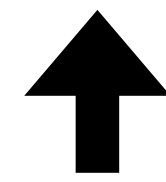
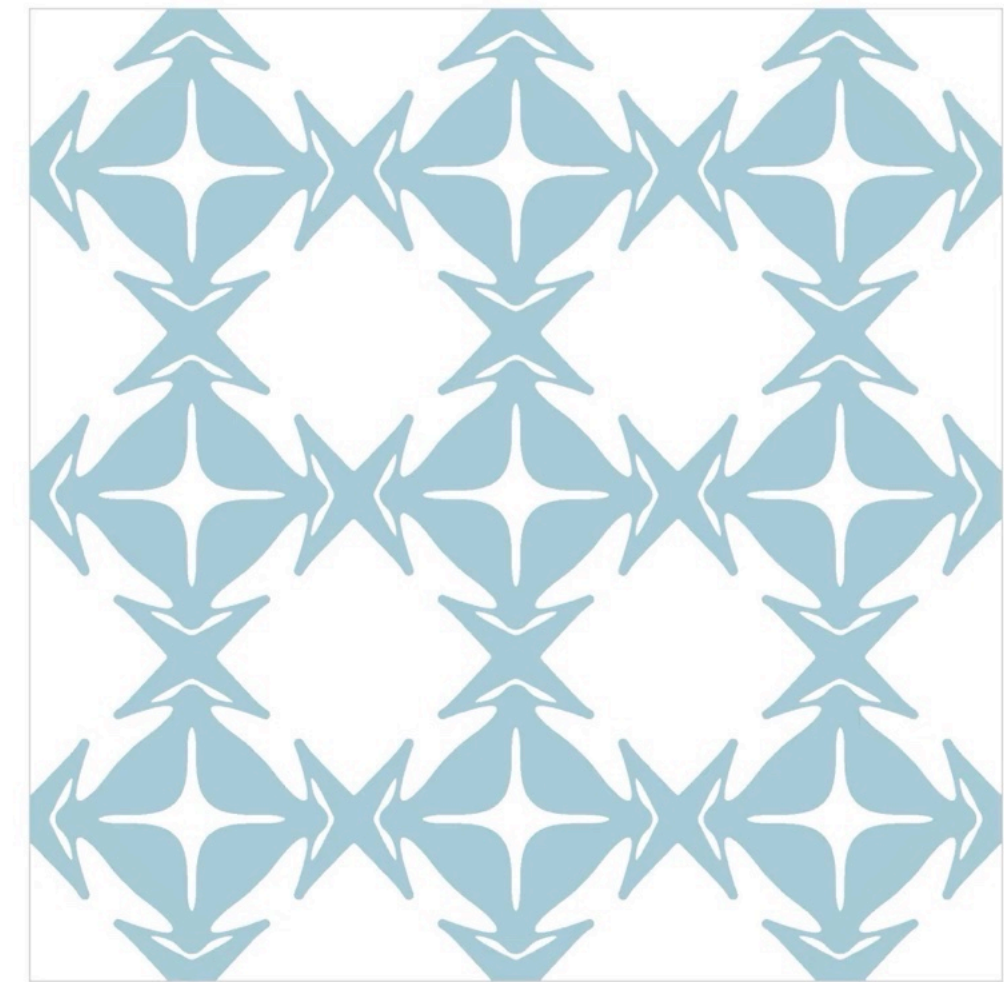
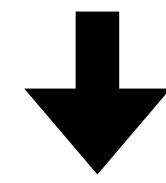
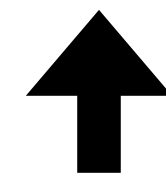
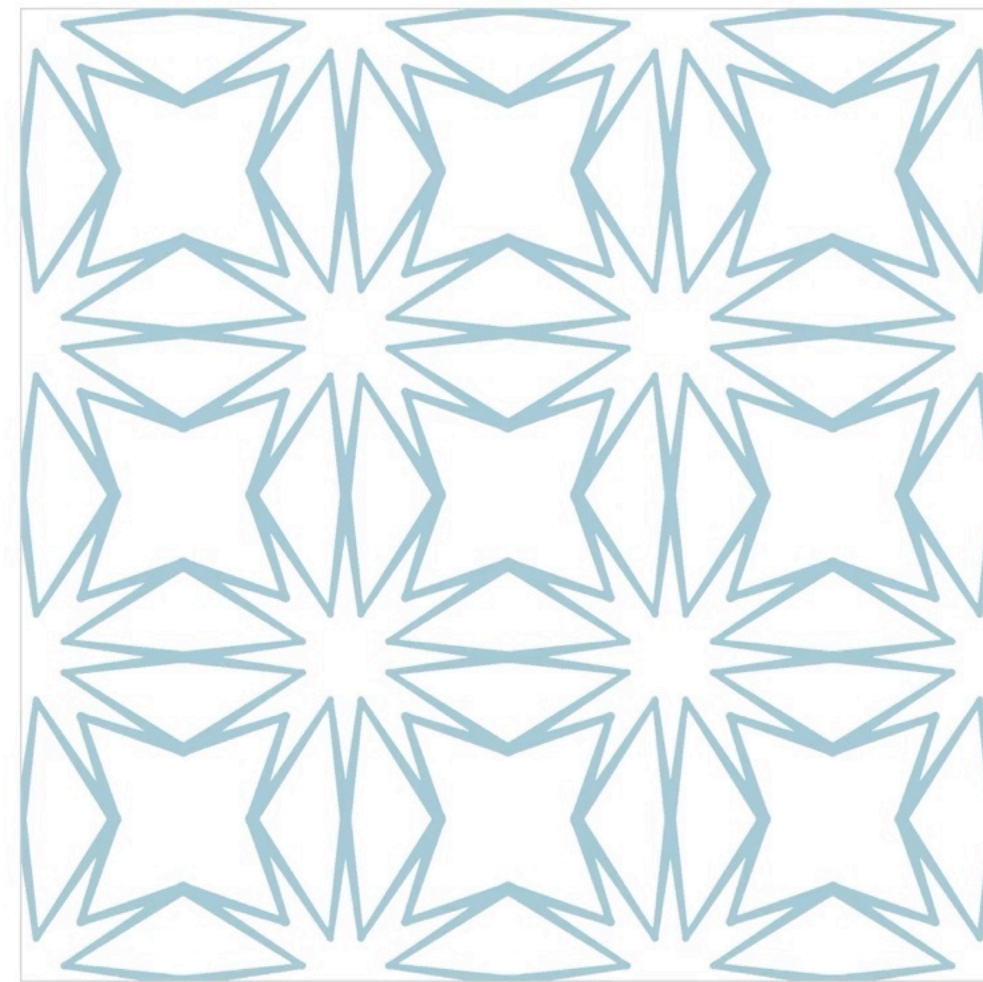
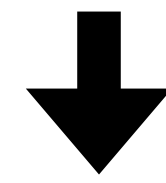
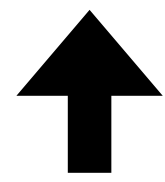
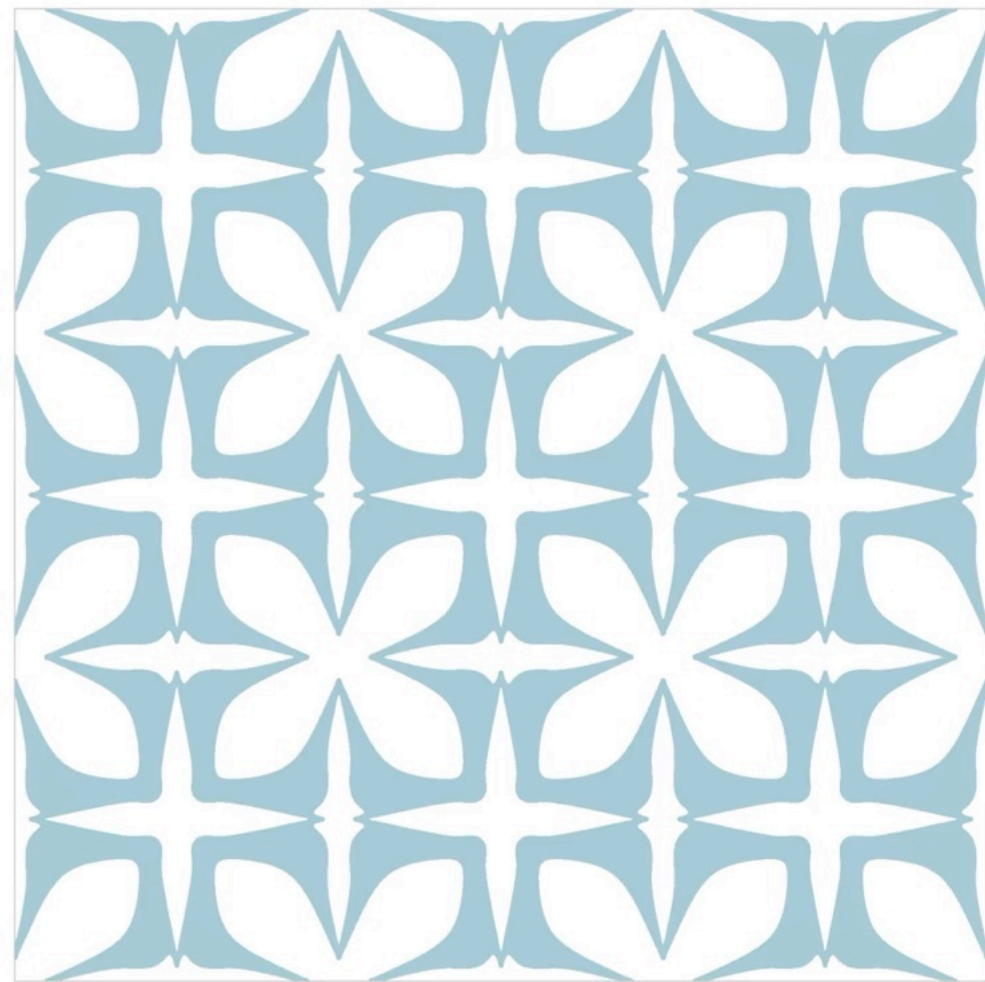
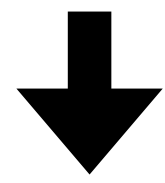


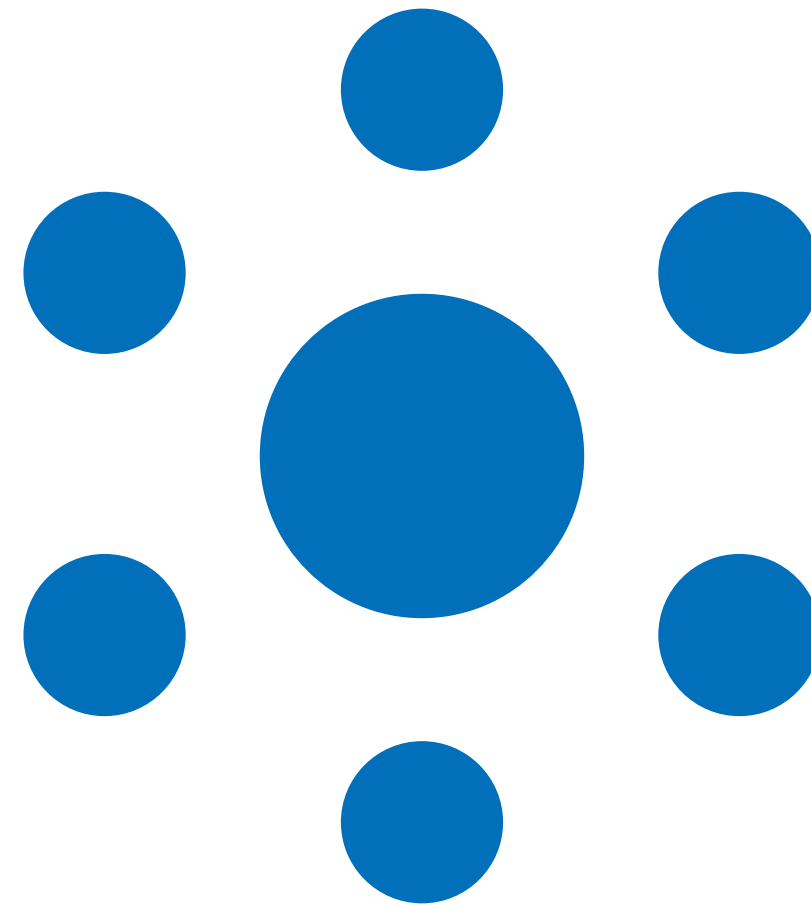
Computational Design of Flexible Planar Microstructures

Zhan Zhang
 Christopher Brandt
 Jean Jouve
 Yue Wang
 Tian Chen
 Mark Pauly
 Julian Panetta

