSIONS

- An adaptive, data-accelerated nonlinear homogenization (with high-order interpolation)
- A shape design algorithm for nonlinear collision-free planar microstructures

Future Work:

- Better understand the shape parameters space
- Expand the range of achievable material properties
- Homogenize with differentiable contact simulation
- Develop 3D computational design framework



ACKNOWLEDGEMENTS

- Collaborators:

Christopher Brandt[2], Jean Jouve[3], Yue Wang[4], Tian Chen[4], Mark Pauly[5], Julian Panetta[1]

1 University of California, Davis

2 1000shapes GmbH

3 University Grenoble Alpes Inria, CNRS, Grenoble INP, LJK

4 University of Houston, Architected Intelligent Matter Laboratory

5 École Polytechnique Fédérale de Lausanne (EPFL)

Thanks!