METAMATERIALS

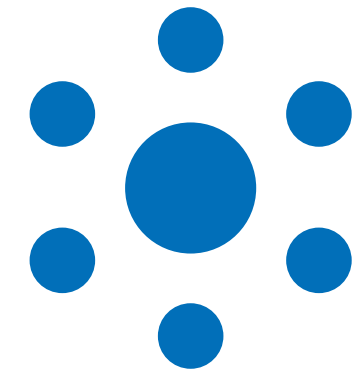


METAMATERIALS



Material

METAMATERIALS

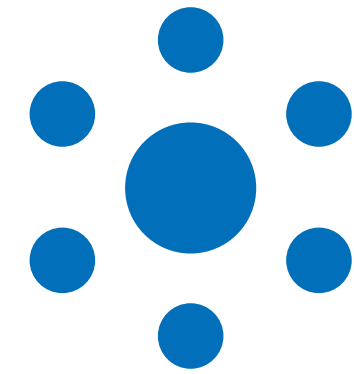


Material



Microstructure

METAMATERIALS

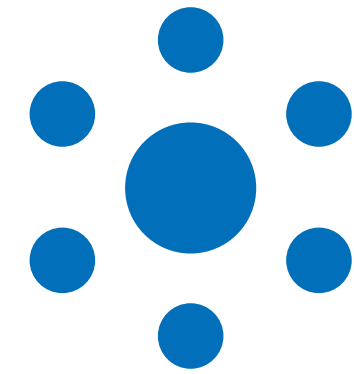


Material



Microstructure

METAMATERIALS

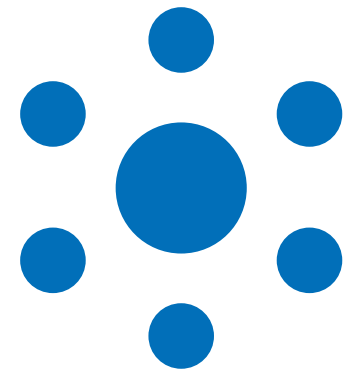


Material



Microstructure

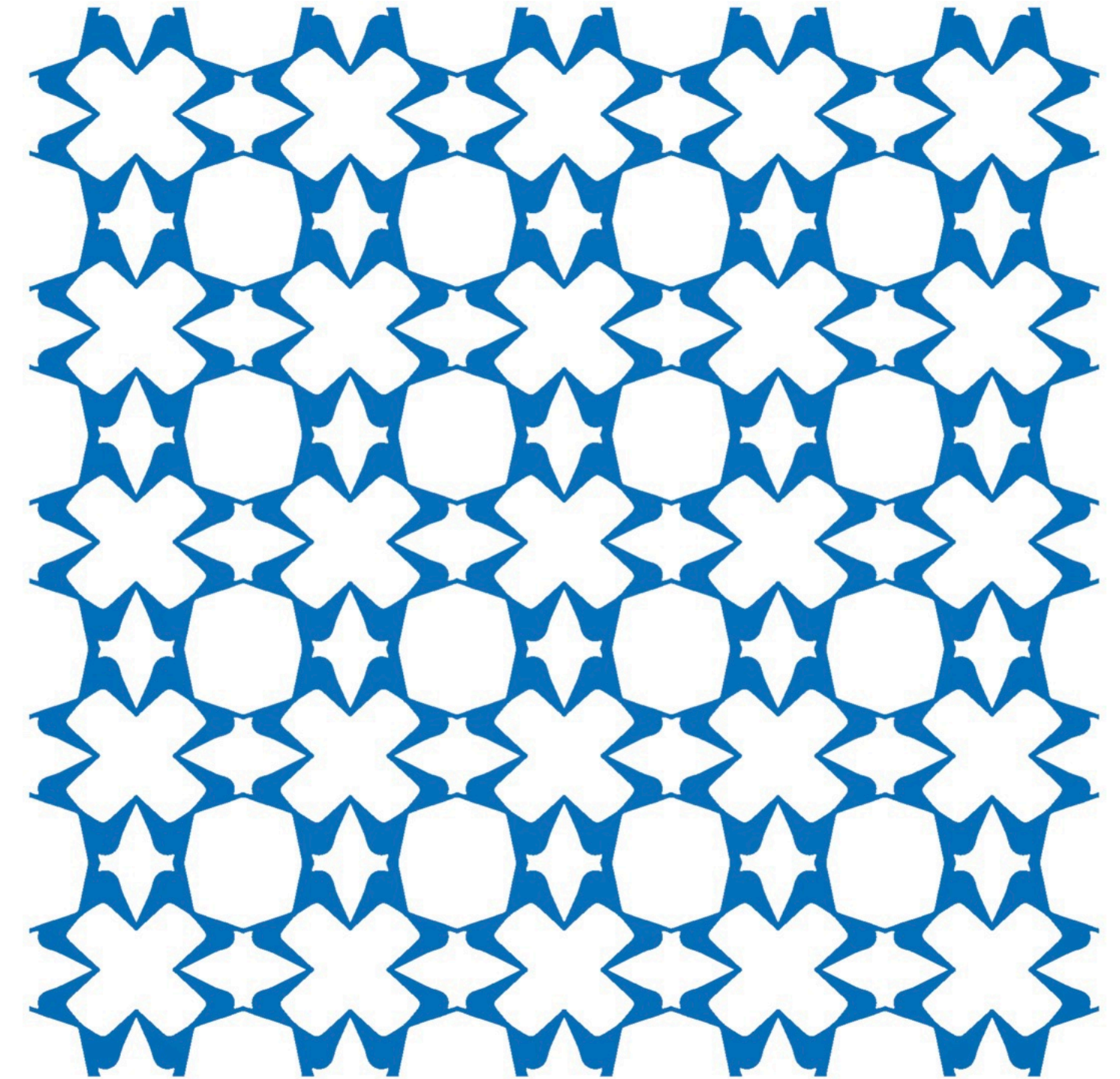
METAMATERIALS



Material

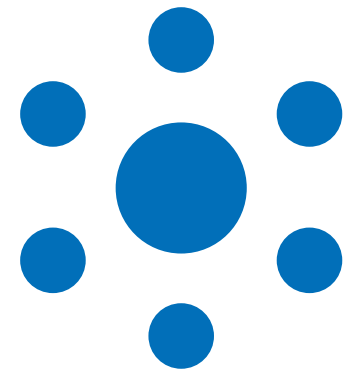


Microstructure

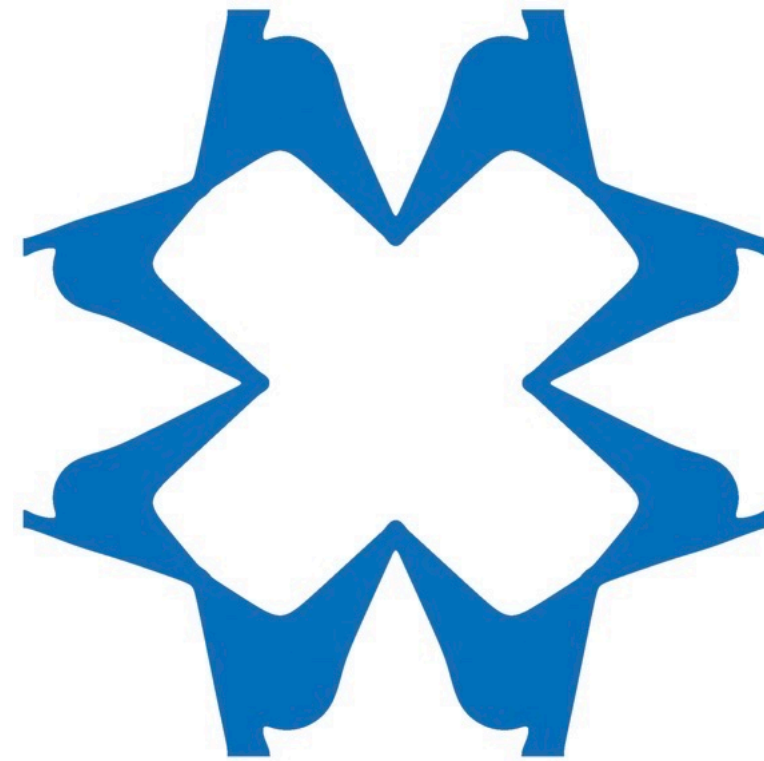


Metamaterial

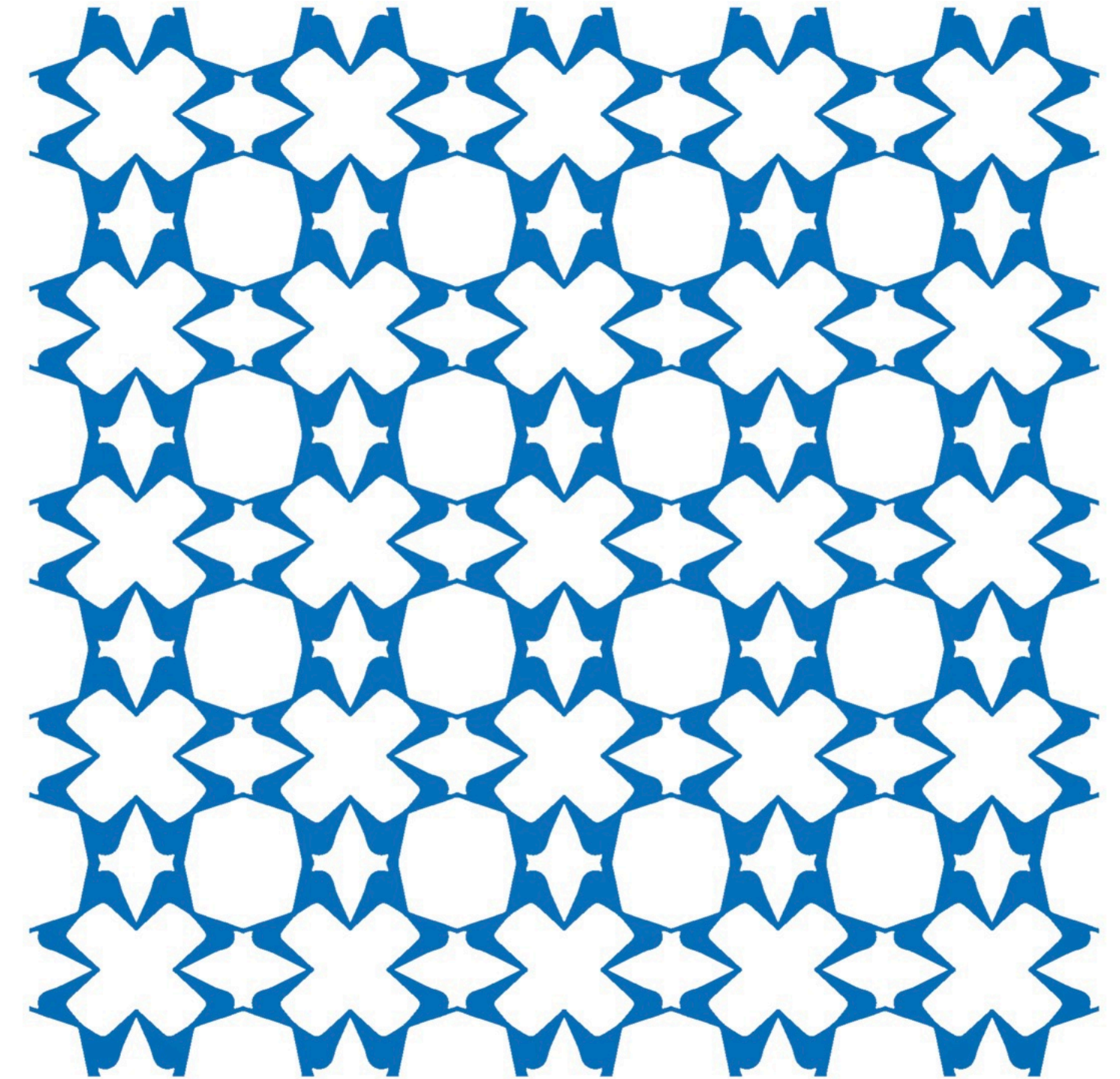
METAMATERIALS



Material

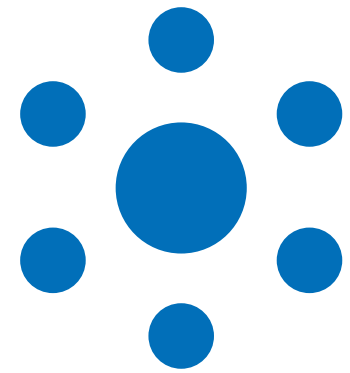


Microstructure

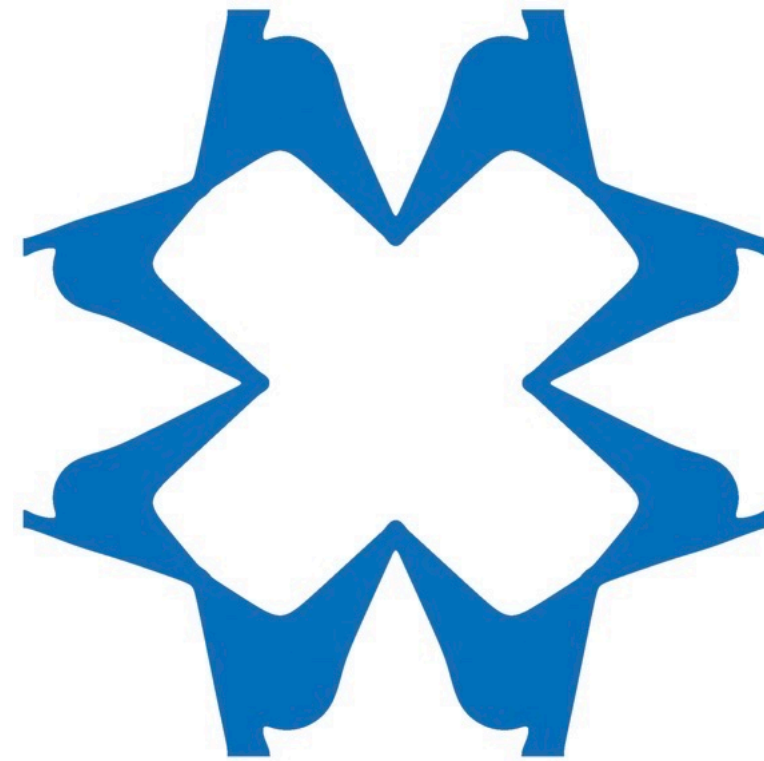


Metamaterial

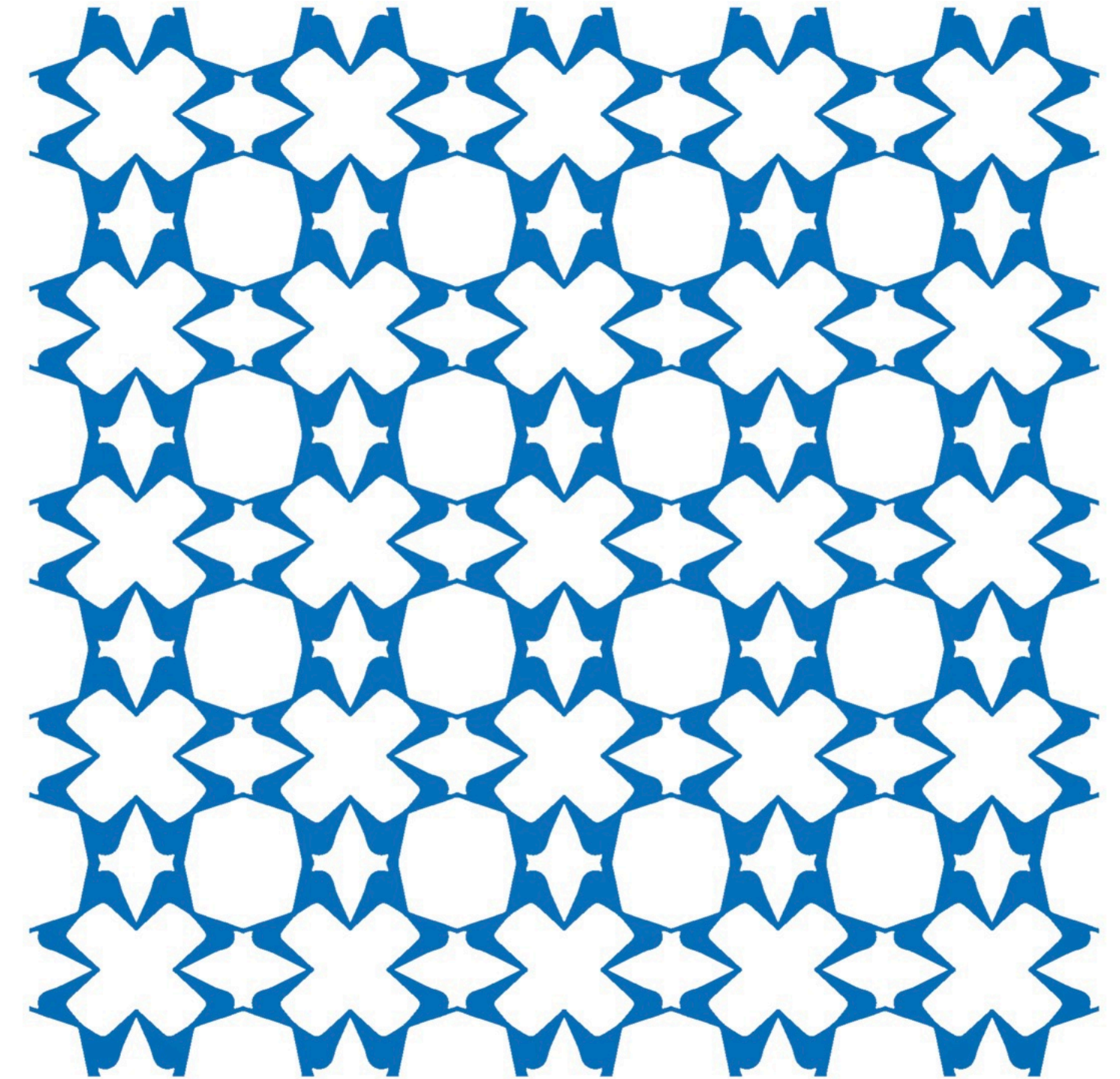
METAMATERIALS



Material

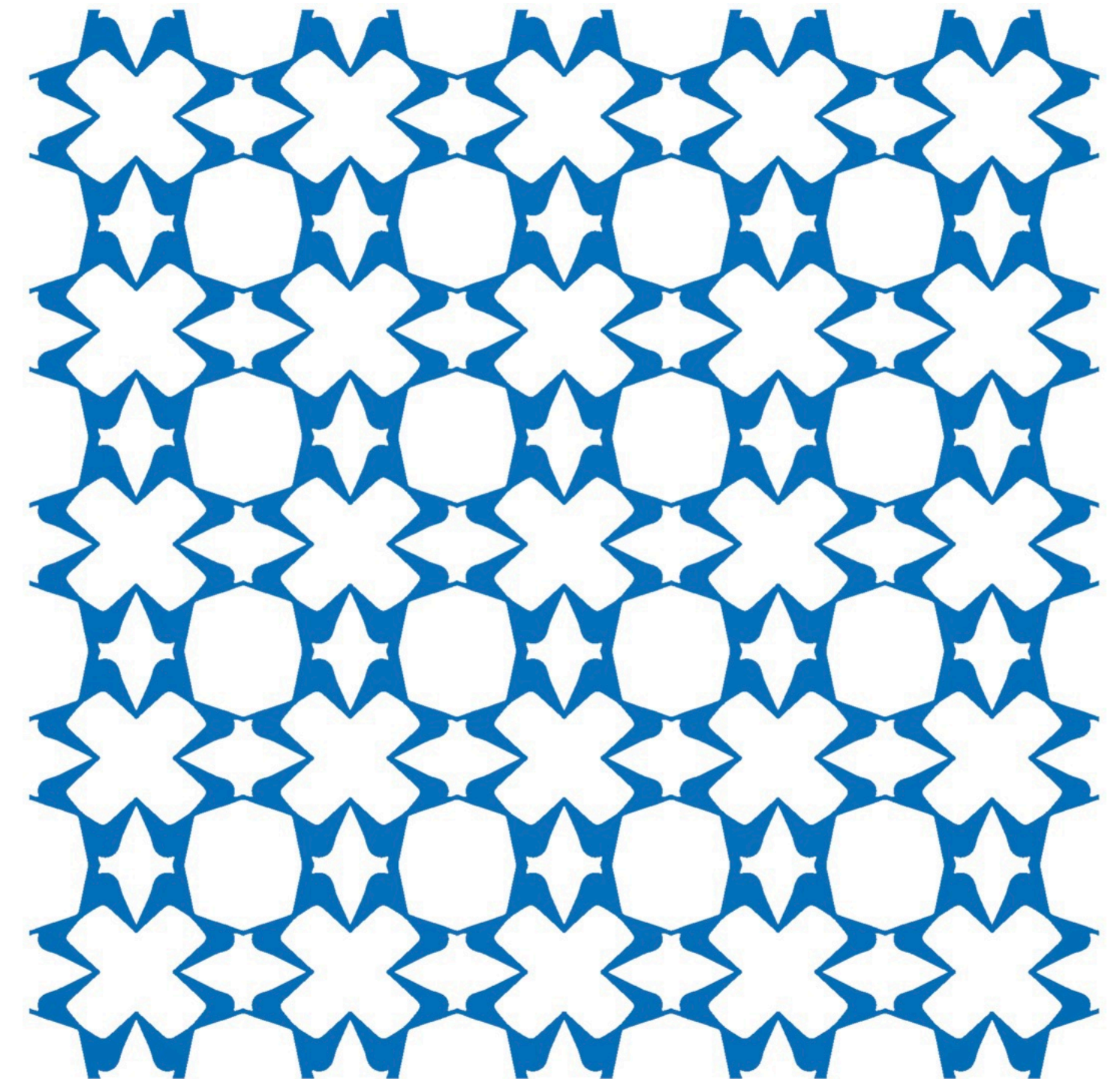
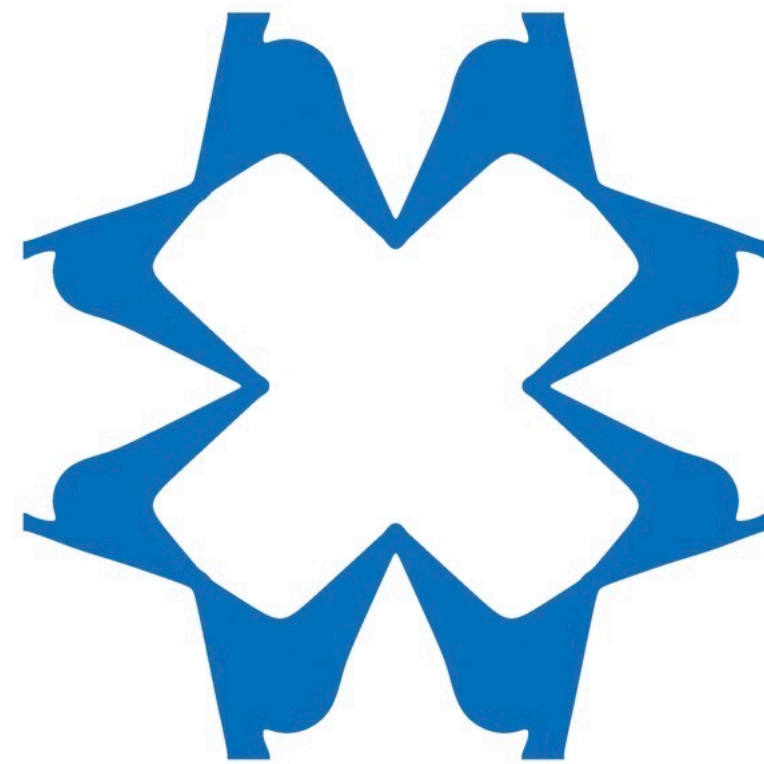
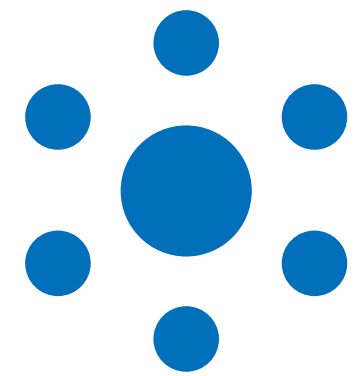


Microstructure



Metamaterial

METAMATERIALS

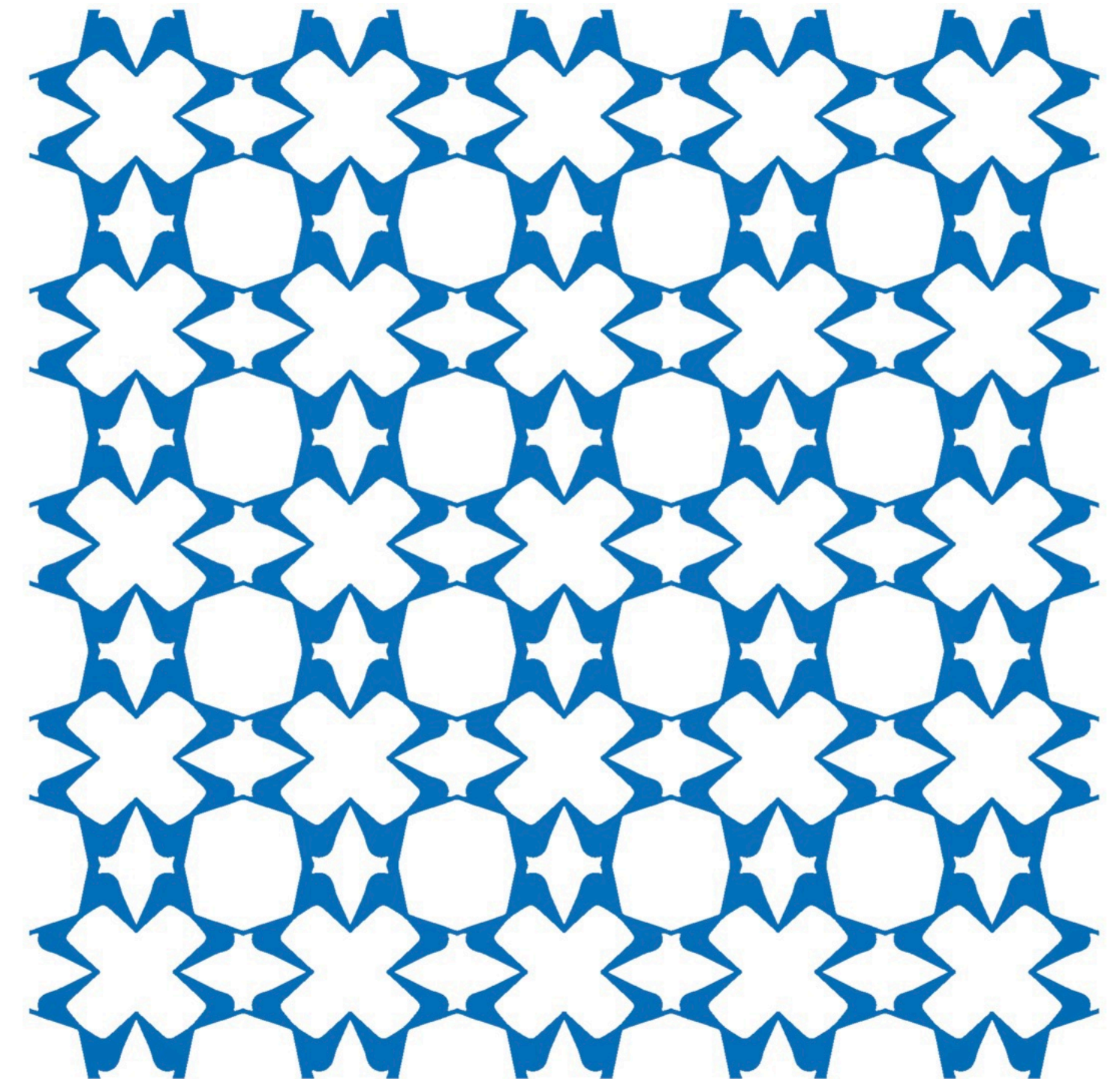
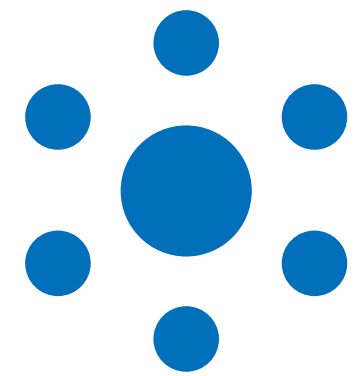


Material

Microstructure

Metamaterial

METAMATERIALS

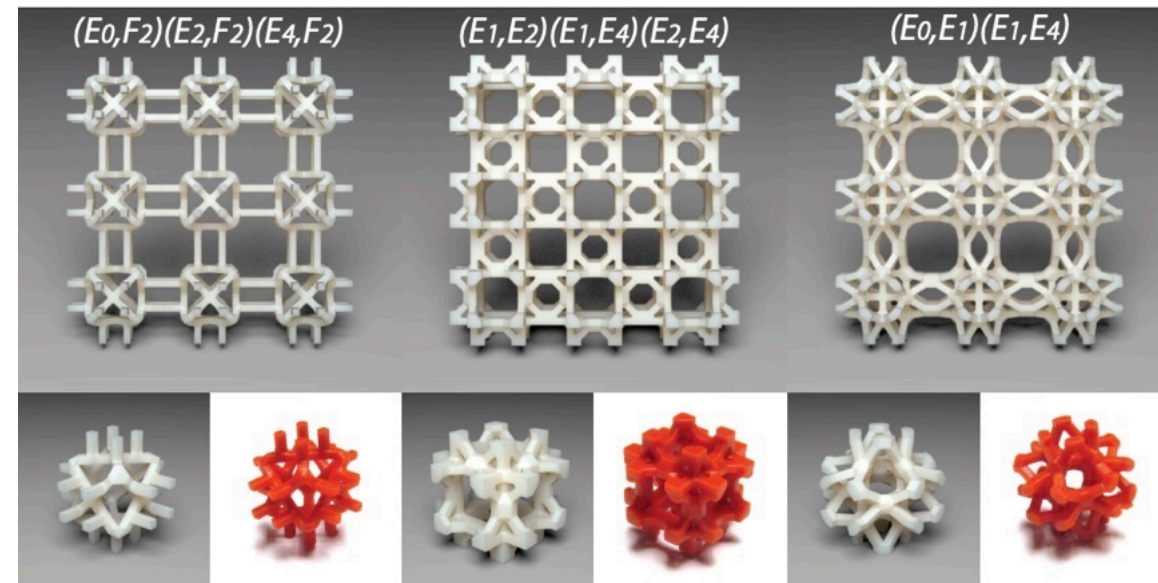


Material

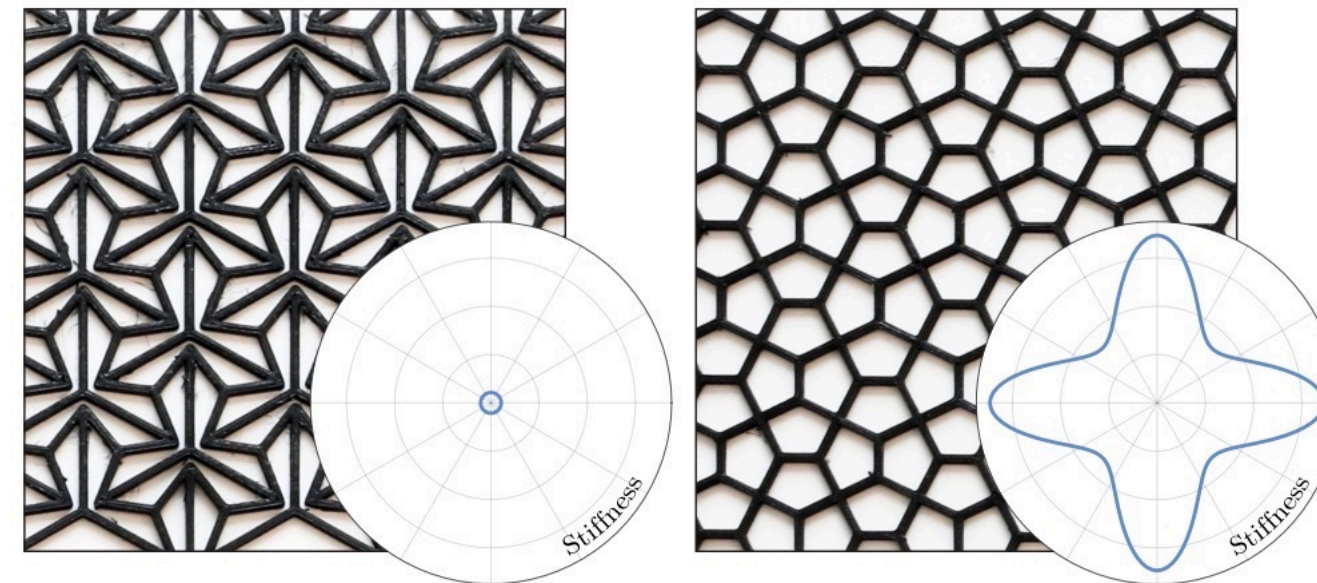
Microstructure

Metamaterial

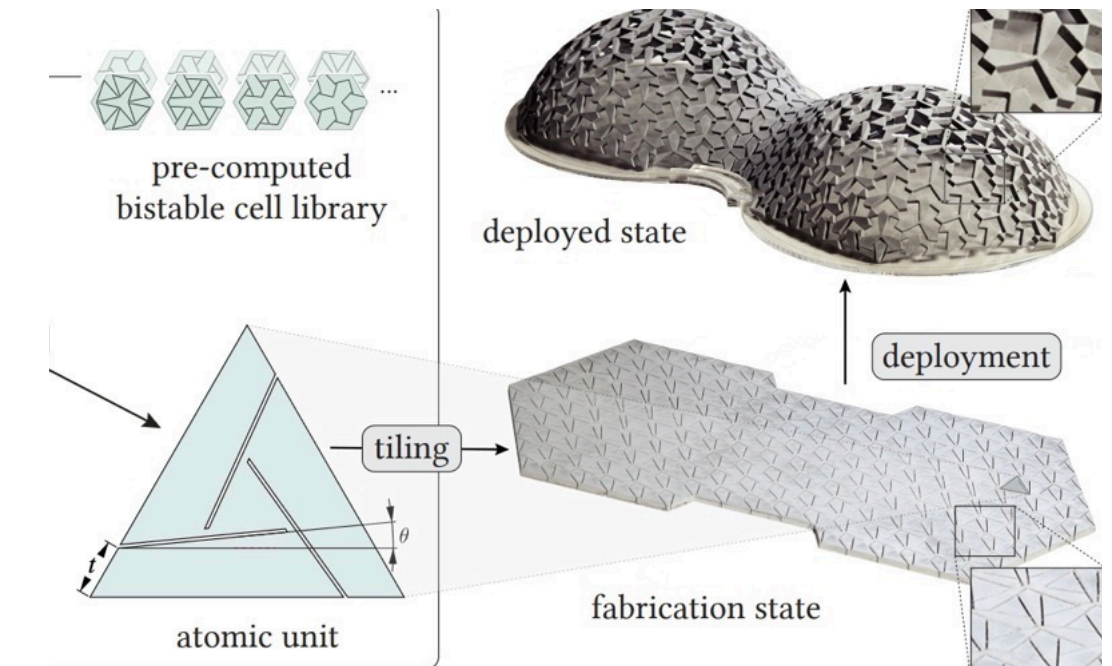
RELATED WORK



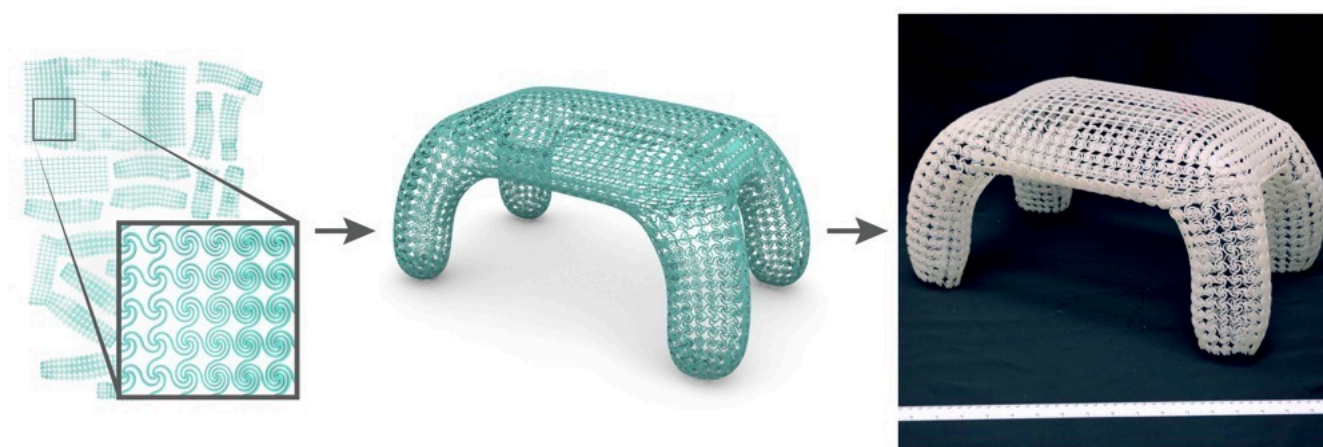
Elastic Textures for Additive Fabrication
[Panetta et al. 2015]



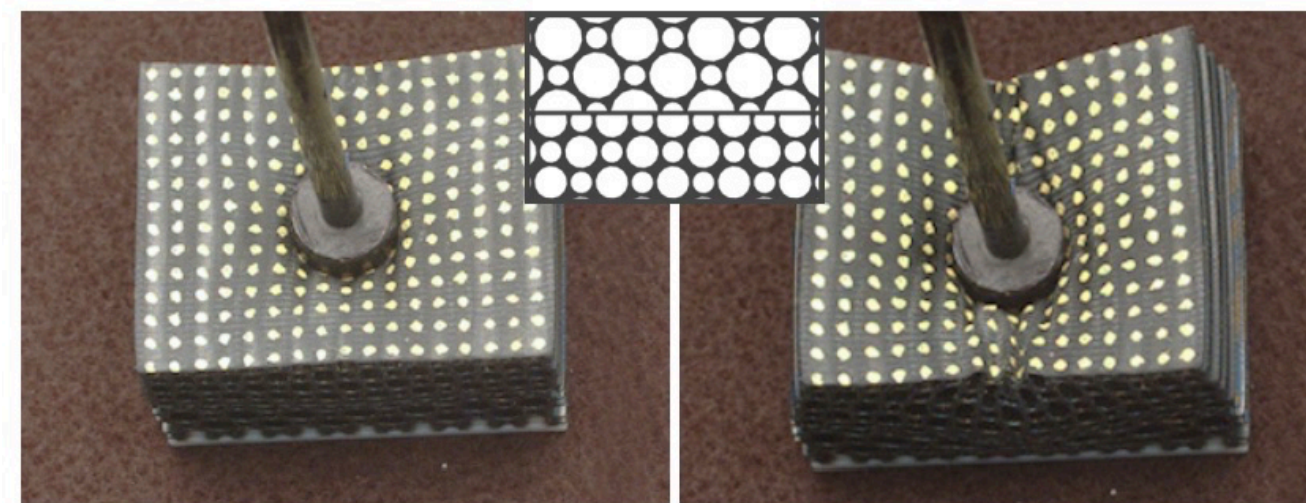
Structured Sheet Materials
[Schumacher et al. 2018]



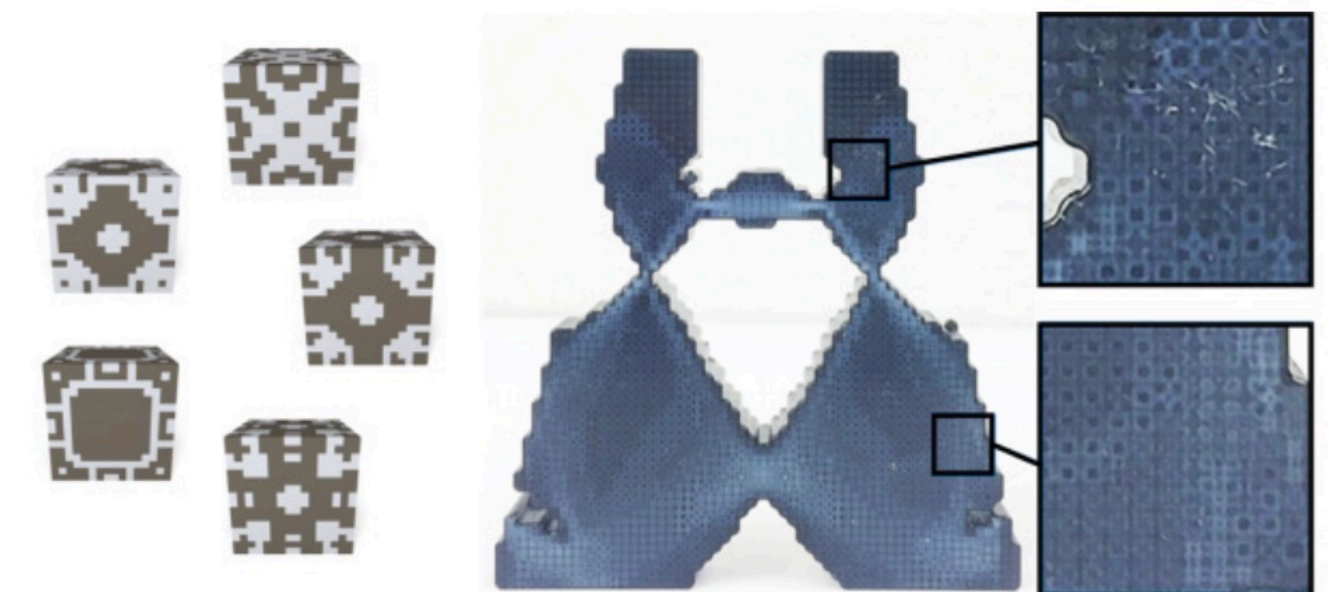
Bistable Auxetic Surface Structures
[Chen et al. 2021]



FlexMaps
[Malomo et al. 2018]

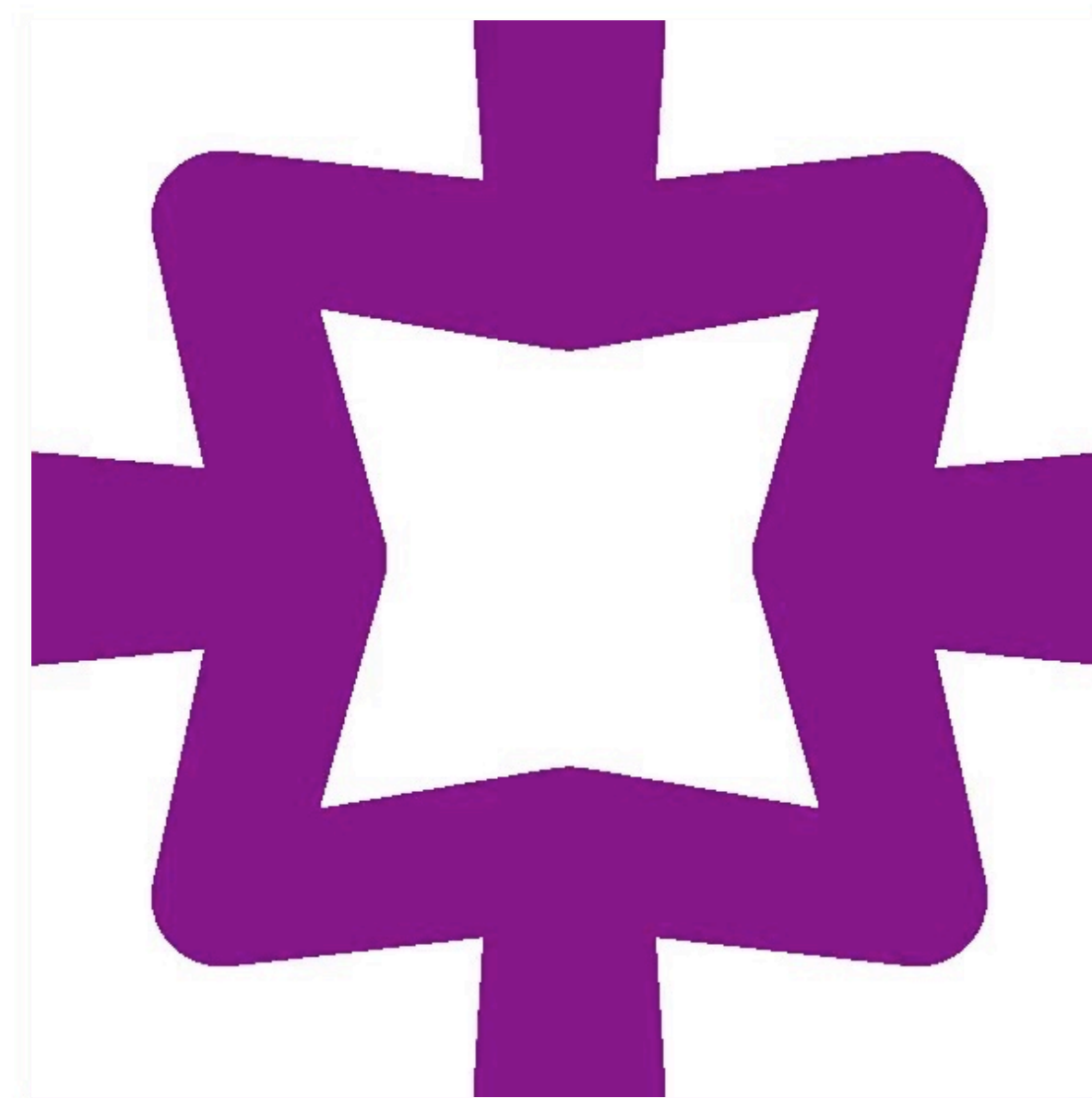


Desired Deformation Material Design
[Bickel et al. 2010]

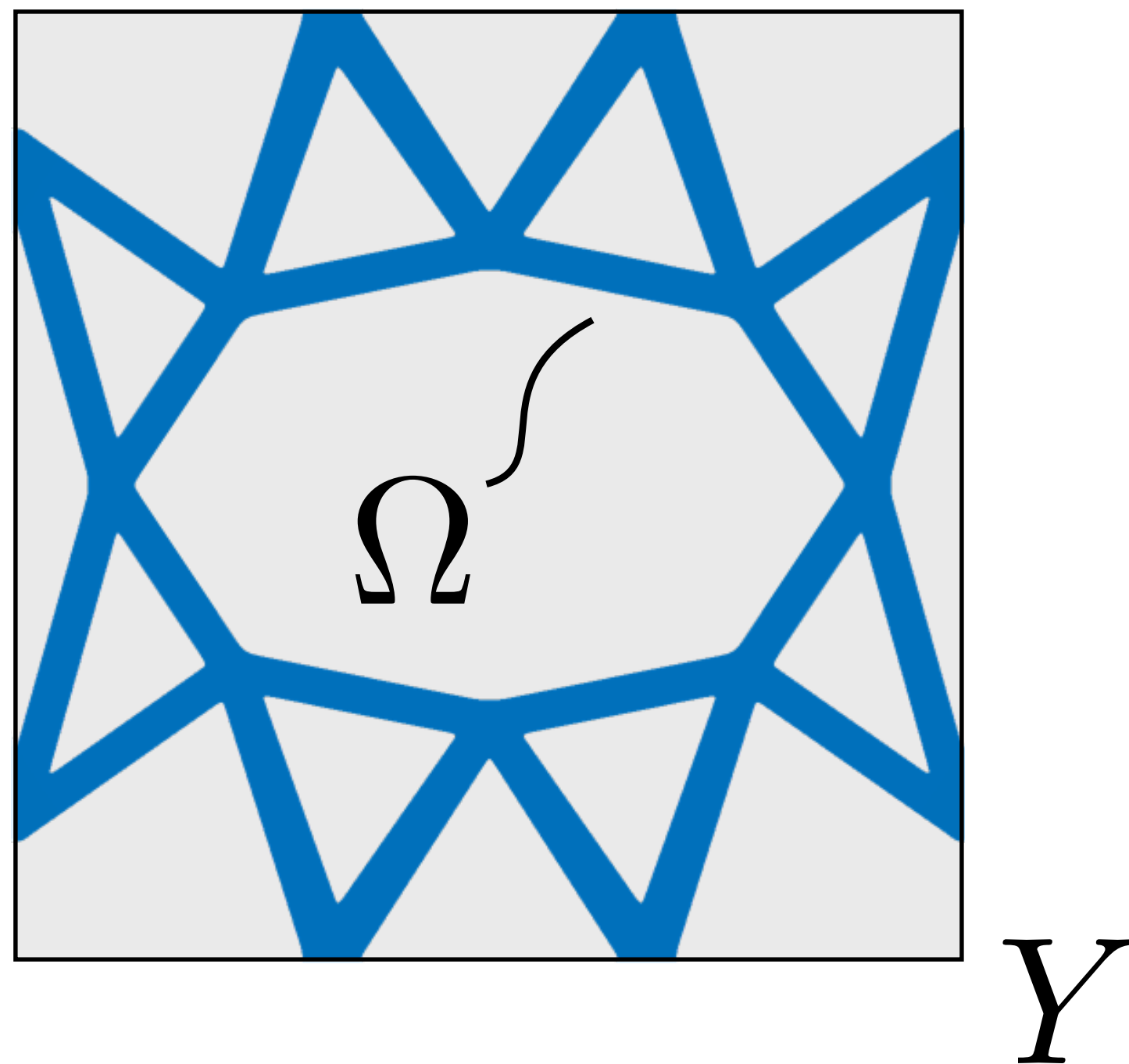


Two-Scale Topology Optimization
[Zhu et al. 2017]

HOMOGENIZATION



HOMOGENIZATION



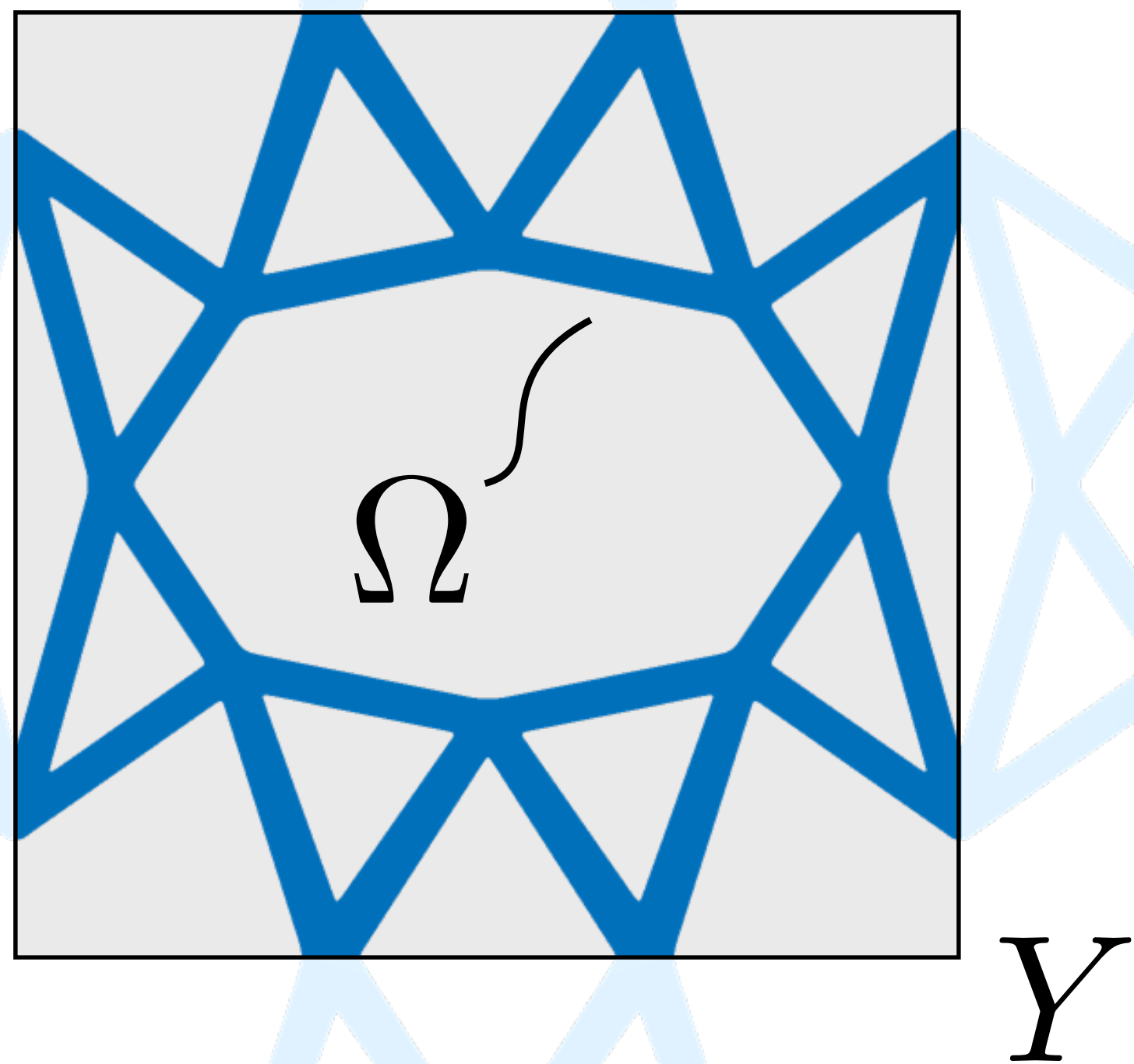
Deformation Function:

$$\Phi(\mathbf{X}) = \bar{F}\mathbf{X} + \omega(\mathbf{X})$$

macro
deformation

micro
deformation

HOMOGENIZATION

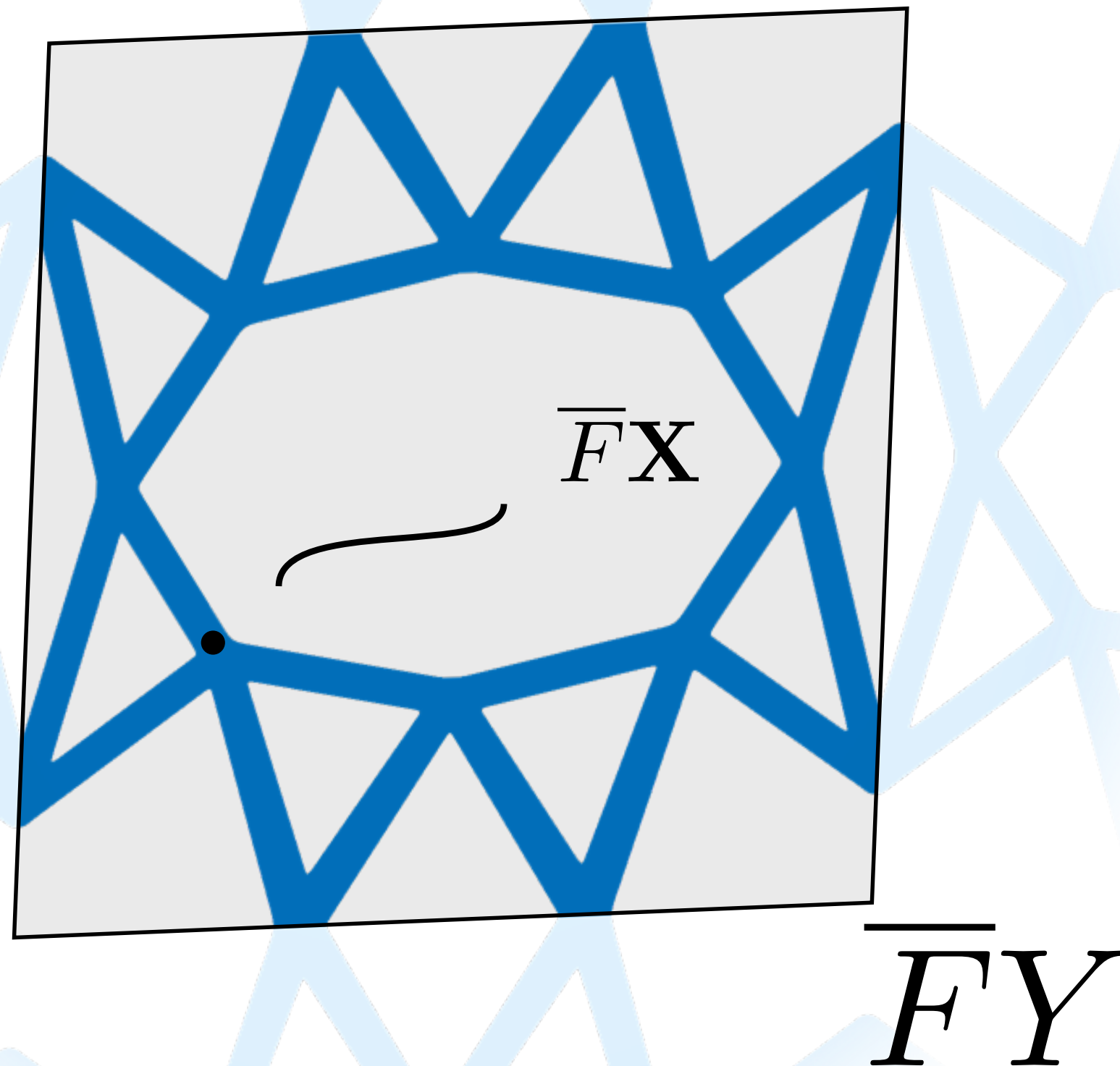


Deformation Function:

$$\Phi(\mathbf{X}) = \bar{F}\mathbf{X}$$

macro
deformation

HOMOGENIZATION

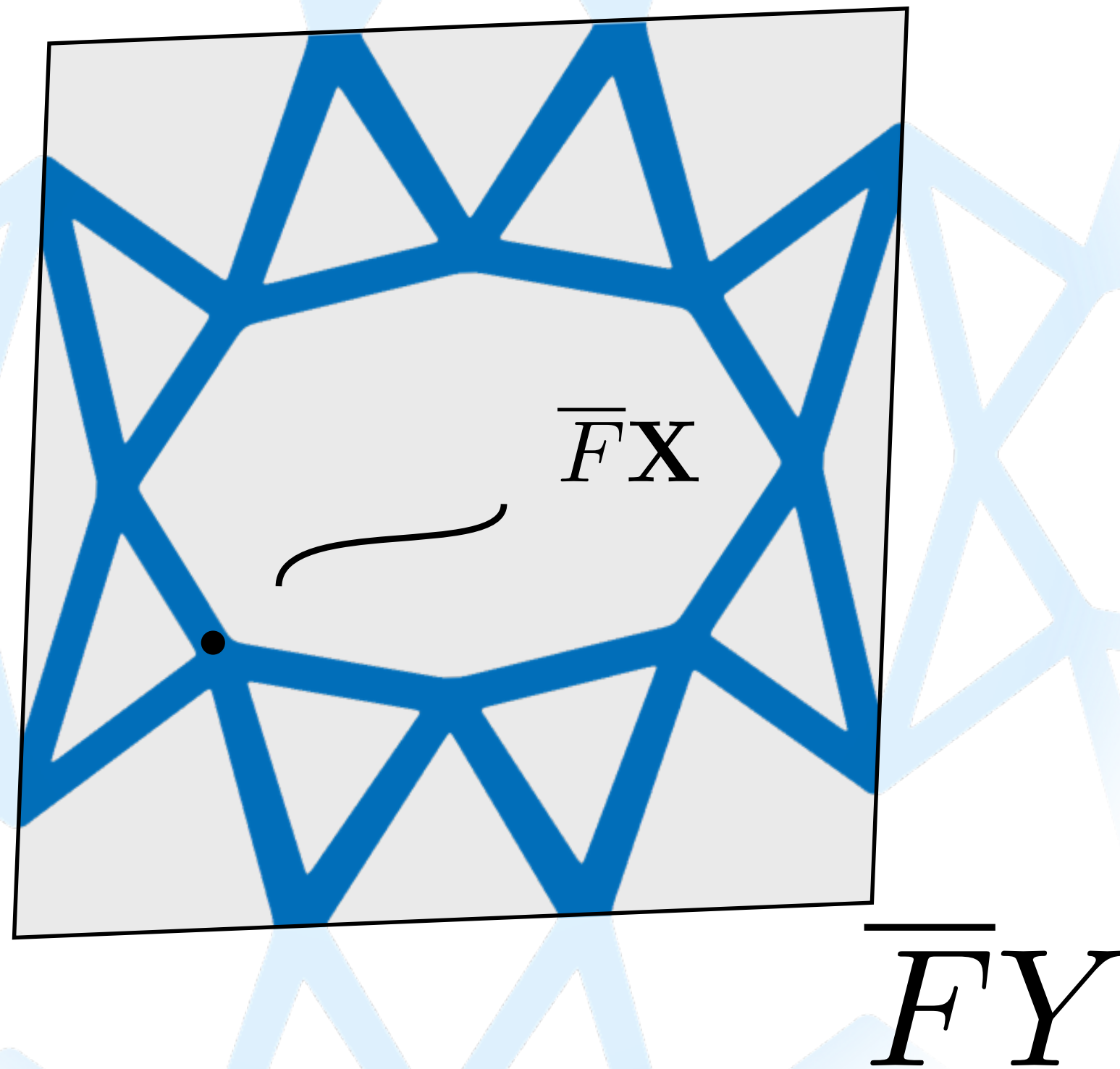


Deformation Function:

$$\Phi(\mathbf{X}) = \bar{F}\mathbf{X}$$

macro
deformation

HOMOGENIZATION



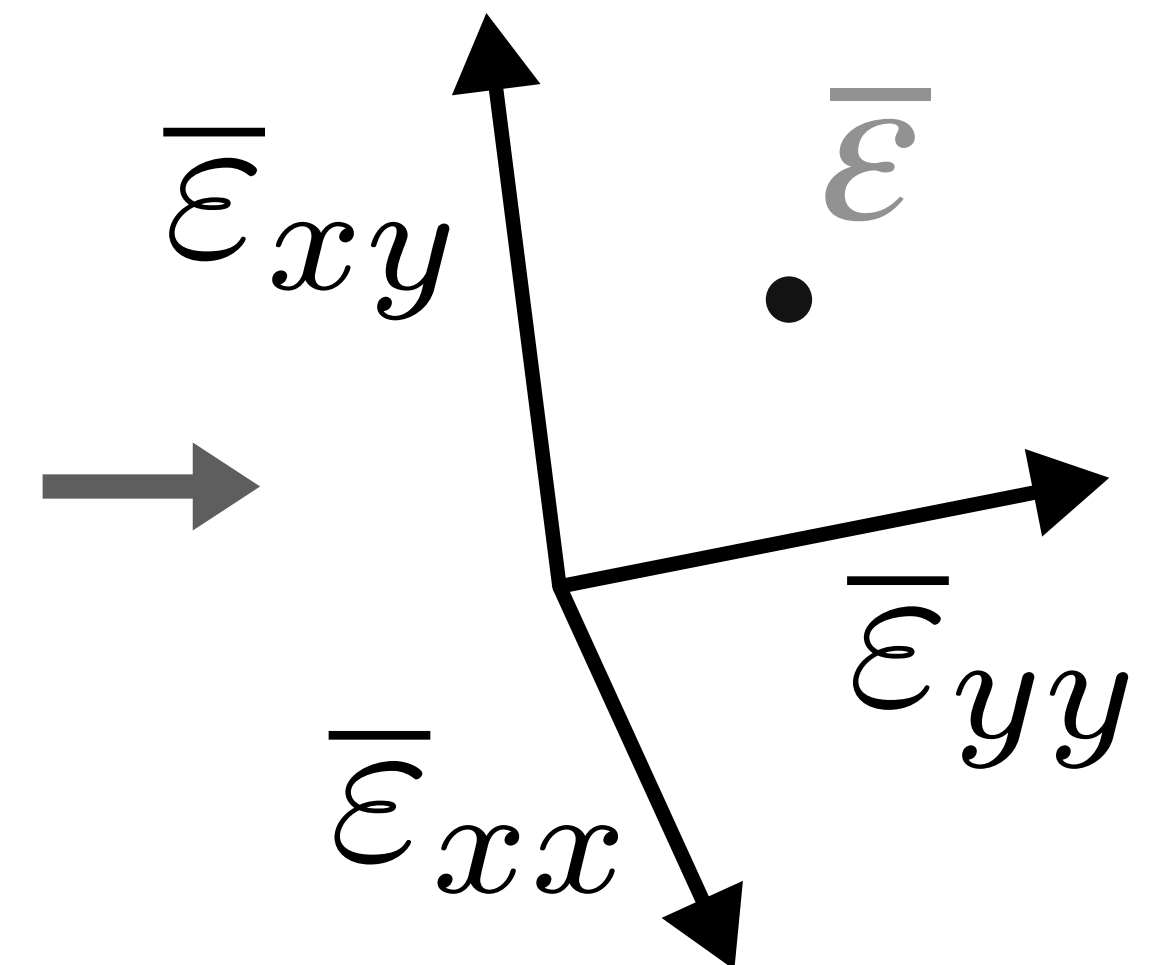
$\bar{F}X$ macro deformation

Symmetric $\bar{F} \in \mathbb{R}^{2 \times 2}$

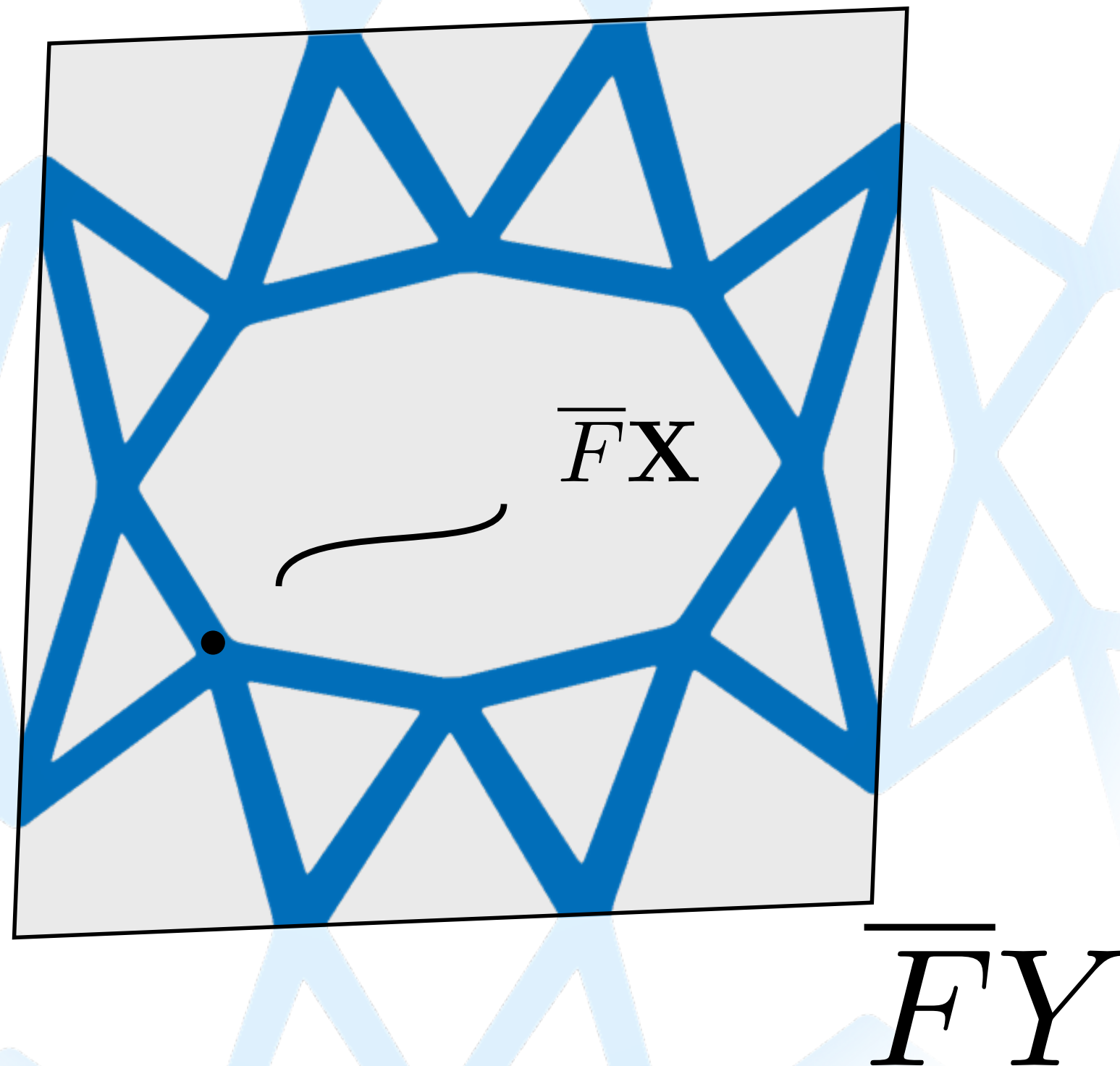
$$\bar{F} = \bar{\varepsilon} + I$$

$$\begin{bmatrix} \bar{\varepsilon}_{xx} & \bar{\varepsilon}_{xy} \\ \bar{\varepsilon}_{xy} & \bar{\varepsilon}_{yy} \end{bmatrix}$$

Strain
(Biot)



HOMOGENIZATION



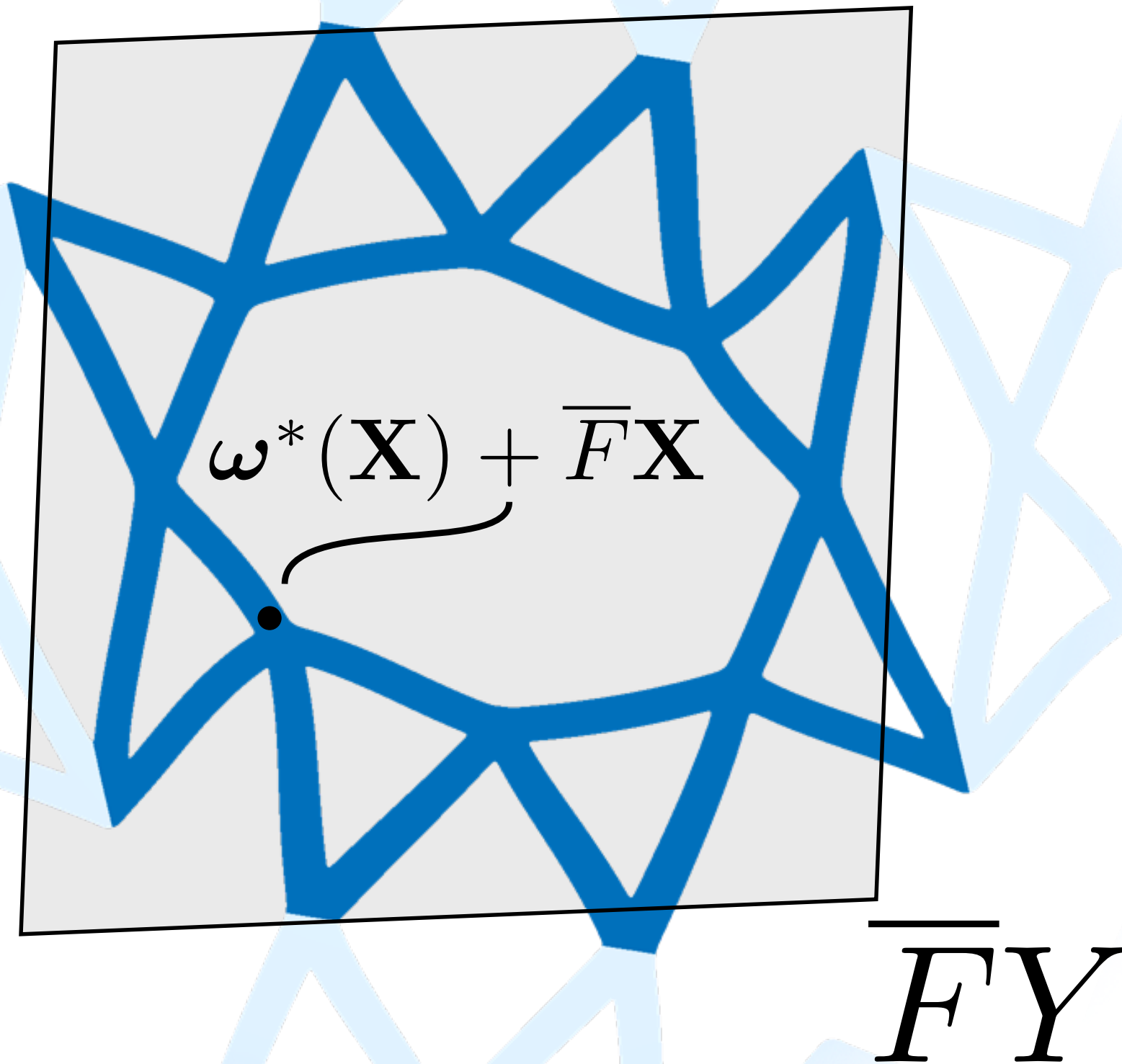
Deformation Function:

$$\Phi(\mathbf{X}) = \bar{F}\mathbf{X} + \omega(\mathbf{X})$$

macro
deformation

micro
deformation

HOMOGENIZATION



Deformation Function:

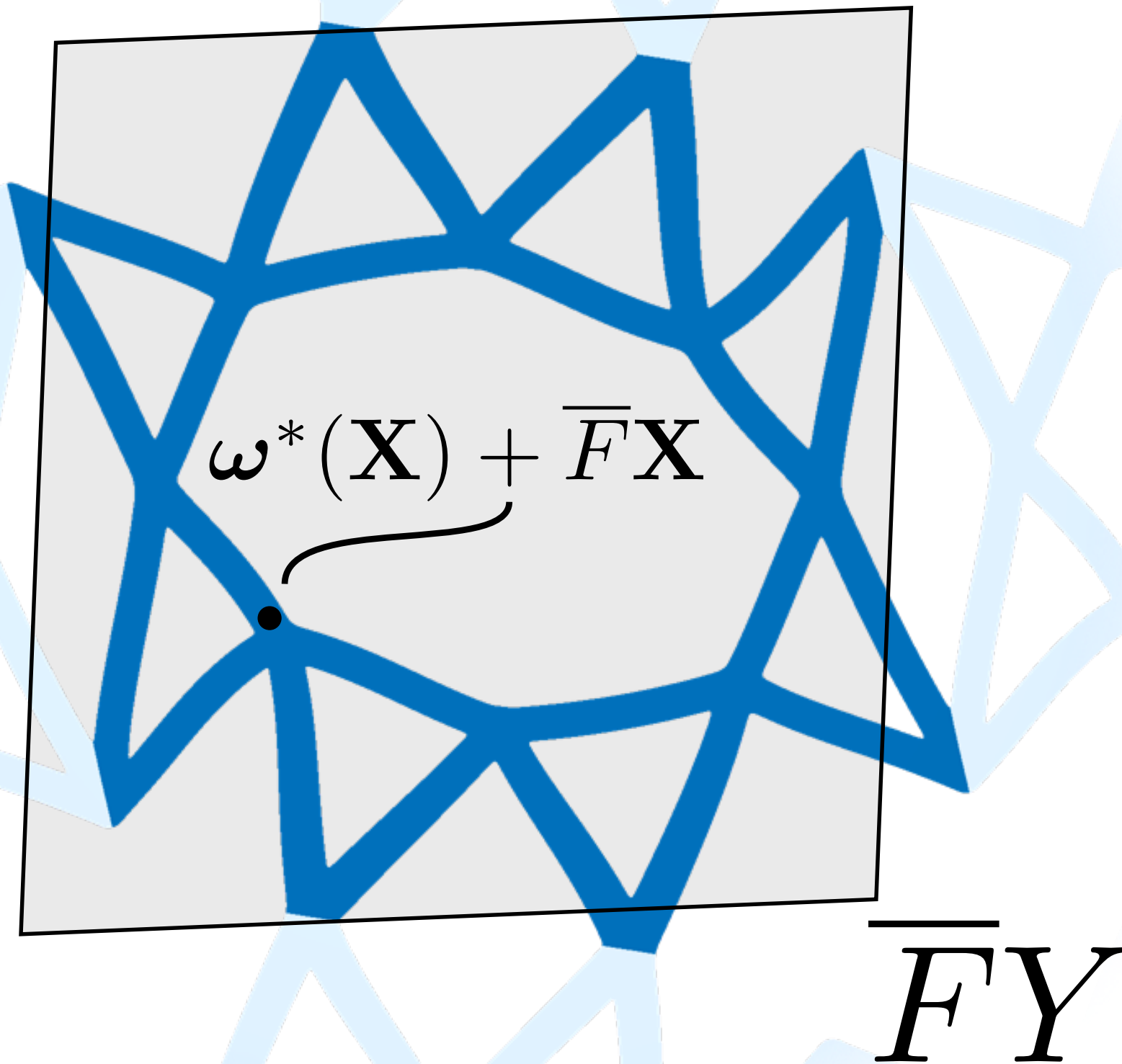
Fluctuation Displacement Field

$$\Phi(\mathbf{X}) = \bar{F}\mathbf{X} + \omega(\mathbf{X}) \quad \omega \text{ periodic}$$

macro
deformation

micro
deformation

HOMOGENIZATION



Deformation Function:

$$\Phi(\mathbf{X}) = \bar{F}\mathbf{X} + \omega(\mathbf{X})$$

$$\min_{\omega \text{ periodic}} \frac{1}{|Y|} \int_{\Omega} \psi(\nabla \omega + \bar{F}) d\mathbf{X}$$

 's elastic energy density

HOMOGENIZATION

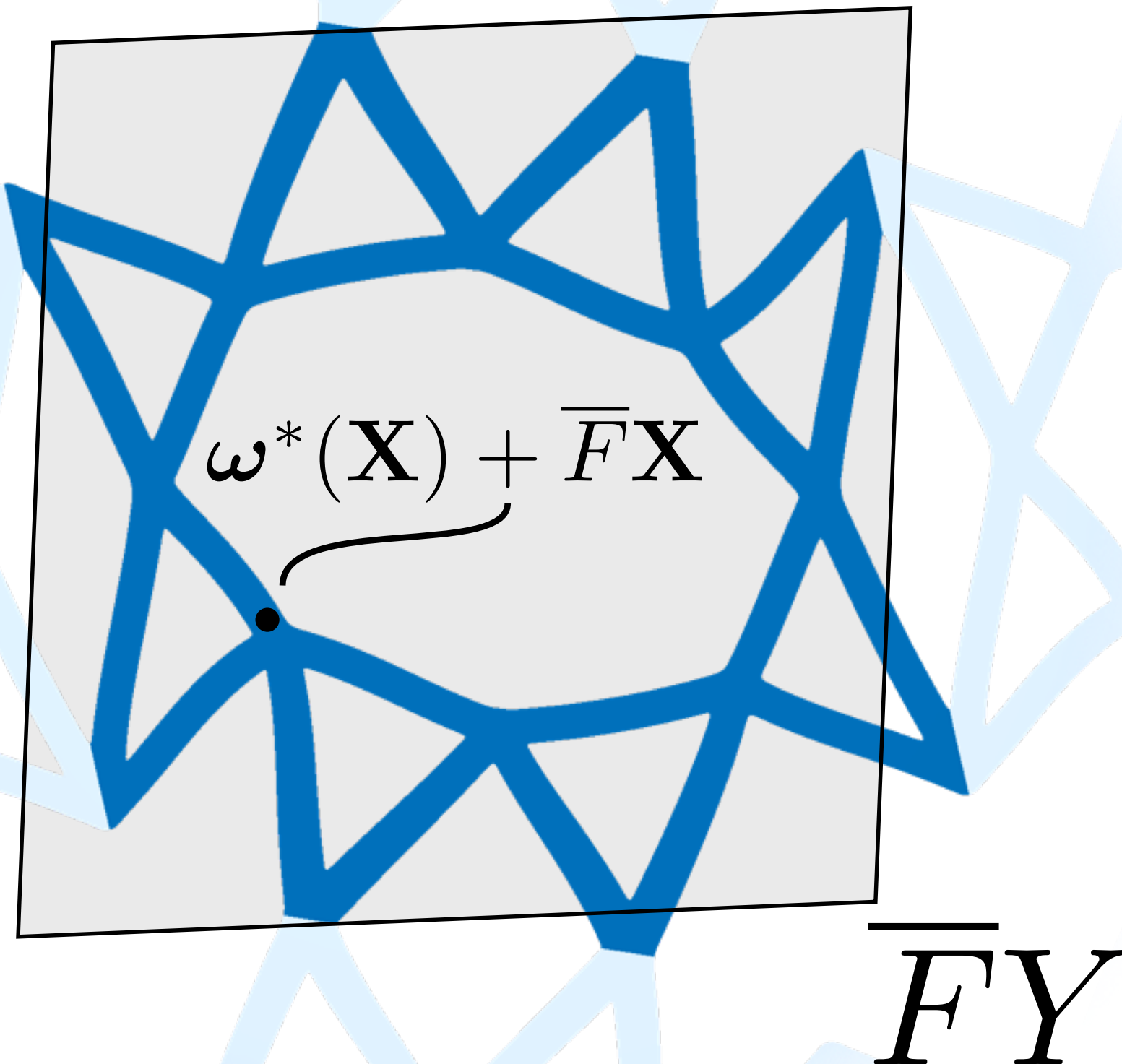
Deformation Function:

$$\Phi(\mathbf{X}) = \bar{F}\mathbf{X} + \omega(\mathbf{X})$$

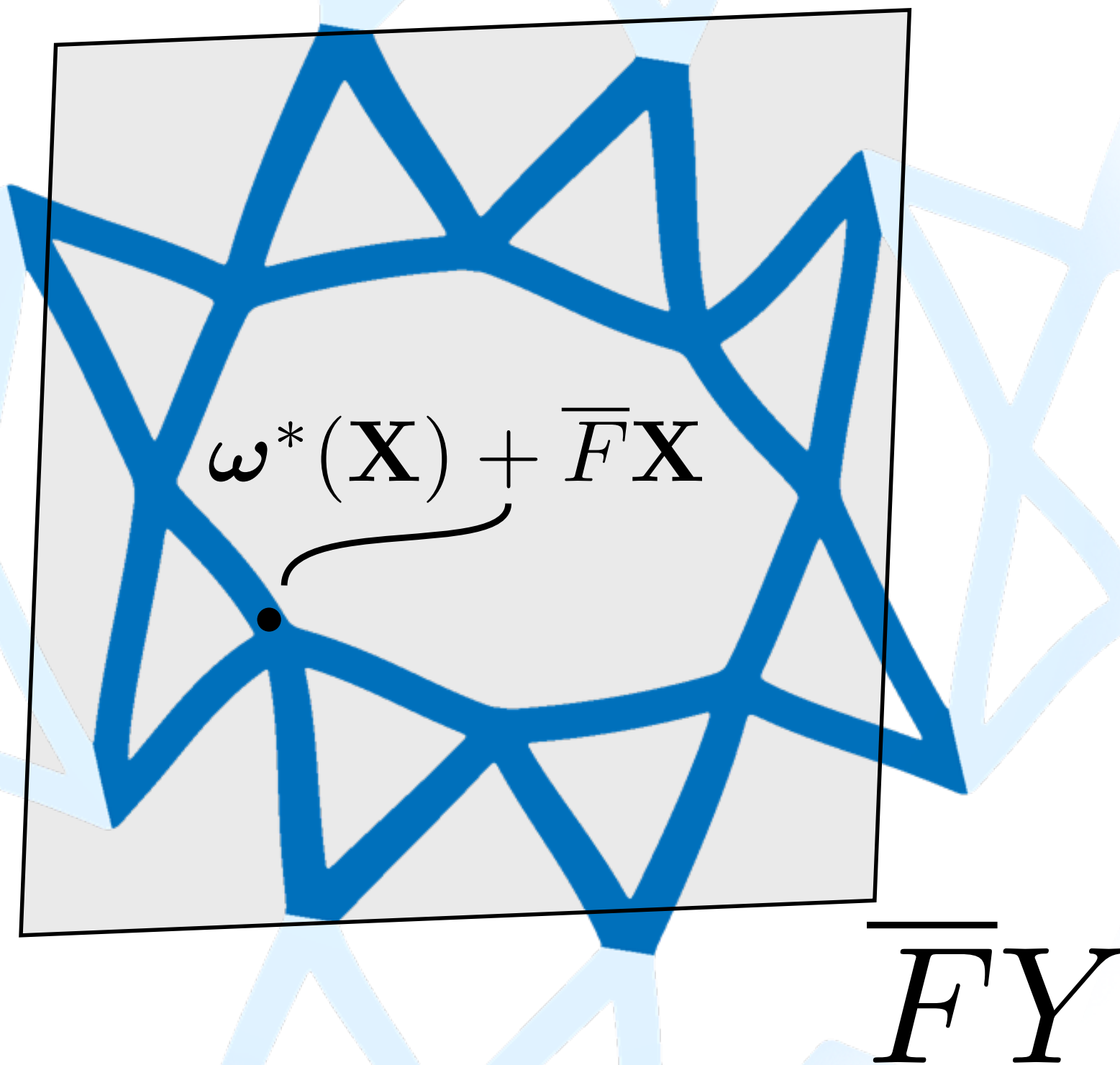
$$\bar{\psi}(\bar{F}) := \min_{\omega \text{ periodic}} \frac{1}{|Y|} \int_{\Omega} \psi(\nabla \omega + \bar{F}) d\mathbf{X}$$

 's elastic energy density  's elastic energy density

Homogenized energy density function



HOMOGENIZATION

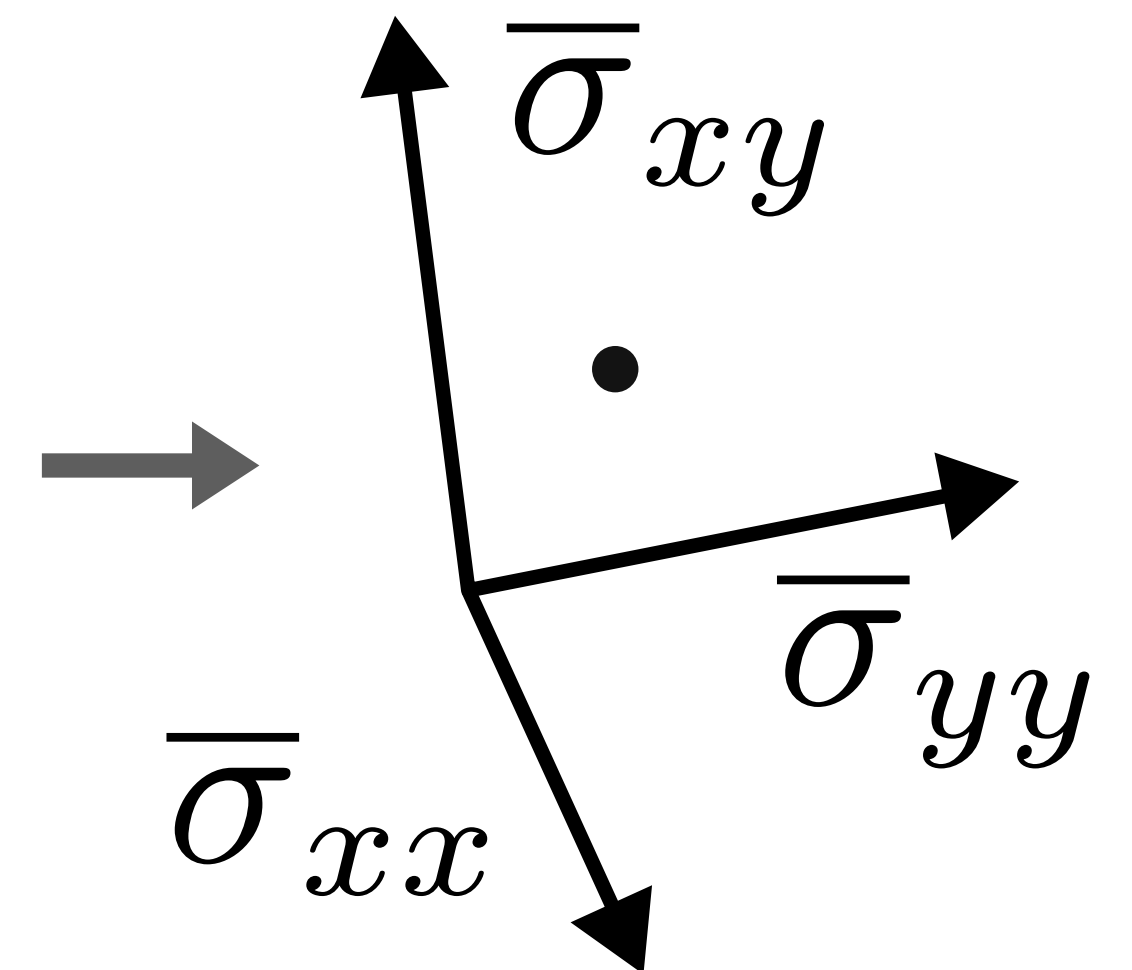


$$\bar{\psi}(\bar{F}) := \min_{\omega \text{ periodic}} \frac{1}{|Y|} \int_{\Omega} \psi(\nabla \omega + \bar{F}) d\mathbf{X}$$

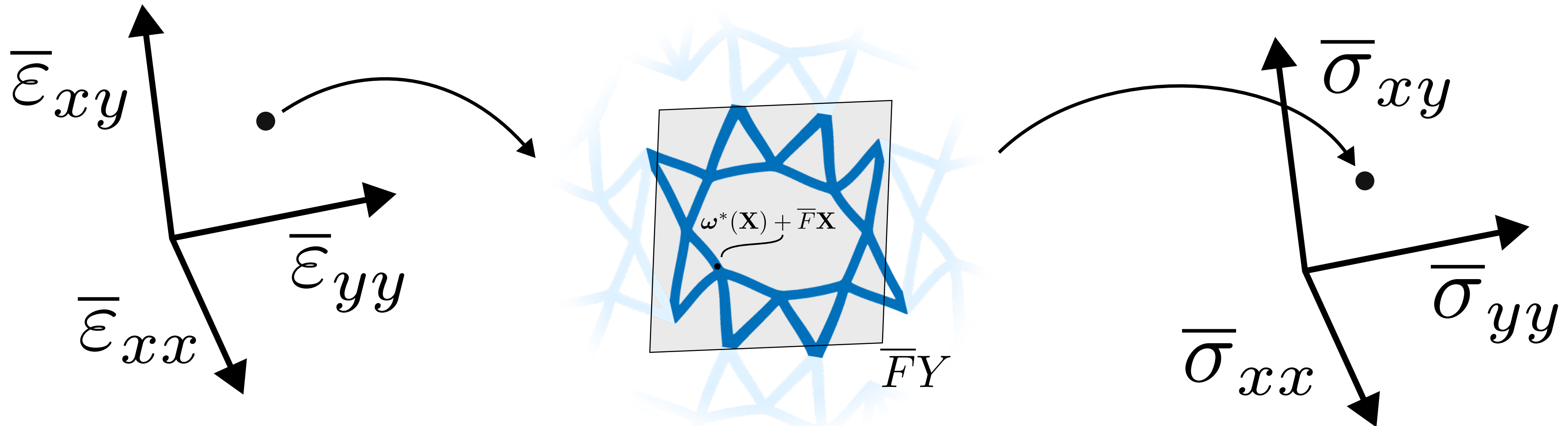
$$\begin{aligned} \bar{\psi}'(\bar{F}) &= \frac{1}{|Y|} \int_{\Omega} \psi'(\nabla \omega^*(\mathbf{X}; \bar{F}) + \bar{F}) d\mathbf{X} \\ &= \bar{\sigma}(\bar{F}) \end{aligned}$$

$$\begin{bmatrix} \bar{\sigma}_{xx} & \bar{\sigma}_{xy} \\ \bar{\sigma}_{xy} & \bar{\sigma}_{yy} \end{bmatrix}$$

Stress



HOMOGENIZATION



Step 1: Apply macro strain

Step 2: Solve displacement field

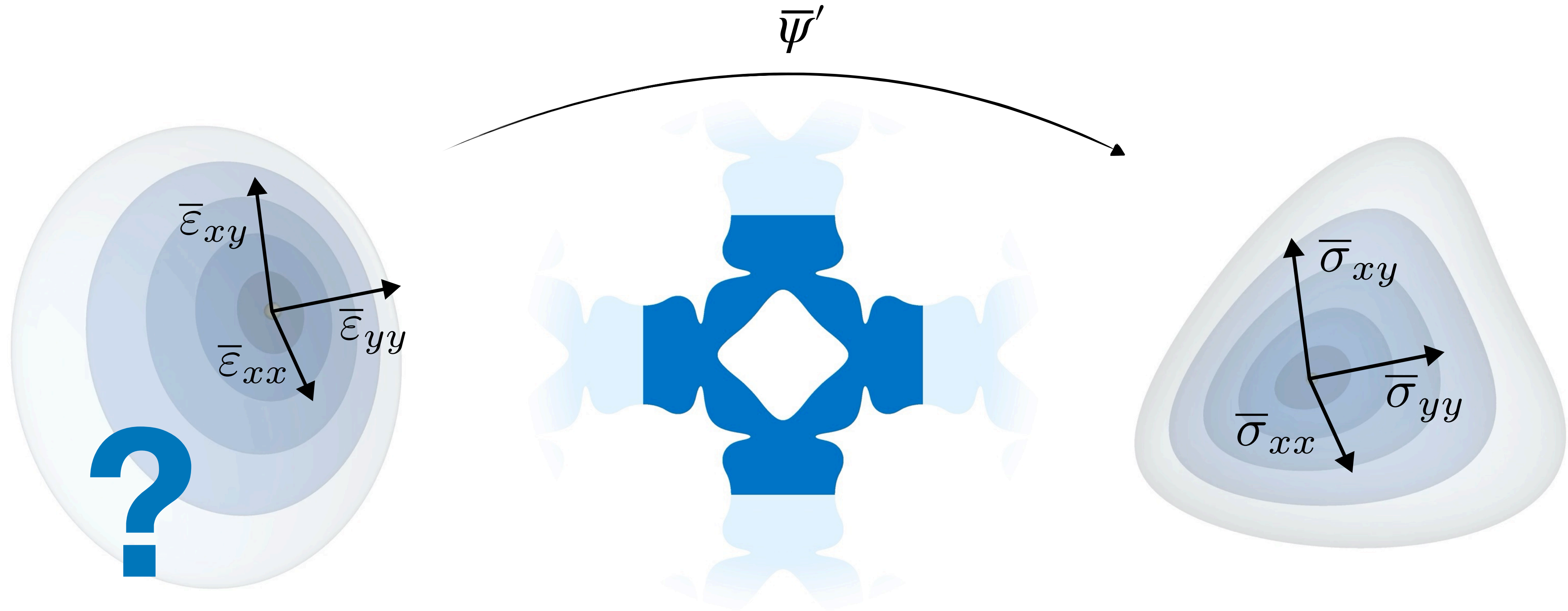
Step 3: Get response stress

Macro Deformation

$$\bar{\psi}(\bar{F}) := \min_{\omega \text{ periodic}} \frac{1}{|Y|} \int_{\Omega} \psi(\nabla \omega + \bar{F}) d\mathbf{X}$$

Response Force

HOMOGENIZATION



Step 1: Apply macro strain

Macro Deformation

Step 2: Solve displacement field

$$\bar{\psi}(\bar{F}) := \min_{\omega \text{ periodic}} \frac{1}{|Y|} \int_{\Omega} \psi(\nabla \omega + \bar{F}) d\mathbf{X}$$

Step 3: Get response stress

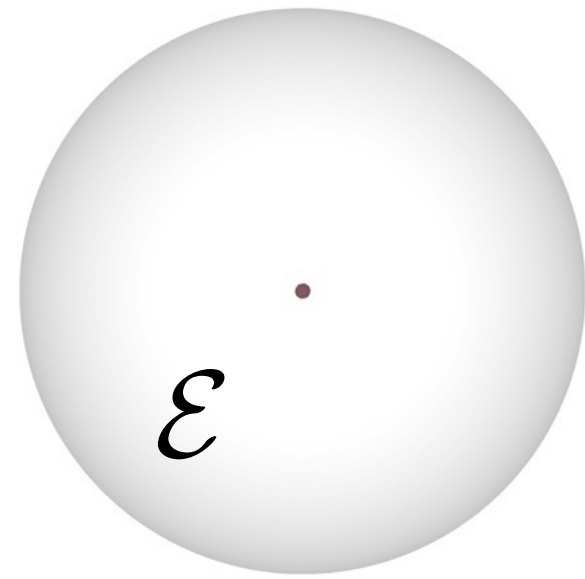
Response Force

HOMOGENIZATION

Past Works

Infinitely small deformation

$$\bar{\psi}(\bar{F}) := \min_{\omega \text{ periodic}} \frac{1}{|Y|} \int_{\Omega} \psi(\nabla \omega + \bar{F}) dX$$



\mathcal{E}
{origin}

ψ Linear Elasticity

[Neves et al. 2000] ...

The collage features several research papers and diagrams. At the top right, a blue logo with a star-like shape and the text " ψ Linear Elasticity" is visible. The papers include:

- Elastic Textures for Additive Fabrication** by Julian Panetta, Qingshan Zhou, Luigi Malomo, Denis Zorin, and Nico Petroni.
- Microstructures to Control Elasticity in 3D Printing** by Christian Schumacher, Bernd Bickel, Jan Rys, Steve Marcher, Chiara Durasio, ETH Zurich, IST Austria, and Concord University.
- Procedural Voronoi Foams for Additive Manufacturing** by Jérôme Dumas, Université de Lorraine, INRIA, and Sylvain Lefebvre, INRIA, Université de Lorraine.
- Two-Scale Topology Optimization with Microstructures** by BO ZHU, MÉLINA SKOURAS, DESAI CHEN, WOJCIECH MATUSIK, MIT CSAIL.
- Star-Shaped Metrics for Mechanical Metamaterial Design** by JONAS MARTINEZ, Université de Lorraine, CNRS, Inria, LORIA; MÉLINA SKOURAS, Université Grenoble Alpes, Inria, CNRS, Grenoble INP, LIG; CHRISTIAN SCHUMACHER, ETH Zurich; SAMUEL HORNIG, Université de Lorraine, CNRS, Inria, LORIA; SYLVAIN LEFEBVRE, Université de Lorraine, CNRS, Inria, LORIA; and BERNHARD THOMASZEWski, Université de Montréal.
- Worst-Case Stress Relief for Microstructures** by JULIAN PANETTA, ARYIN RAHMANI, and DENIS ZORIN, New York University.
- Orthotropic k-nearest foams for additive manufacturing** by JONAS MARTINEZ and HAICHUAN SONG, INRIA; JÉRÉMIE DUMAS, Université de Lorraine, INRIA; and SYLVAIN LEFEBVRE, INRIA, Université de Lorraine.

Diagrams include a circular diagram with a central dot and the Greek letter epsilon (ϵ), a 3D model of a chair-like structure with a grid overlay, and various microstructural patterns and stress distribution visualizations.

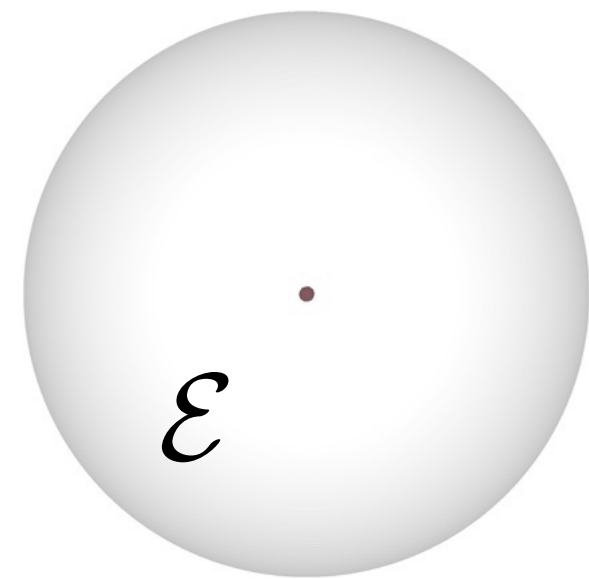
HOMOGENIZATION

Past Works

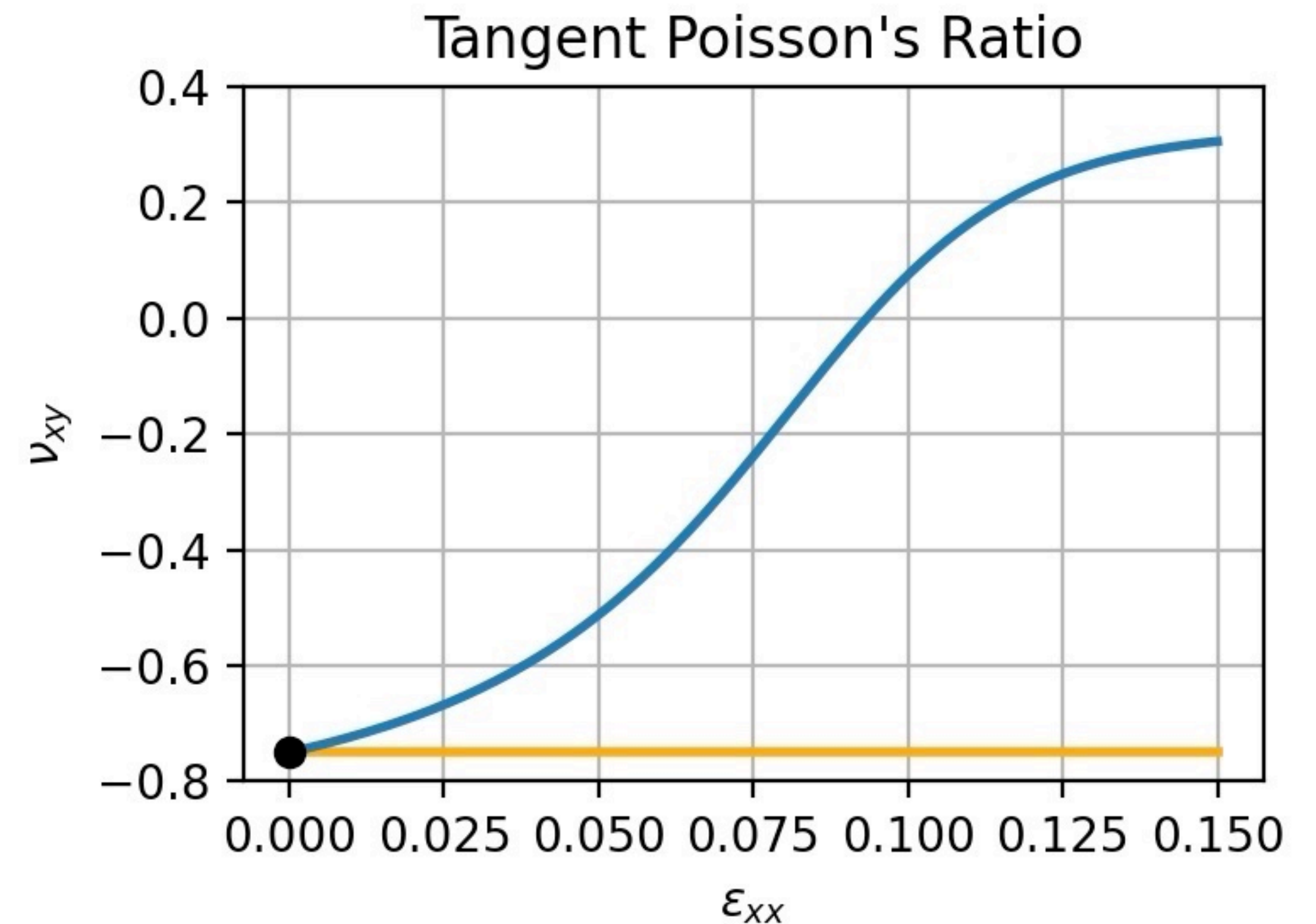
Infinitely small deformation

$$\bar{\psi}(\bar{F}) := \min_{\omega \text{ periodic}} \frac{1}{|Y|} \int_{\Omega} \psi(\nabla \omega + \bar{F}) d\mathbf{X}$$

 ψ Linear Elasticity



\mathcal{E}
{origin}

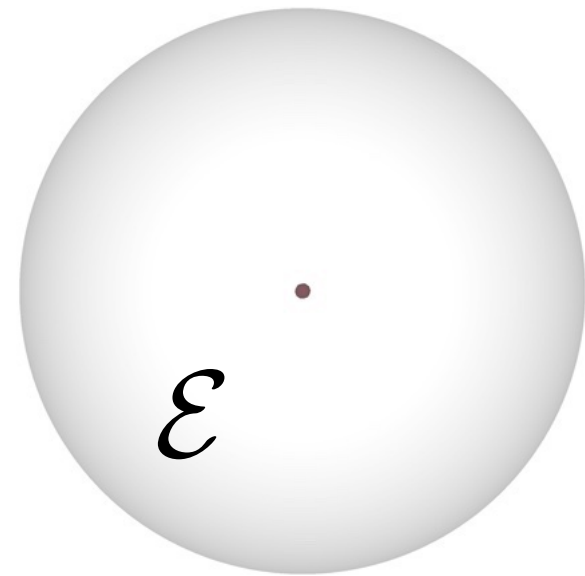


[Neves et al. 2000]

HOMOGENIZATION

Past Works

Infinitely small deformation



\mathcal{E}
{*origin*}

[Neves et al. 2000]

$$\bar{\psi}(\bar{F}) := \min_{\omega \text{ periodic}} \frac{1}{|Y|} \int_{\Omega} \psi(\nabla \omega + \bar{F}) dX$$

 ψ **Nonlinear Elasticity Model**

- Corotated
- Saint Venant-Kirchhoff
- Neo-Hookean
- ...

Flexible

HOMOGENIZATION

Past Works

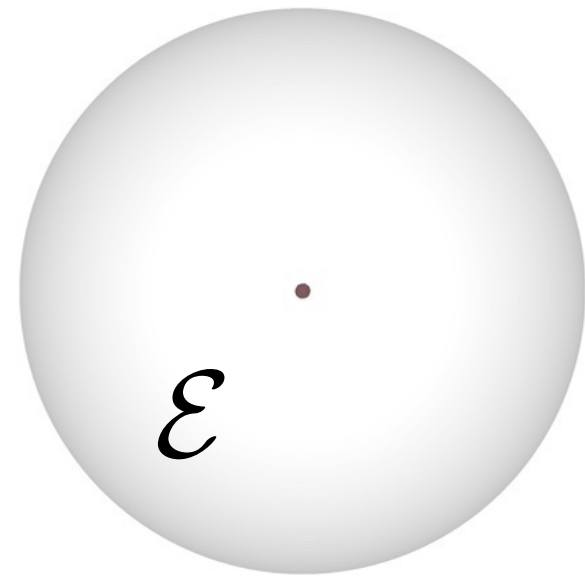
Flexible

Infinitely small deformation

A few sampled biaxial strains

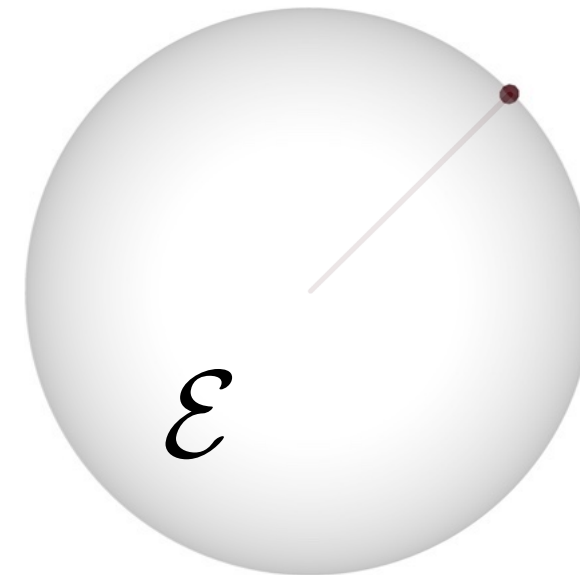
Along uniaxial stretch path

Trajectories through strain space



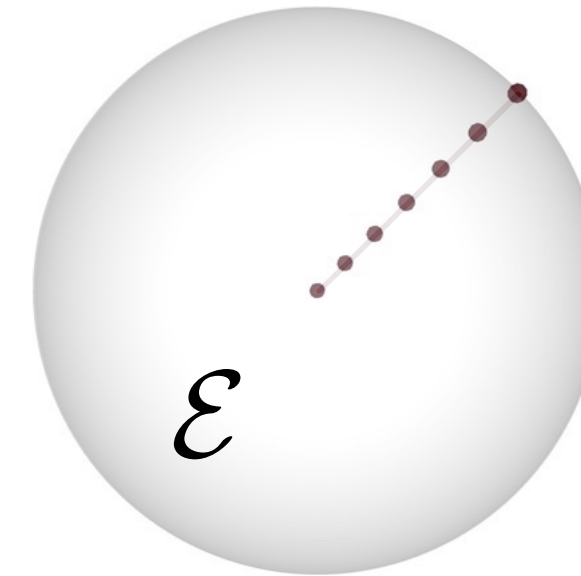
\mathcal{E}
{*origin*}

[Neves et al. 2000]



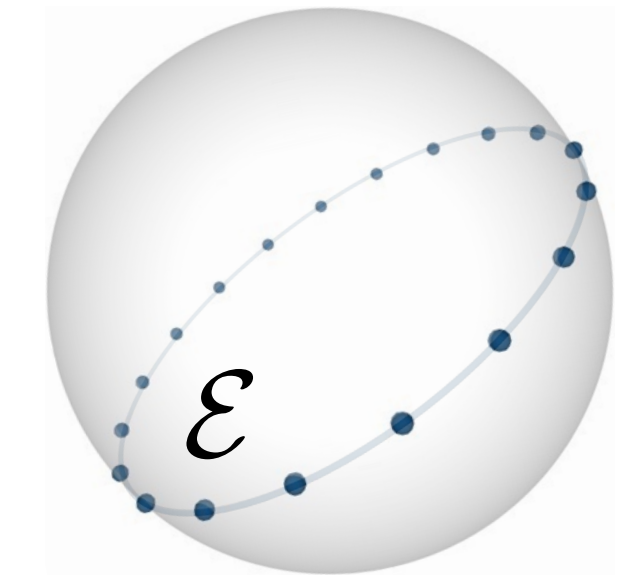
\mathcal{E}
{*points*}

[Behrou et al. 2021]



\mathcal{E}
{*a line*}

[Clausen et al. 2015]



\mathcal{E}
{*a circle*}

[Schumacher et al. 2018]

HOMOGENIZATION

Past Works

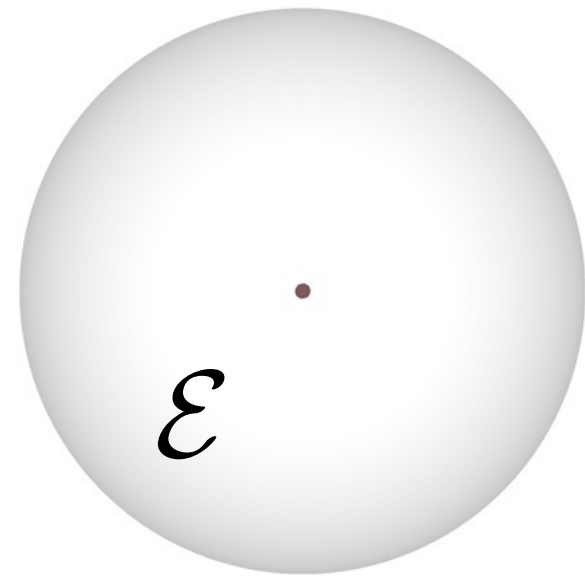
Flexible

Infinitely small deformation

A few sampled biaxial strain

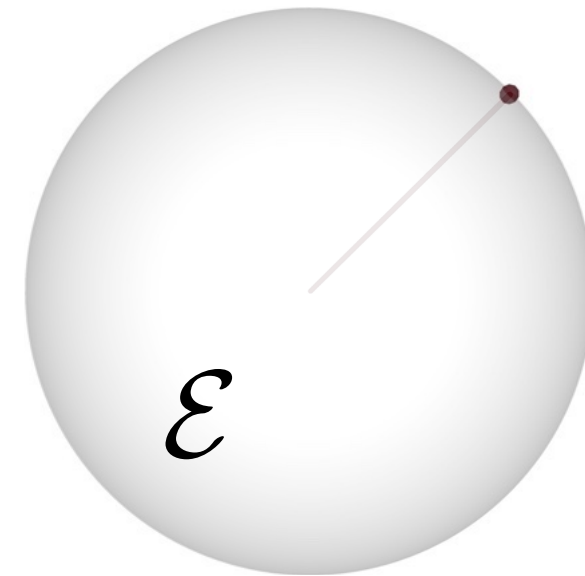
Along uniaxial stretch path

Trajectories through strain space



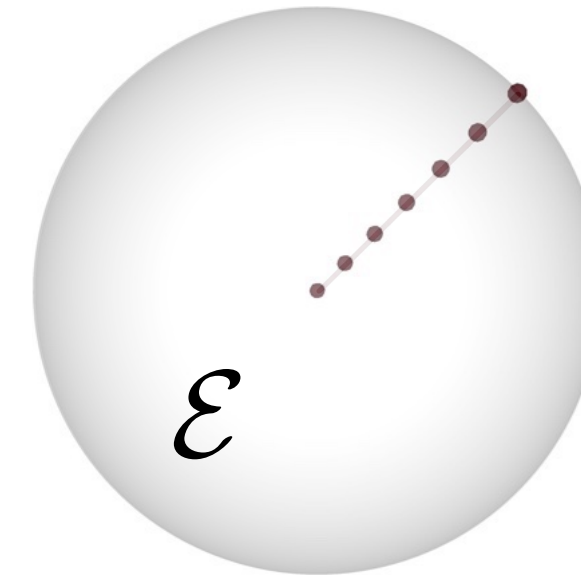
\mathcal{E}
{*origin*}

[Neves et al. 2000]



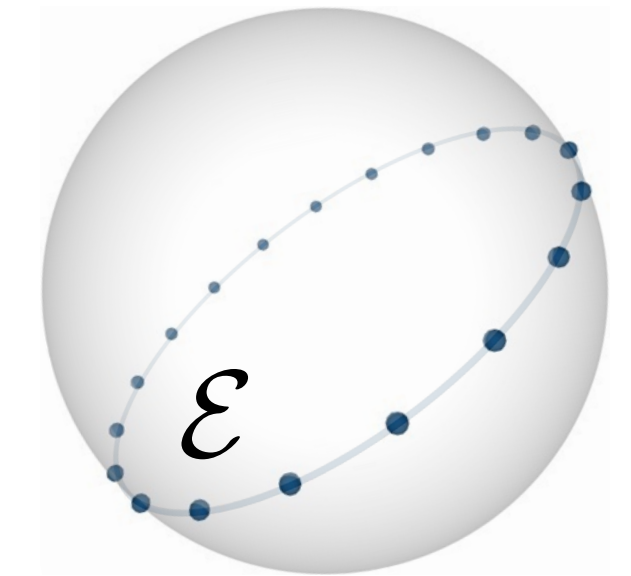
\mathcal{E}
{*points*}

[Behrou et al. 2021]



\mathcal{E}
{*a line*}

[Clausen et al. 2015]

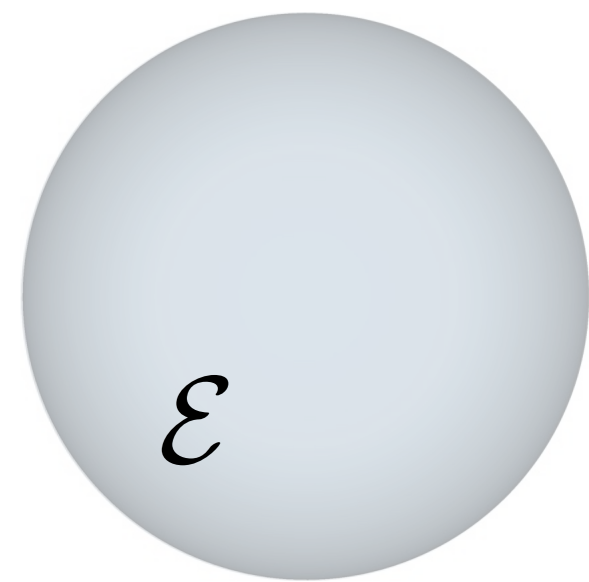


\mathcal{E}
{*a circle*}

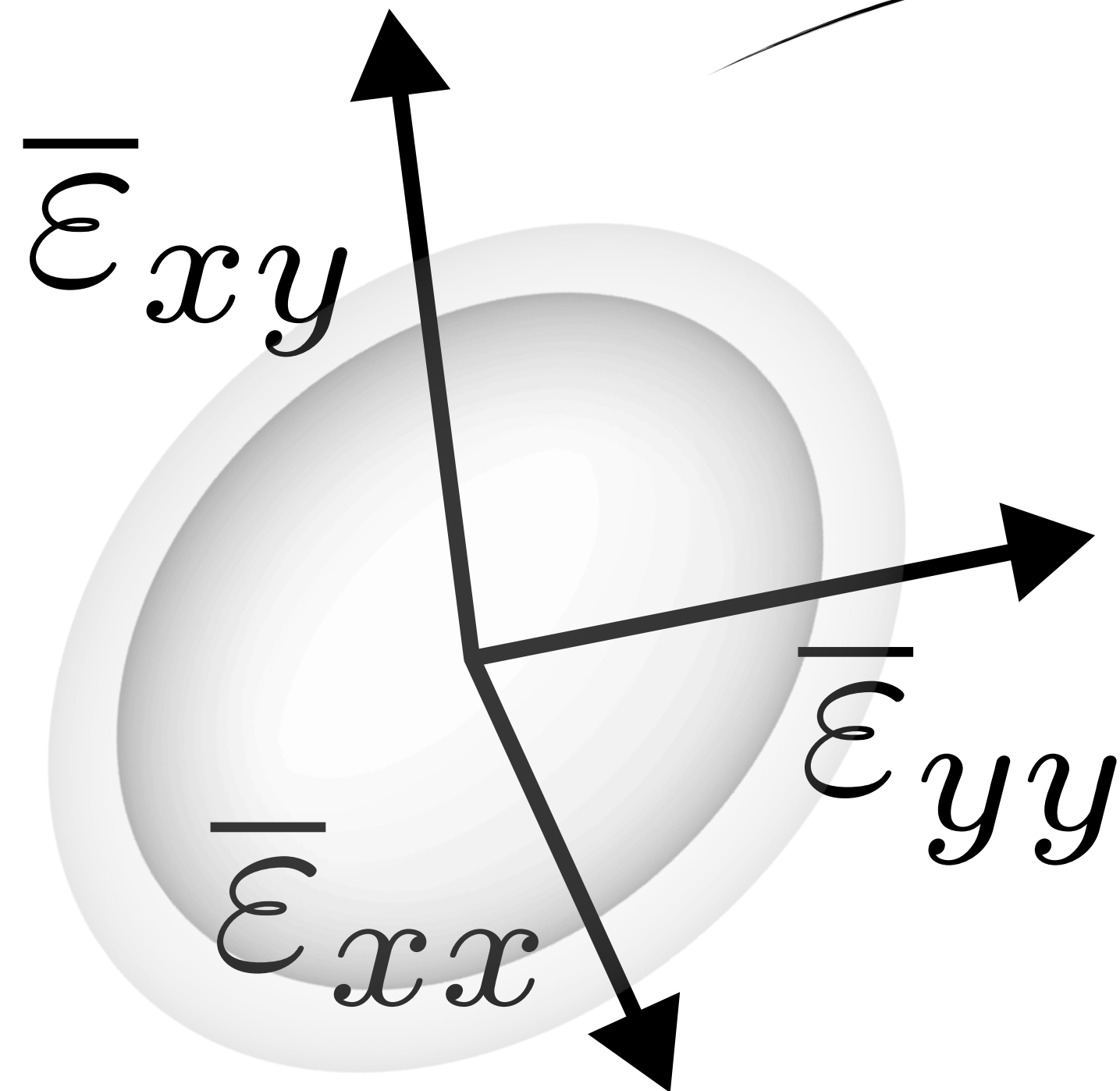
[Schumacher et al. 2018]

HOMOGENIZATION

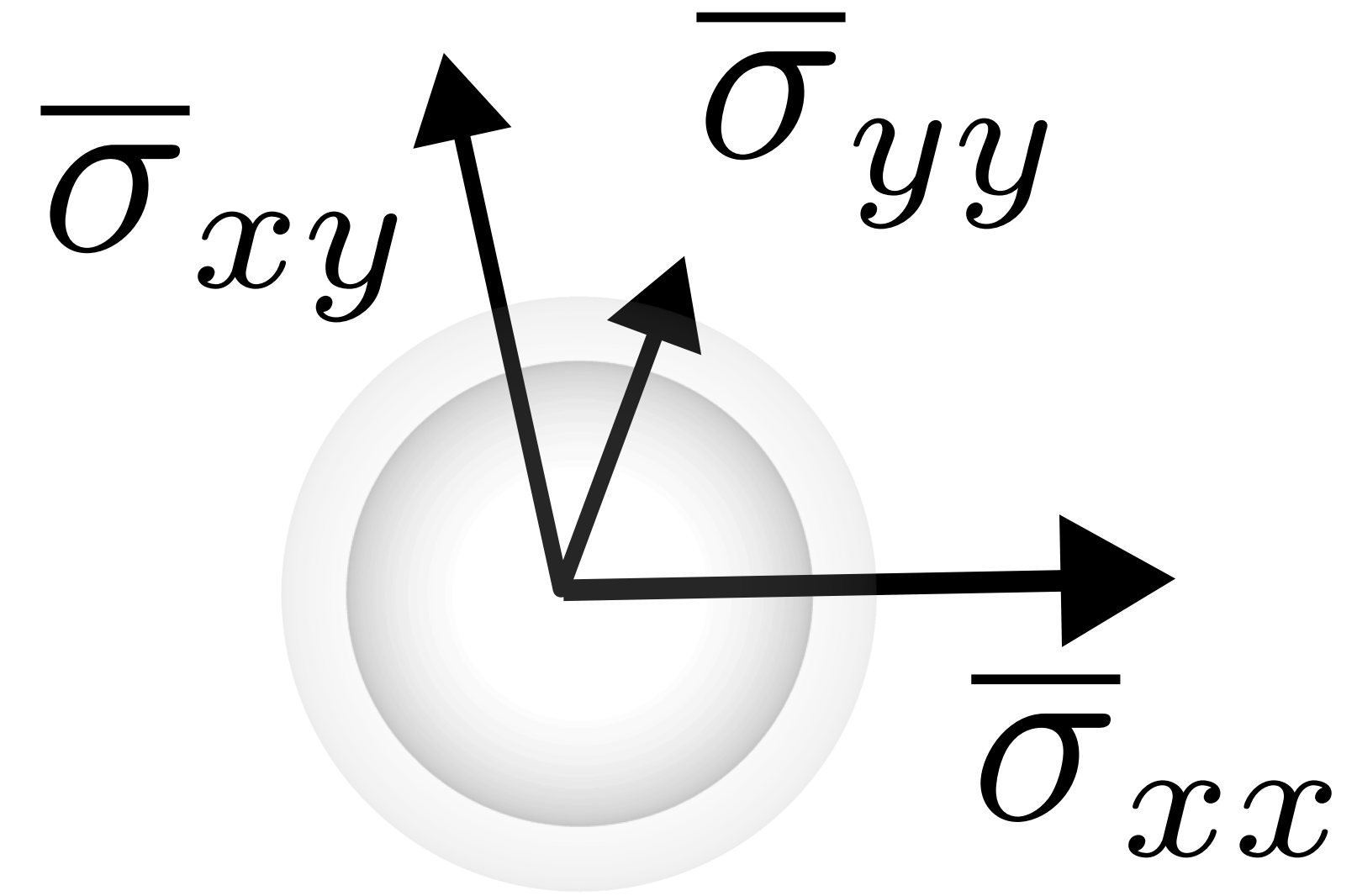
Choose Strain Domain



\mathcal{E}
{volume}



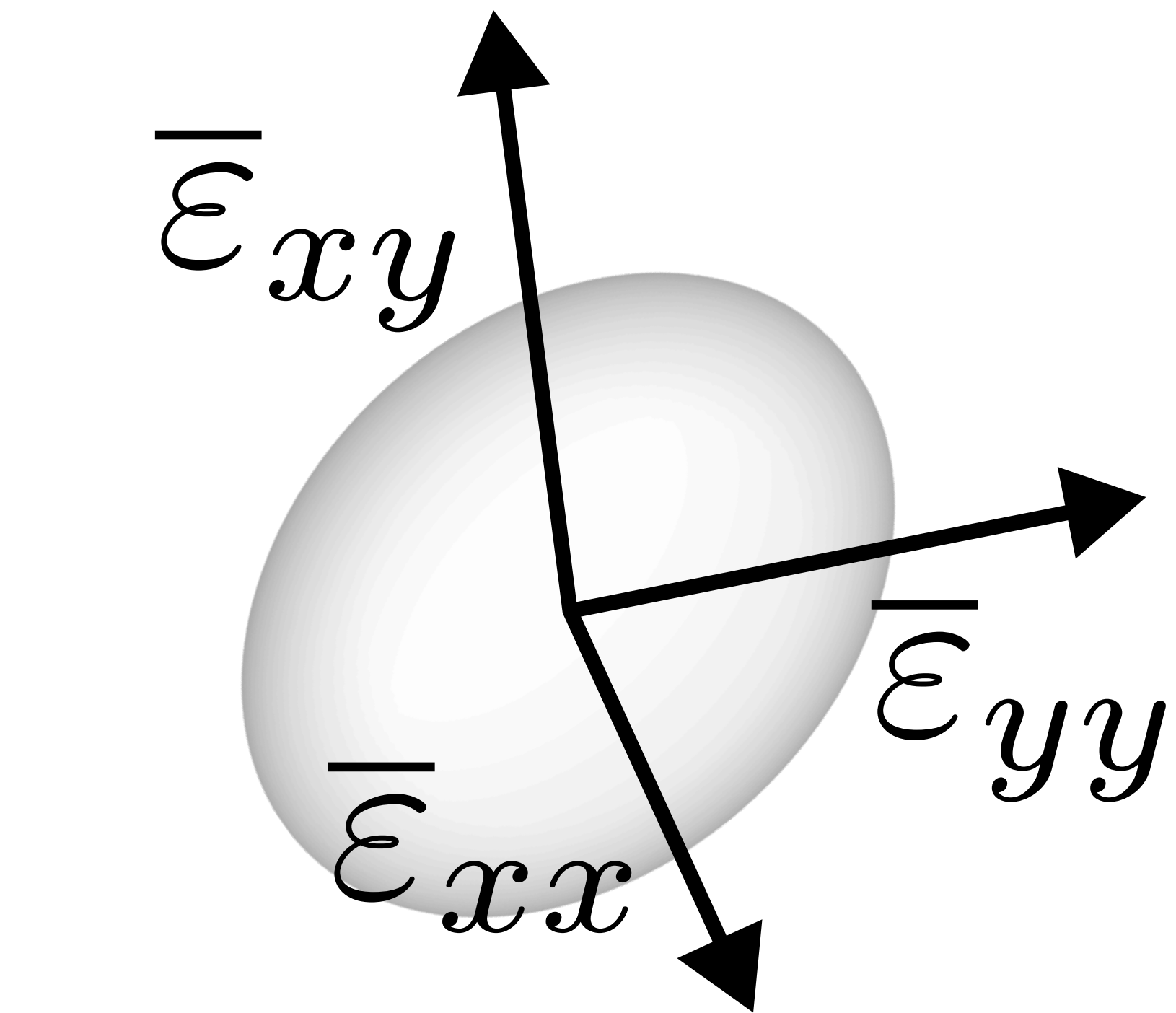
$\bar{\psi}'_{linear}$



Strain 10%
Strain 15%

HOMOGENIZATION

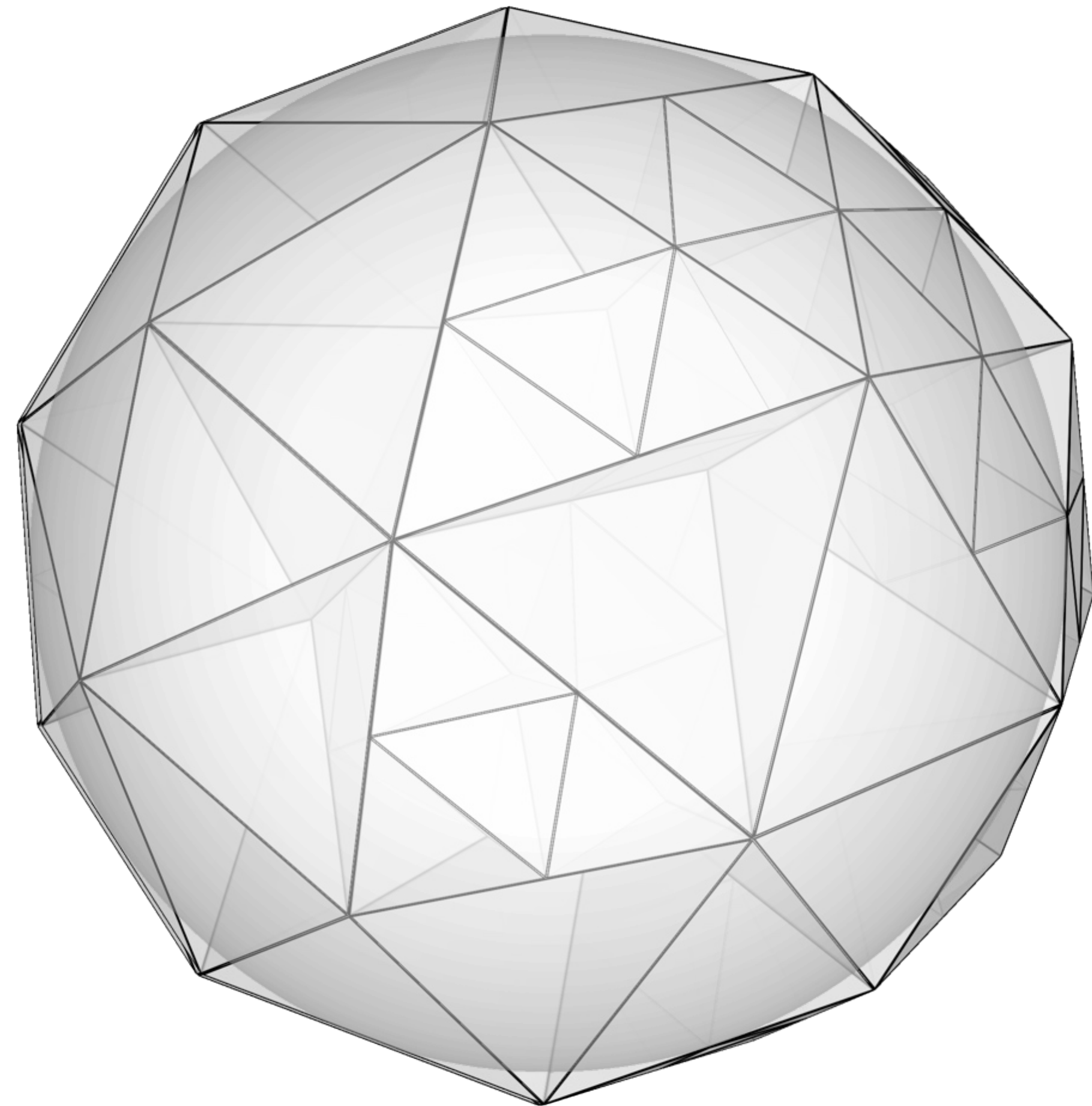
Chiral Domain



\mathcal{E}

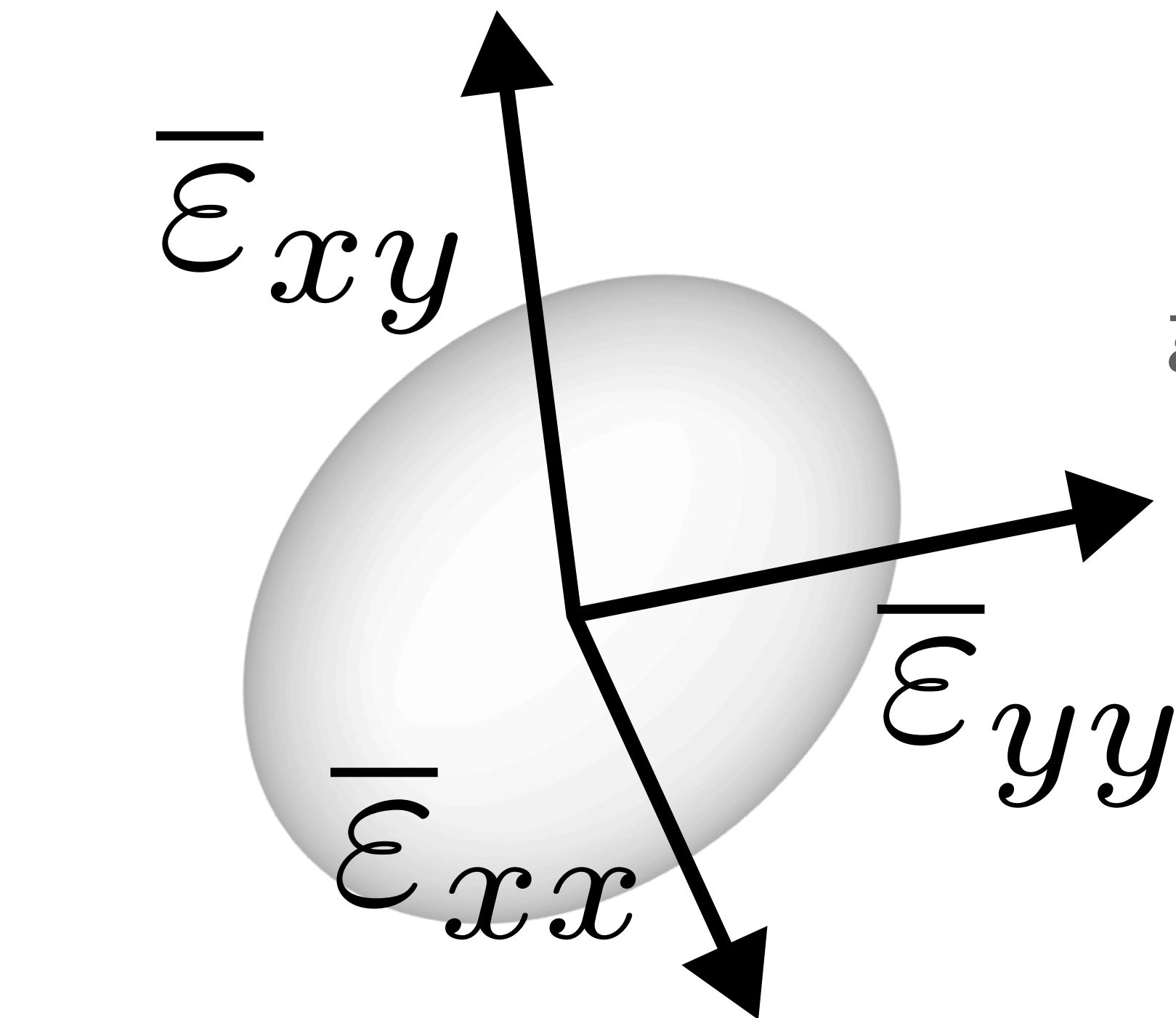
Strain 10%

Adaptive Subdivision



HOMOGENIZATION

Interpolation



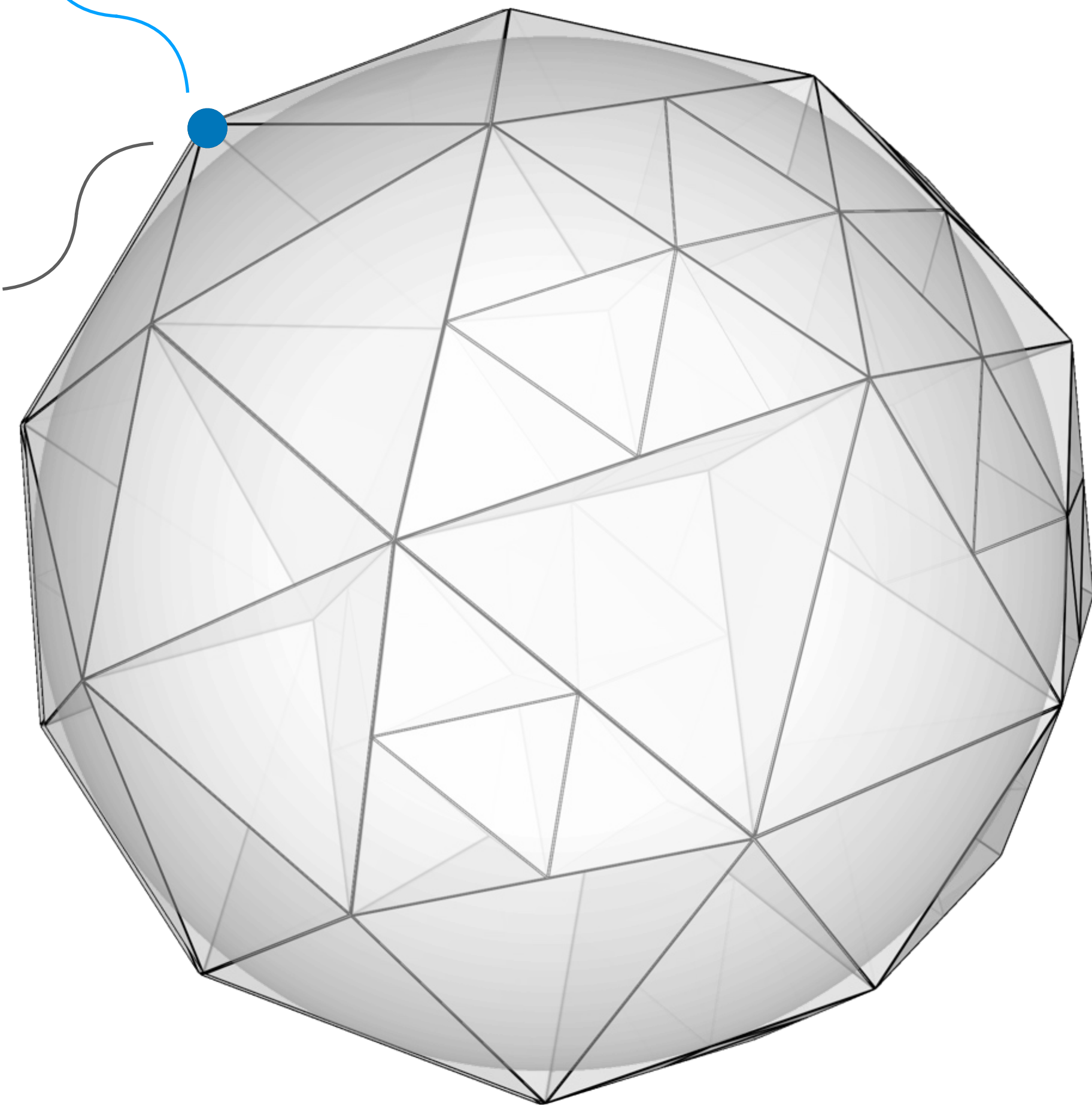
\mathcal{E}

Strain 10%

Interpolate displacement field ω^*

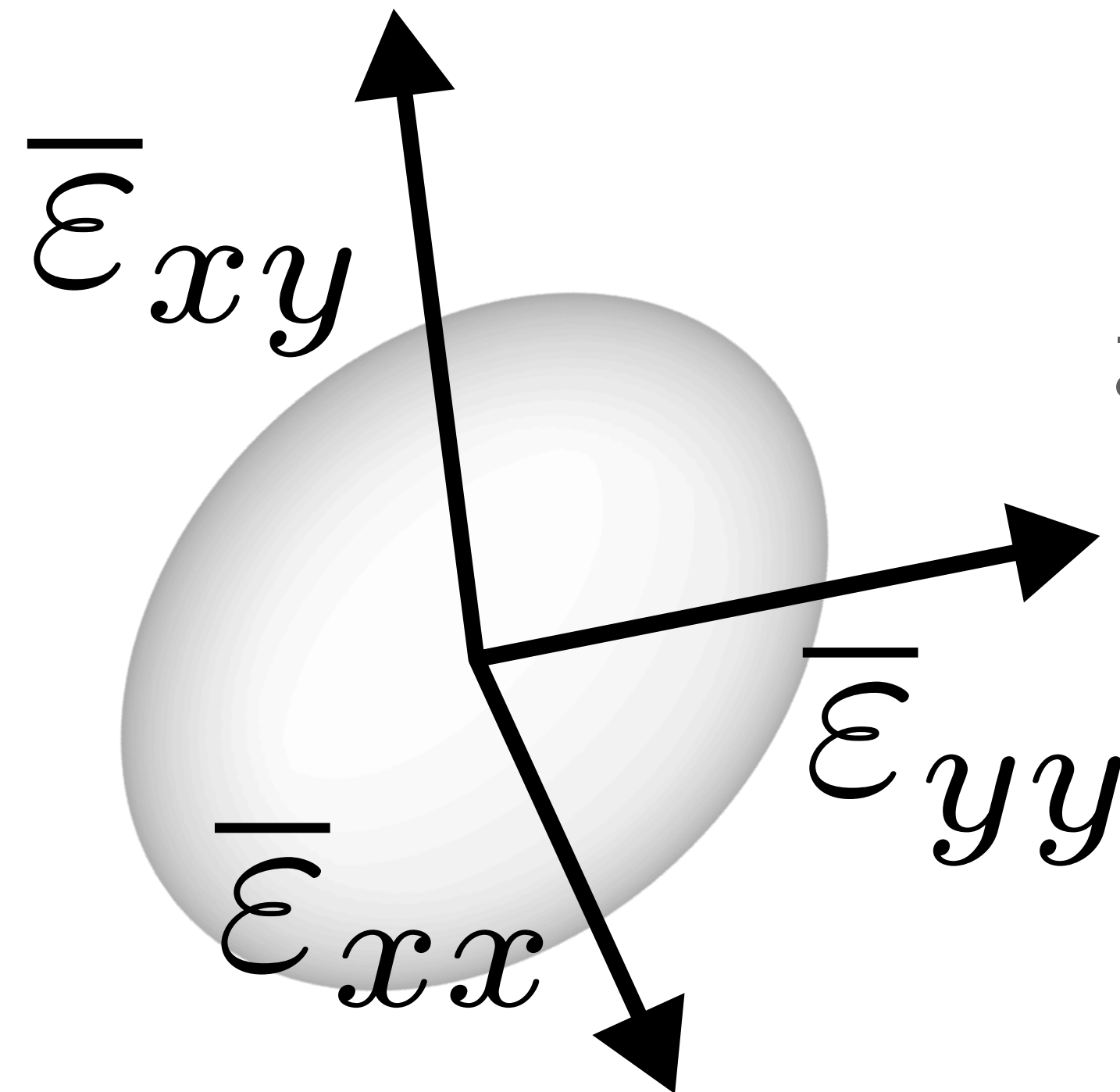
$$\omega^* = \arg \min_{\omega \text{ periodic}} \frac{1}{|Y|} \int_{\Omega} \psi(\nabla \omega + \bar{F}) d\mathbf{X}$$

$$\bar{\epsilon} + I = \bar{F}$$



HOMOGENIZATION

Interpolation



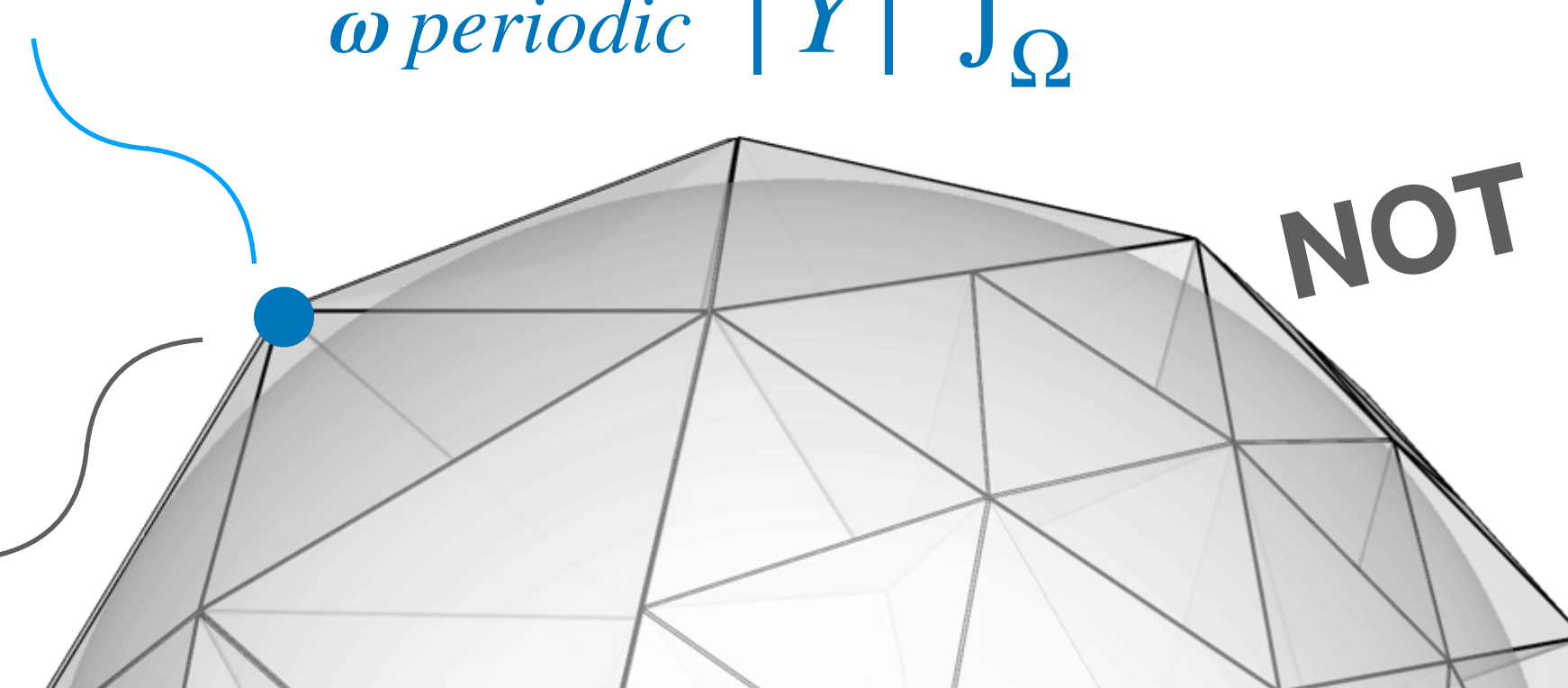
\mathcal{E}

Strain 10%

Interpolate displacement field ω^*

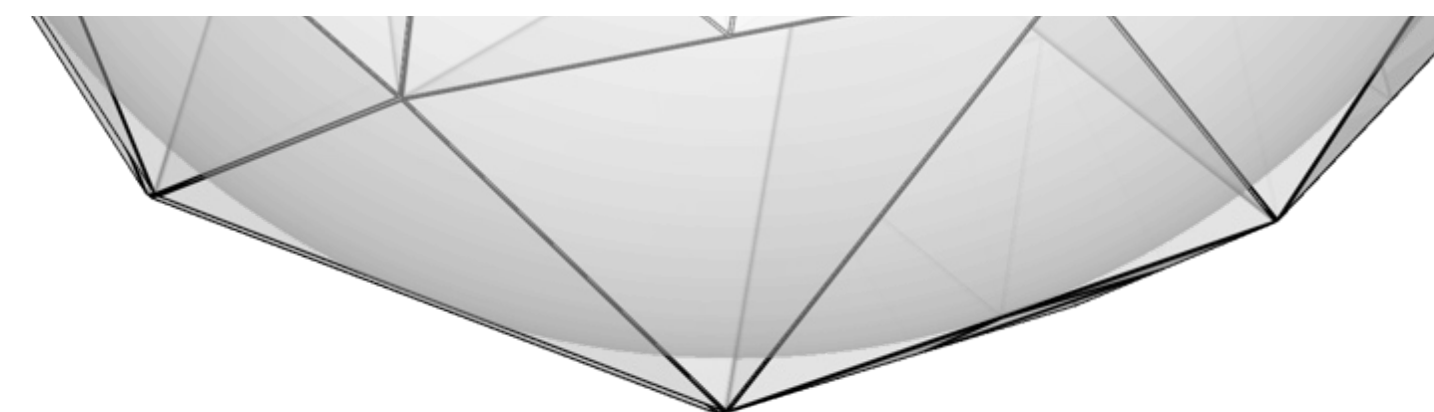
$$\omega^* = \arg \min_{\omega \text{ periodic}} \frac{1}{|Y|} \int_{\Omega} \psi(\nabla \omega + \bar{F}) dX$$

$$\bar{\epsilon} + I = \bar{F}$$



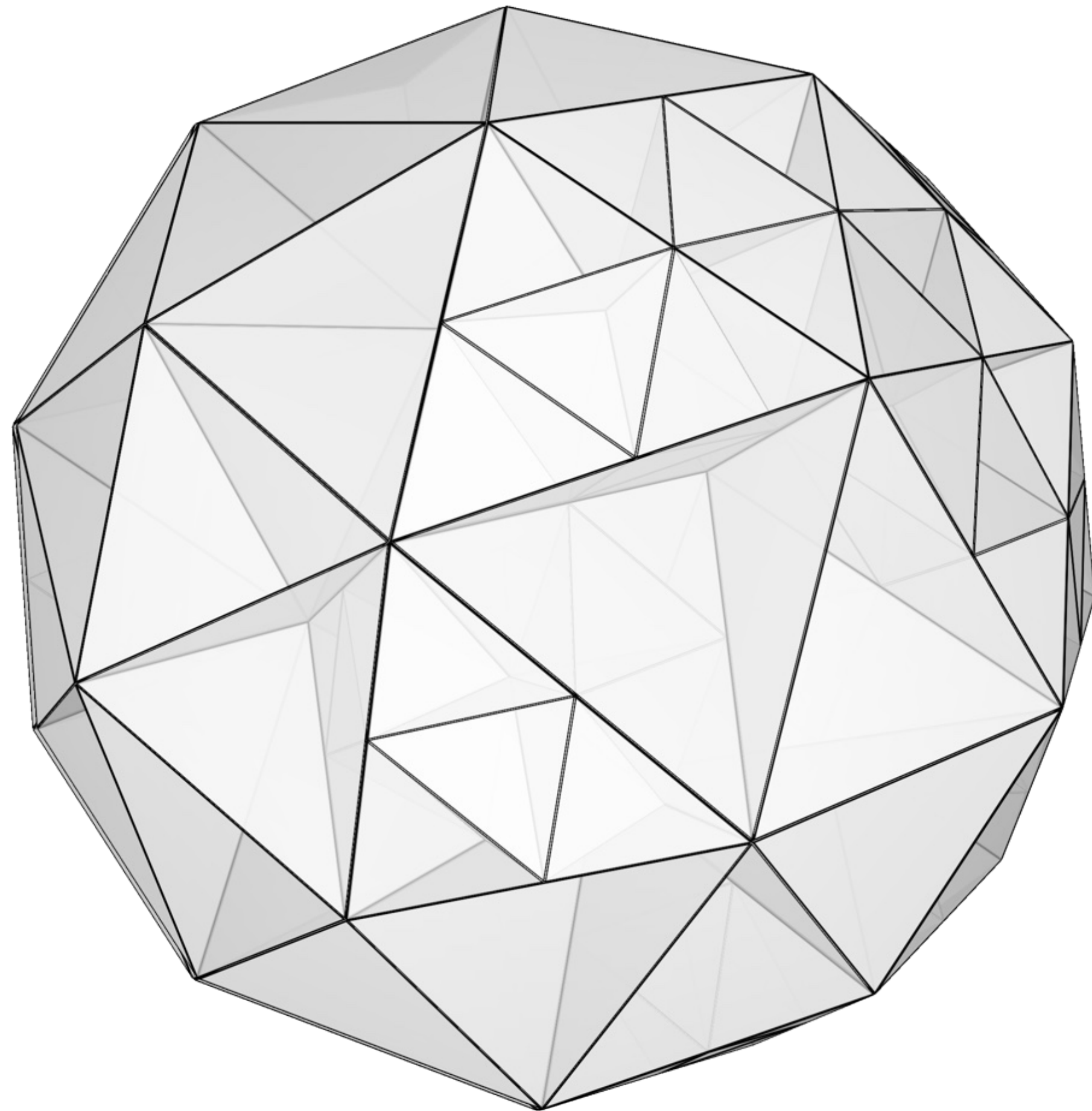
NOT energy $\bar{\Psi}$!

1. Efficient to evaluate the accuracy (no need for ground truth)
2. Accelerates nonlinear solves (provides high-quality initialization)



HOMOGENIZATION

Interpolation



Interpolate displacement field ω^*

$$\omega^* = \arg \min_{\omega \text{ periodic}} \frac{1}{|Y|} \int_{\Omega} \psi(\nabla \omega + \bar{F}) d\mathbf{X}$$

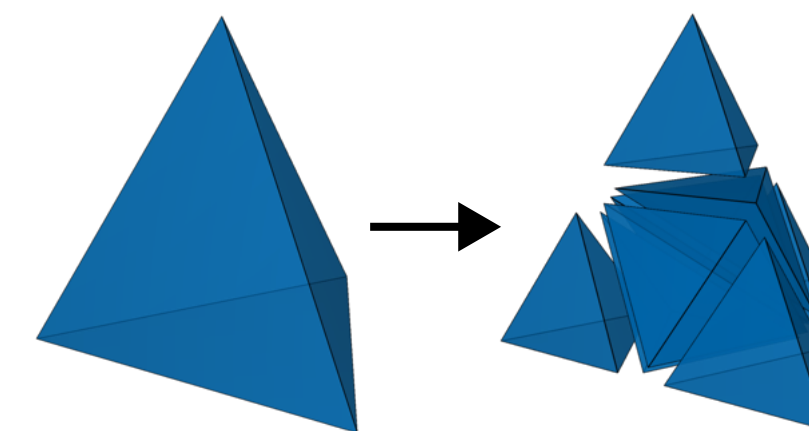
Linear Interpolation

ω^*

C^0

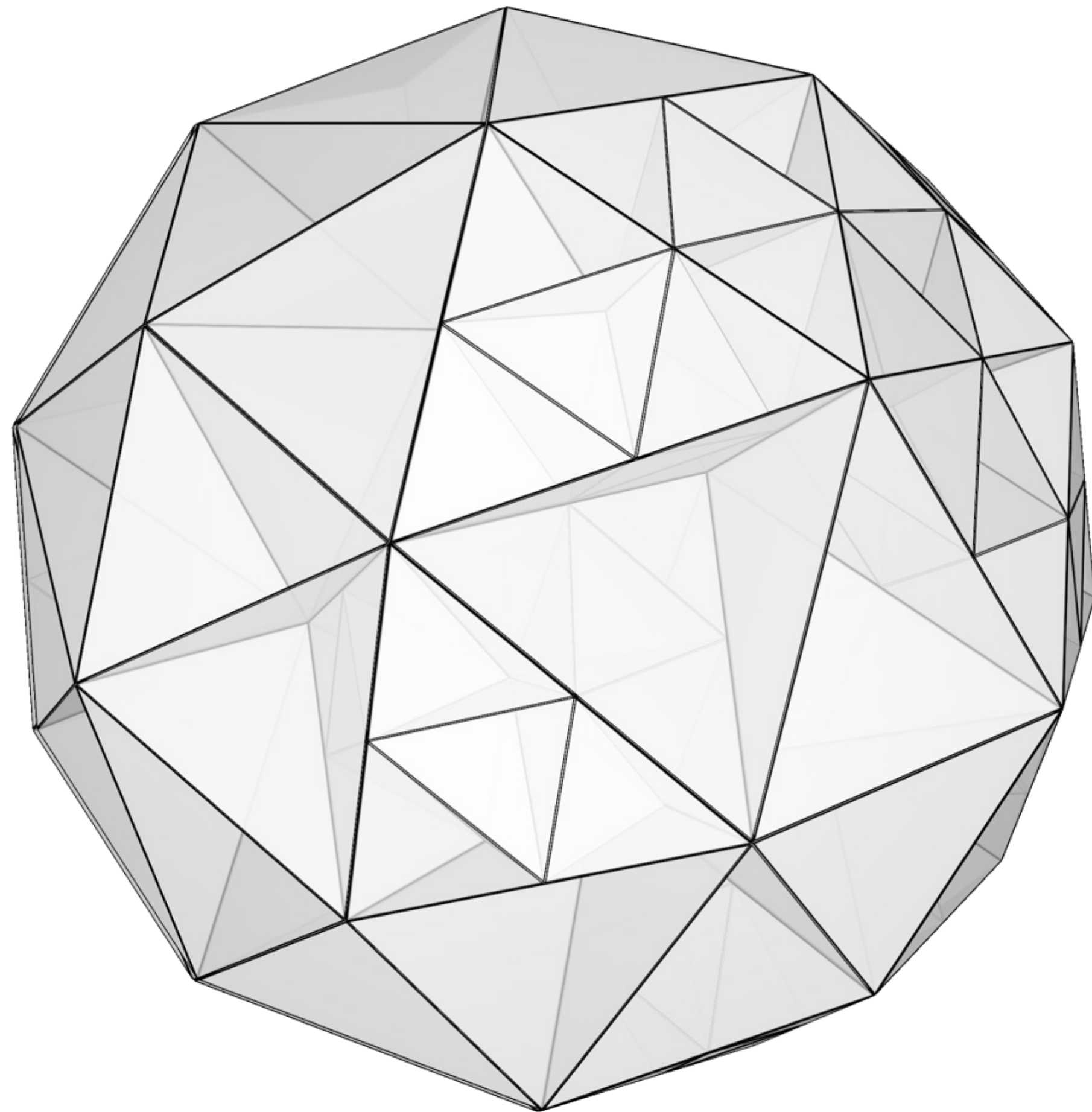
Evaluate at every center of tet

Check for the residual only



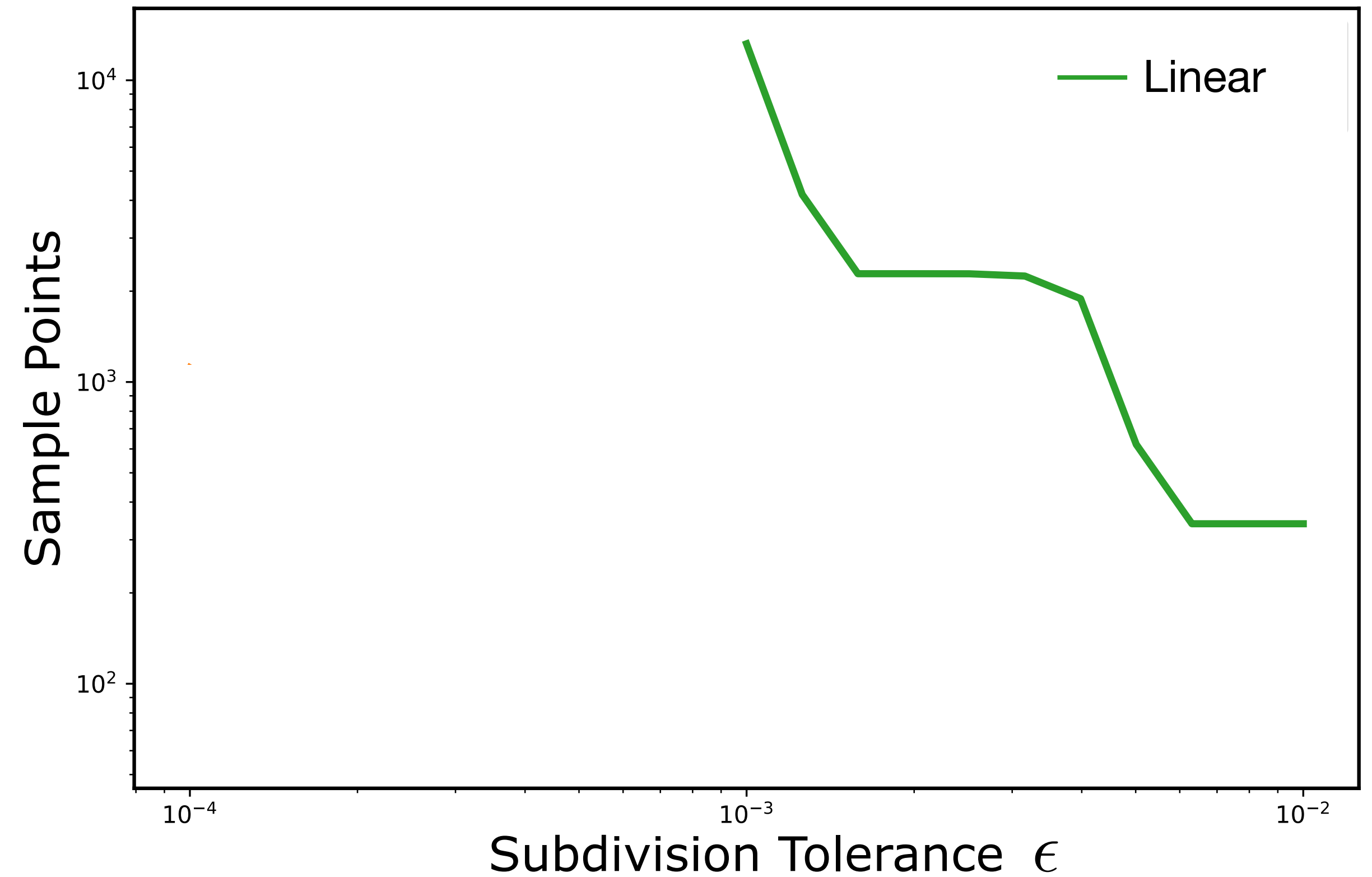
HOMOGENIZATION

Interpolation



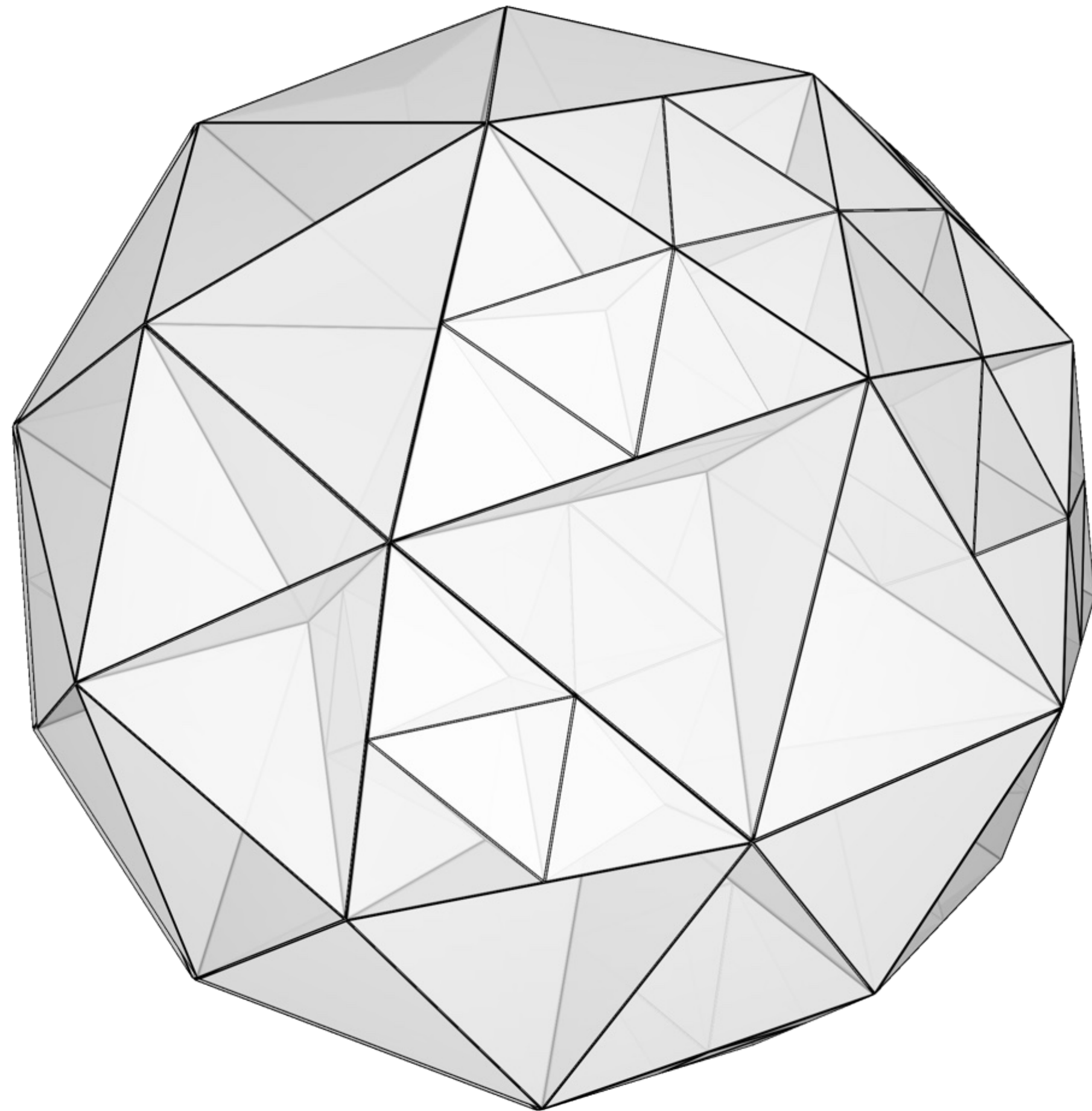
Interpolate displacement field ω^*

$$\omega^* = \arg \min_{\omega \text{ periodic}} \frac{1}{|Y|} \int_{\Omega} \psi(\nabla \omega + \bar{F}) d\mathbf{X}$$



HOMOGENIZATION

Interpolation



Interpolate displacement field ω^*

$$\omega^* = \arg \min_{\omega \text{ periodic}} \frac{1}{|Y|} \int_{\Omega} \psi(\nabla \omega + \bar{F}) d\mathbf{X}$$

Powell-Sabin Interpolation C^1

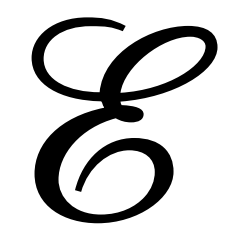
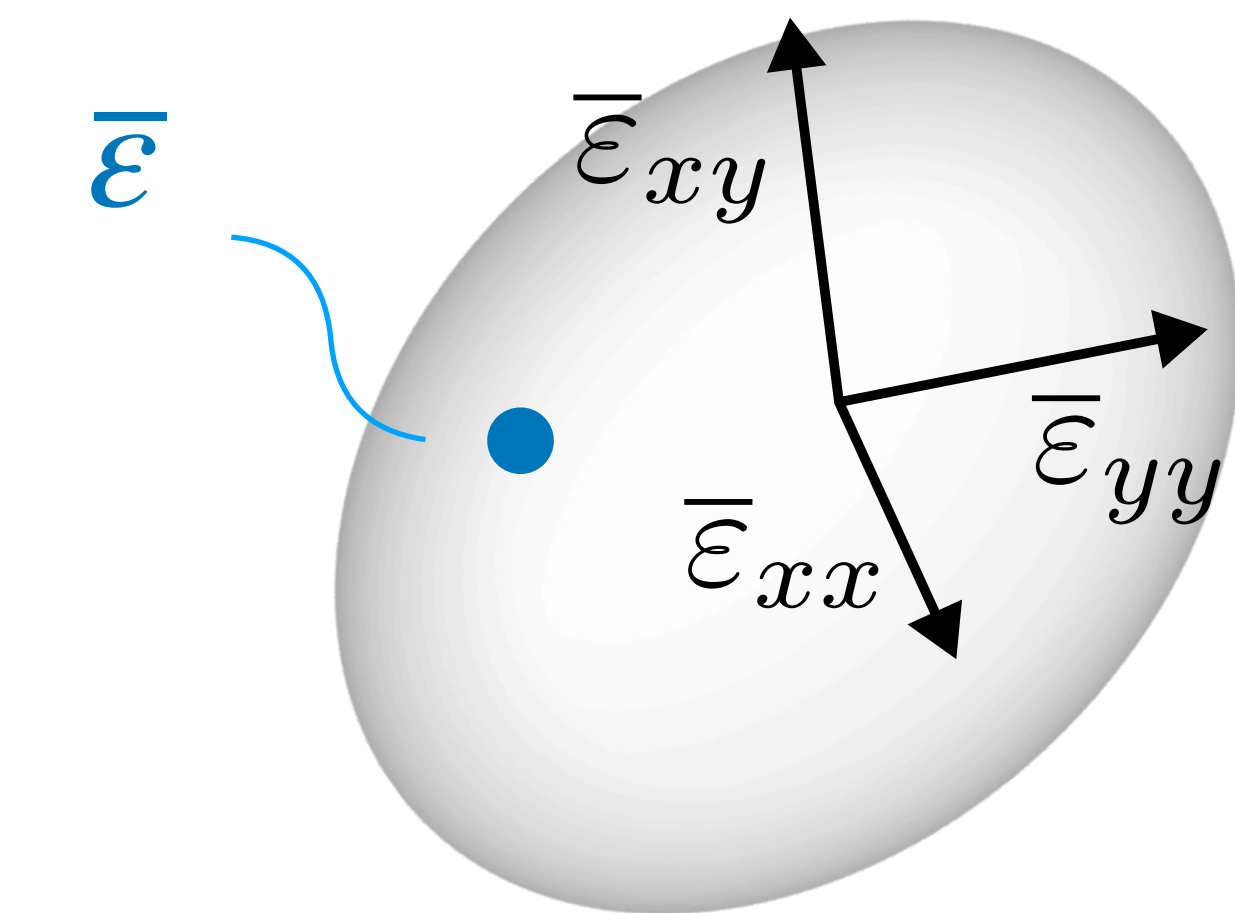
$$\left(\omega^*, \frac{\partial \omega^*}{\partial \bar{F}} \right)$$



derivative with respect to
the macroscopic strain

HOMOGENIZATION

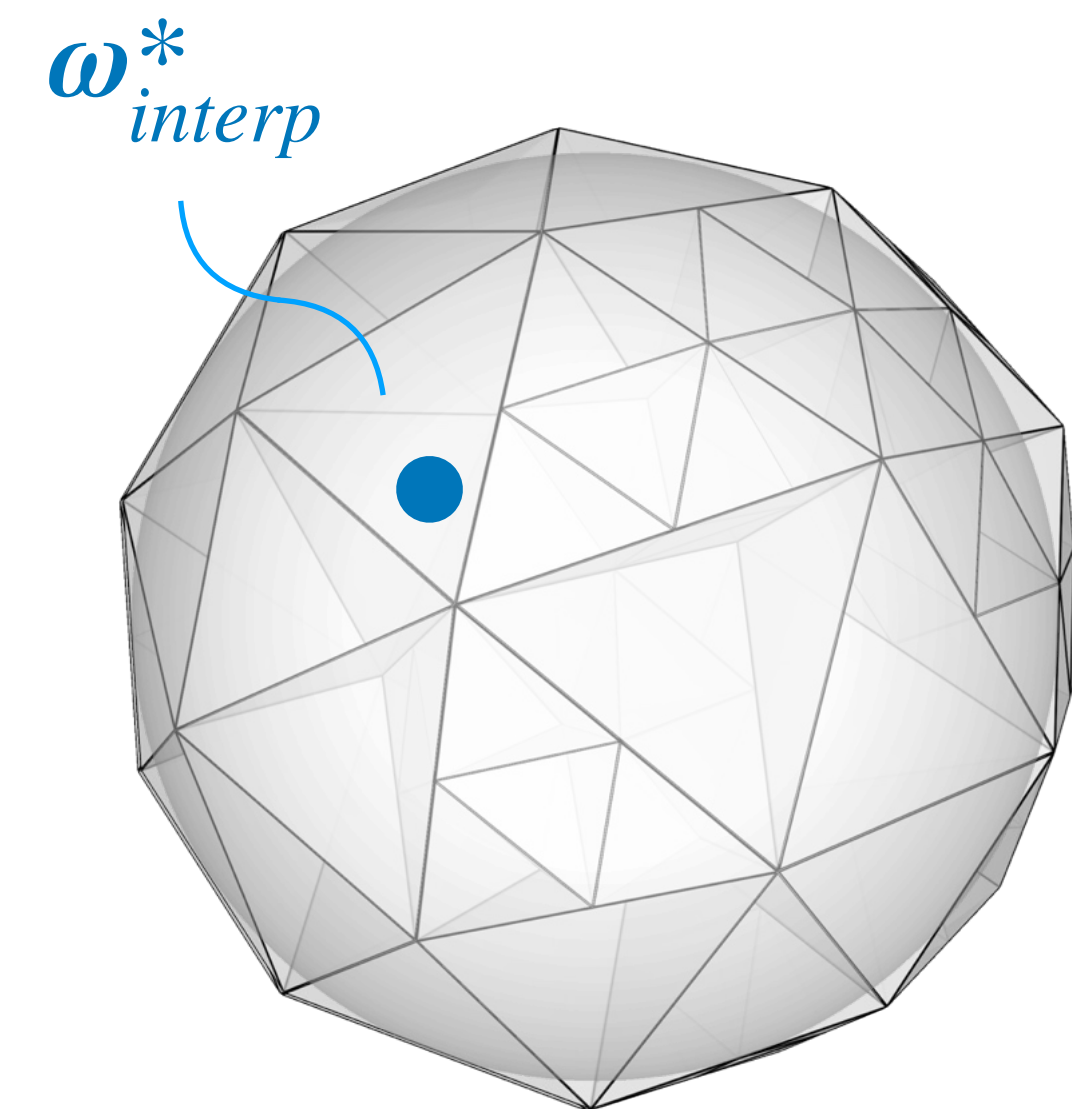
Interpolation



Strain 10%

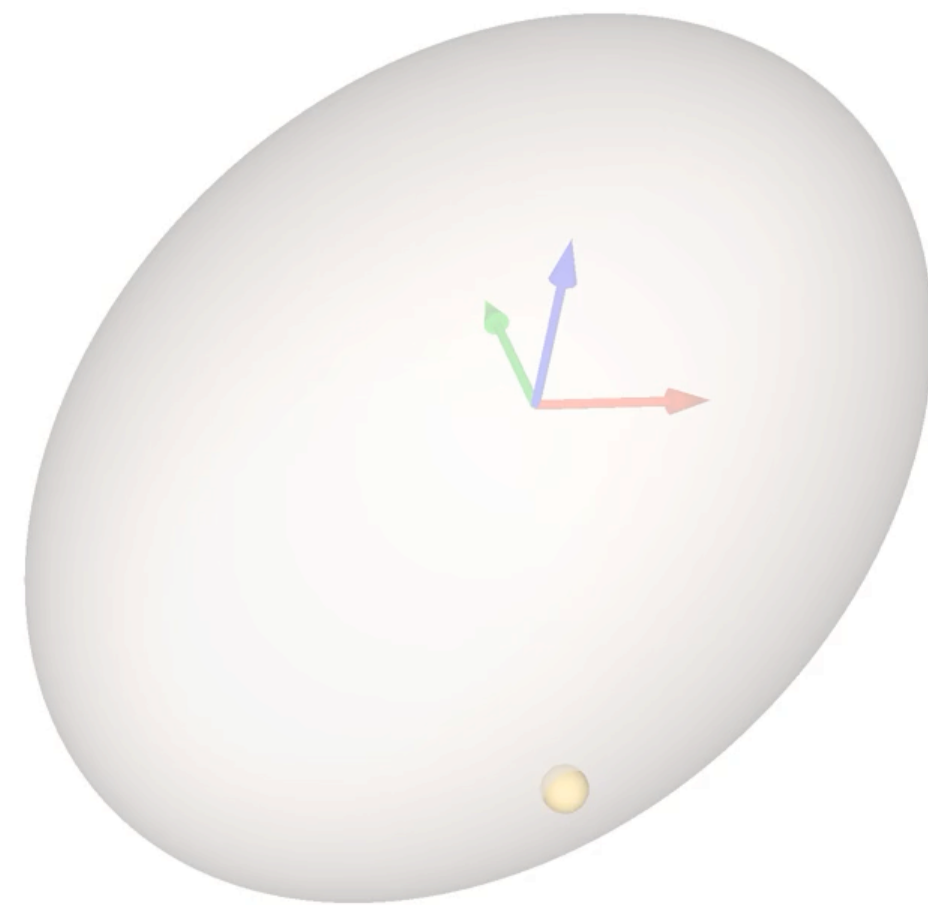


Adaptive Subdivision



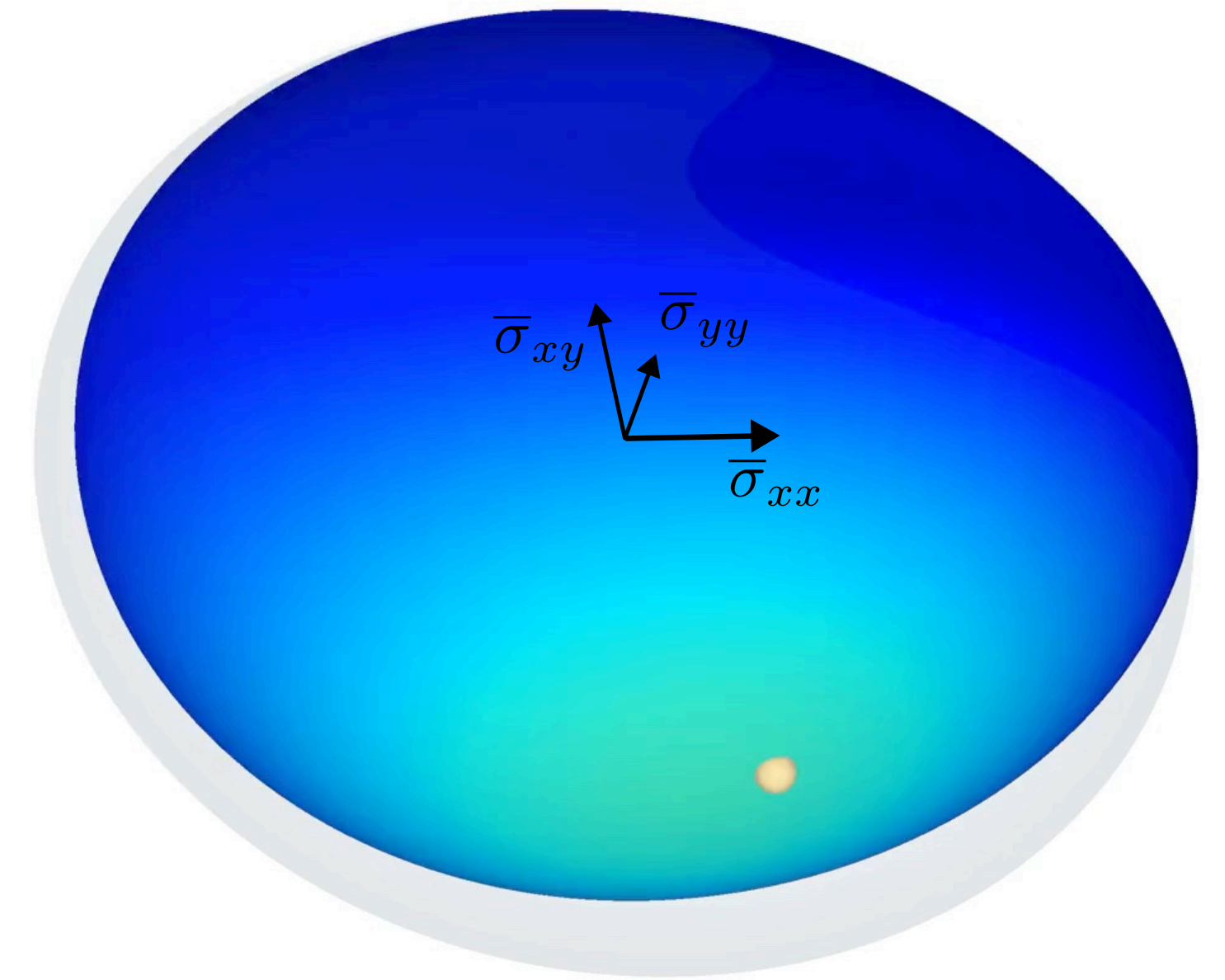
HOMOGENIZATION

Interpolation



\mathcal{E}

Strain 10%



Stress Domain

DESIGN PROBLEM

$\bar{\Psi}$



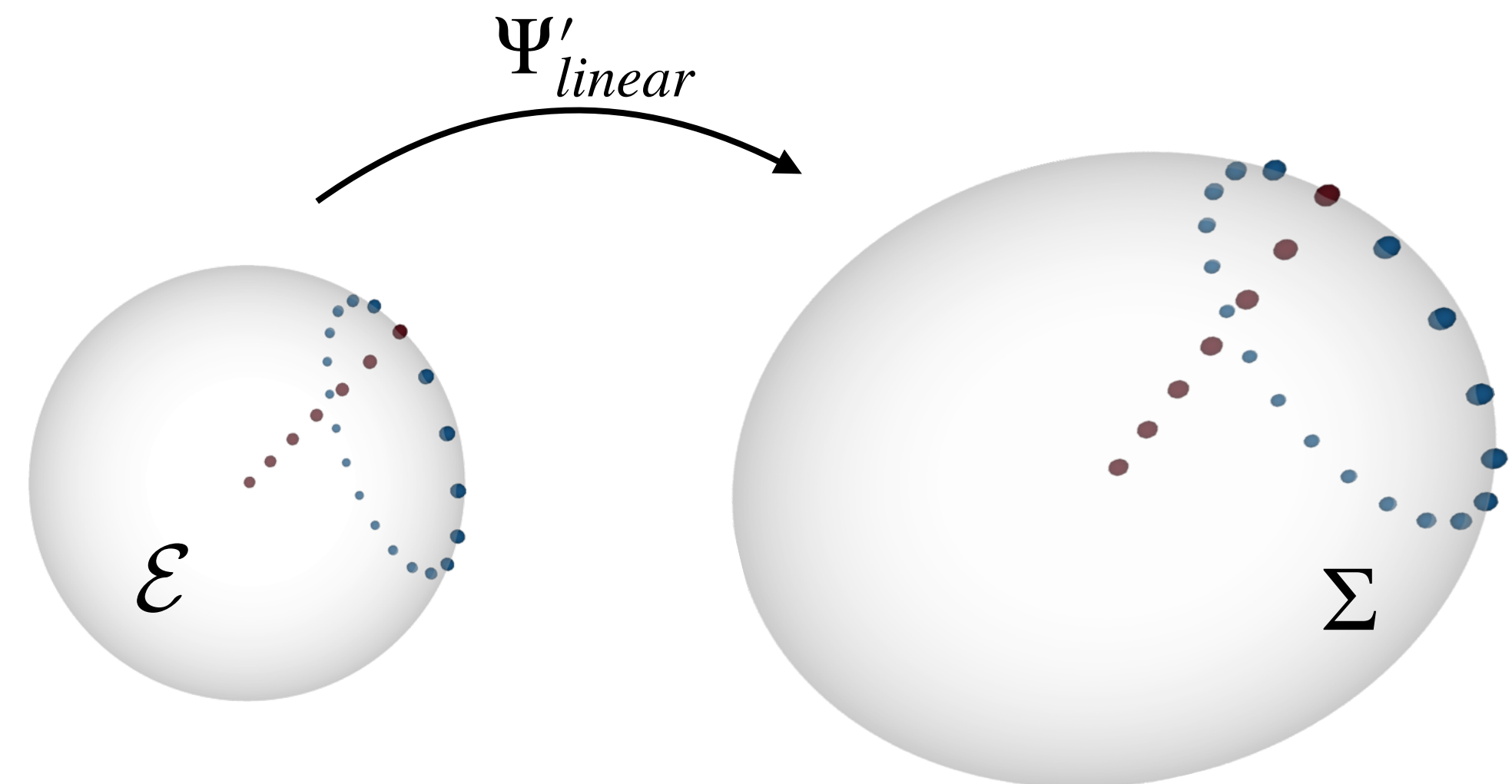
$\bar{\Psi}_{tgt}$

✳ $\bar{\Psi}_{tgt}$ Linear Elasticity

$$\Psi_{linear}(\bar{F}) = \frac{1}{2} \bar{\varepsilon} : C : \bar{\varepsilon}$$

$$\Psi'_{linear}(\bar{F}) = \bar{\sigma} = C : \bar{\varepsilon}$$

fourth-order elasticity tensor



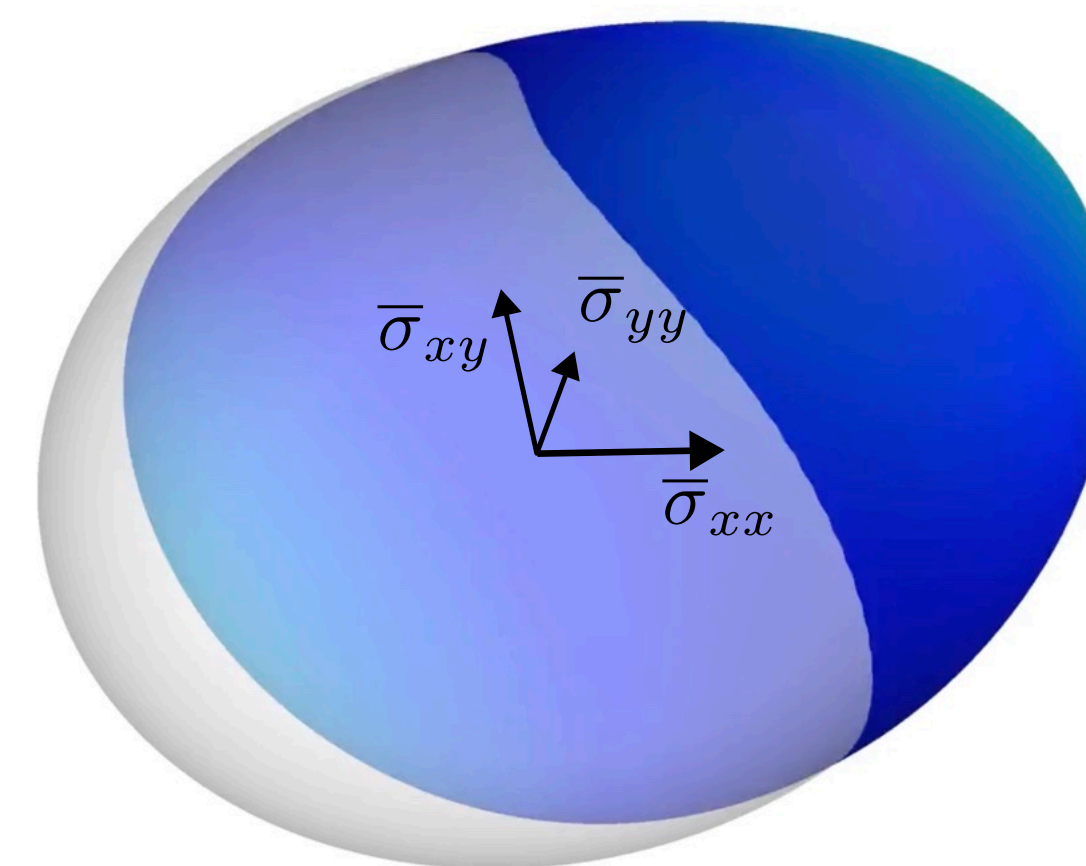
DESIGN PROBLEM

$\bar{\psi}$



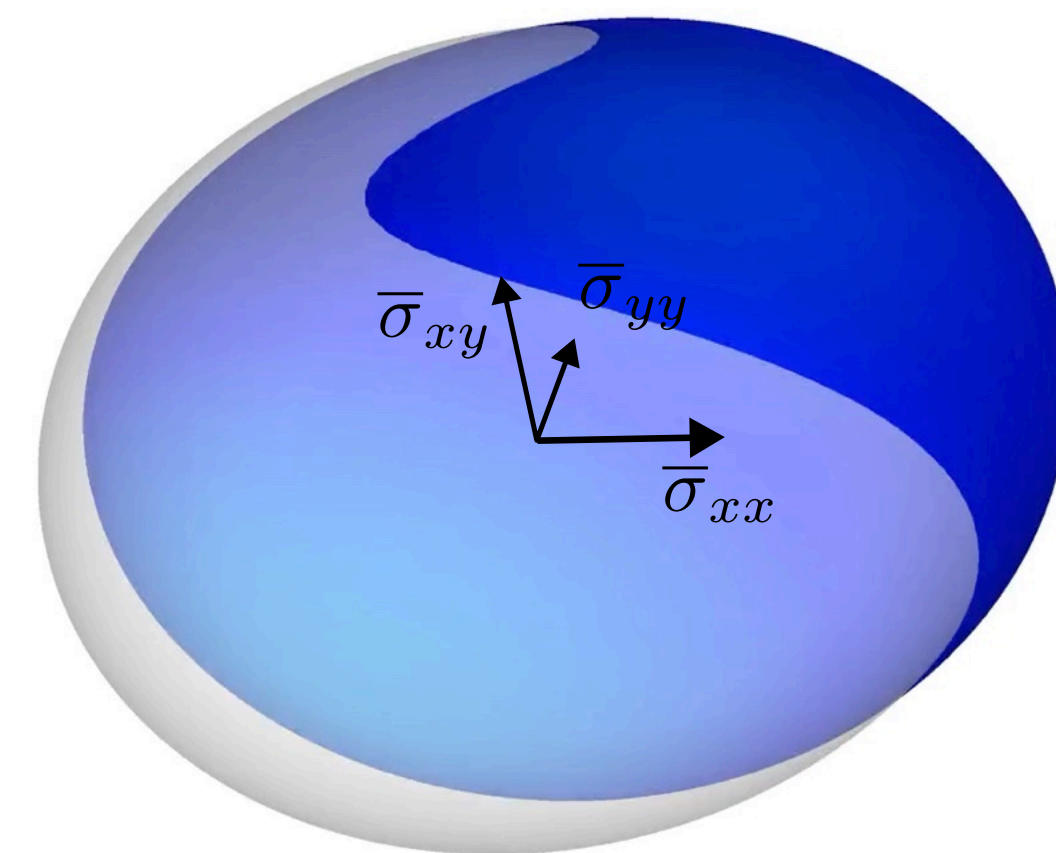
Change geometry Ω

$\bar{\psi}_{tgt}$



DESIGN PROBLEM

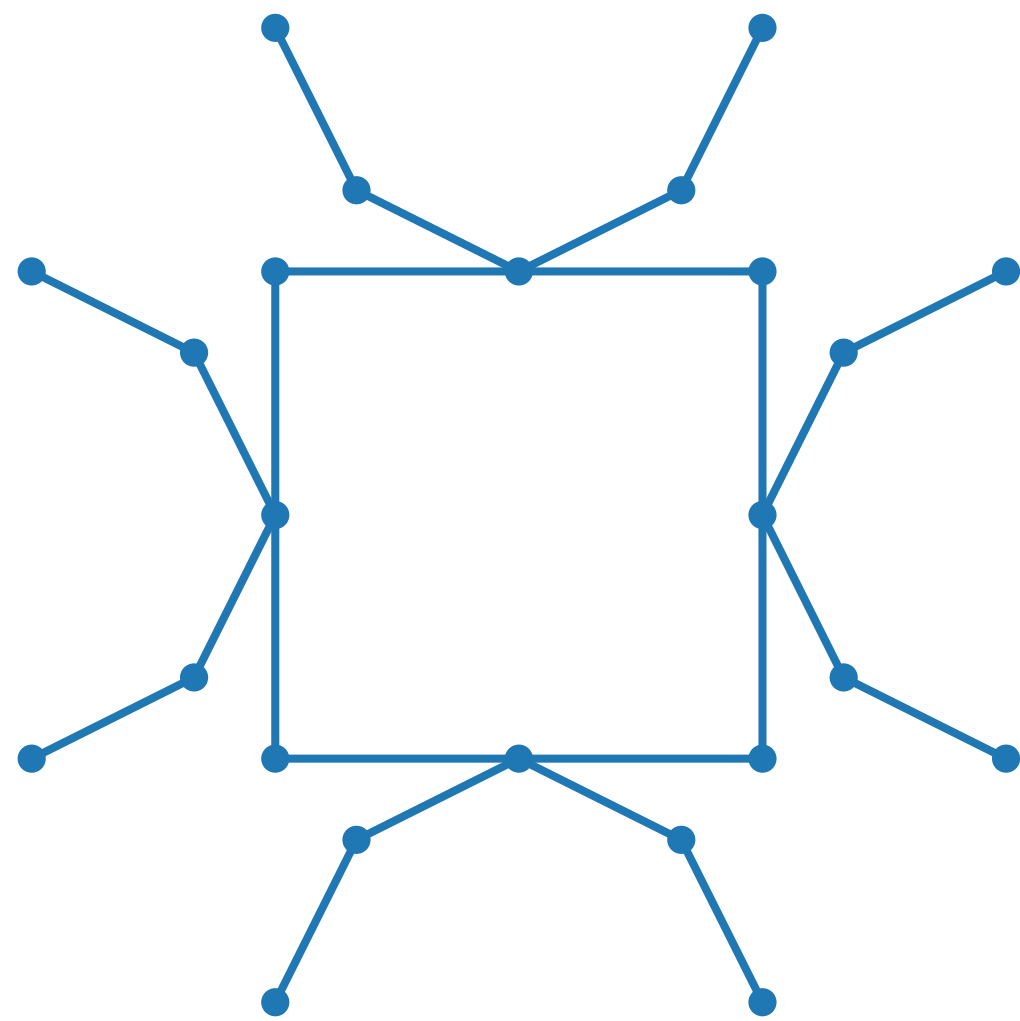
$$\min_{\Omega} \frac{1}{|\mathcal{F}|} \int_{\mathcal{F}} \mathbf{w}_e \frac{(\bar{\psi} - \bar{\psi}_{tgt})^2}{\bar{\psi}_{tgt}^2} + \mathbf{w}_\sigma \frac{\|\bar{\psi}' - \bar{\psi}'_{tgt}\|^2}{\|\bar{\psi}'_{tgt}\|^2} d\bar{F}$$



DESIGN PROBLEM

Shape Derivative

$$\min_{\Omega} \frac{1}{|\mathcal{F}|} \int_{\mathcal{F}} \mathbf{w}_e \left(\bar{\psi} - \bar{\psi}_{tgt} \right)_{rel}^2 + \mathbf{w}_{\sigma} \left\| \bar{\psi}' - \bar{\psi}'_{tgt} \right\|_{rel}^2 d\bar{F}$$



Topology



$\Omega(p)$

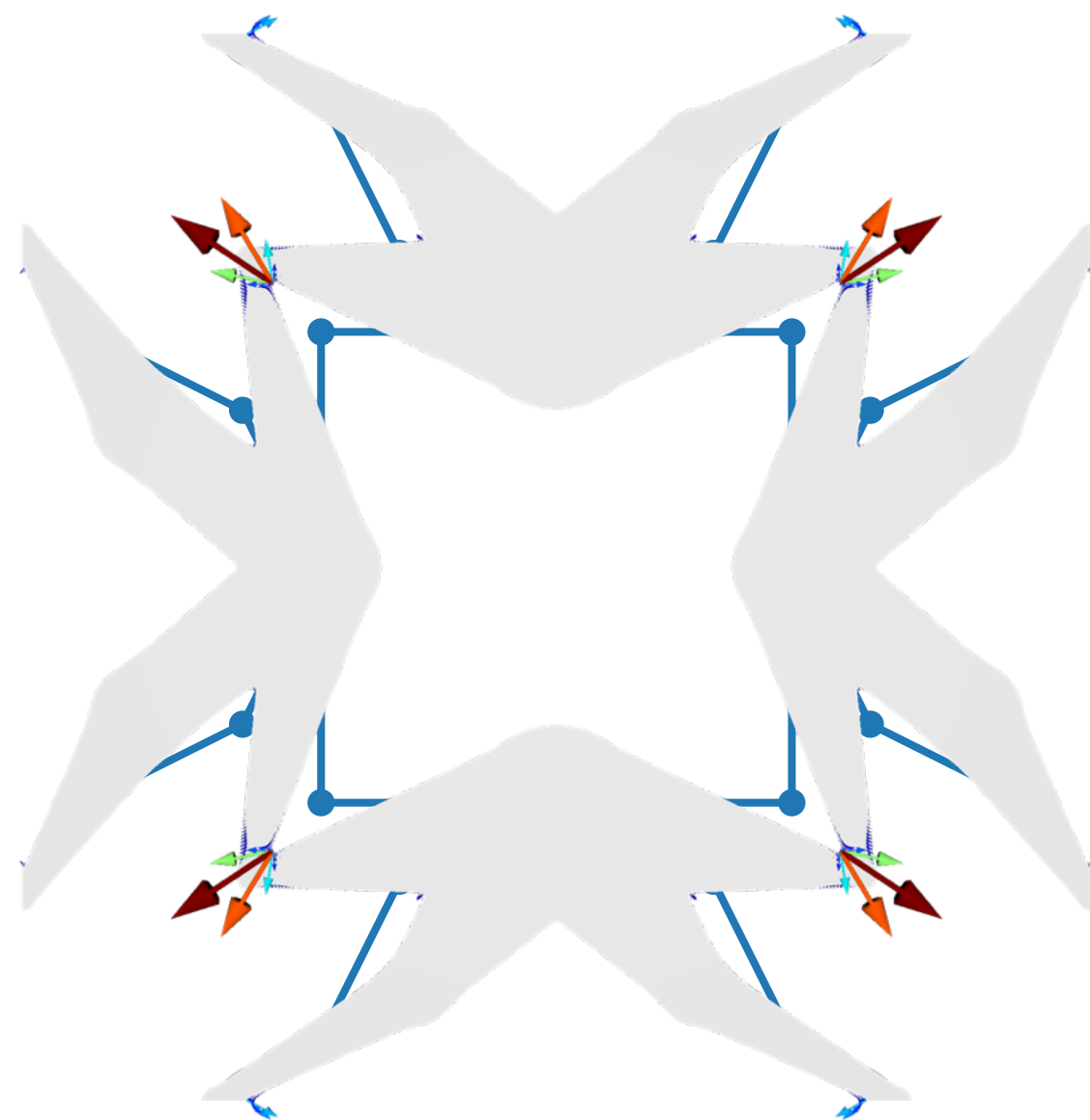
$$p \in \mathbb{R}^{20 \sim 25}$$

DESIGN PROBLEM

Shape Derivative

$$\min_{\Omega} \frac{1}{|\mathcal{F}|} \int_{\mathcal{F}} \mathbf{w}_e \left(\bar{\psi} - \bar{\psi}_{tgt} \right)_{rel}^2 + \mathbf{w}_\sigma \left\| \bar{\psi}' - \bar{\psi}'_{tgt} \right\|_{rel}^2 d\bar{F}$$

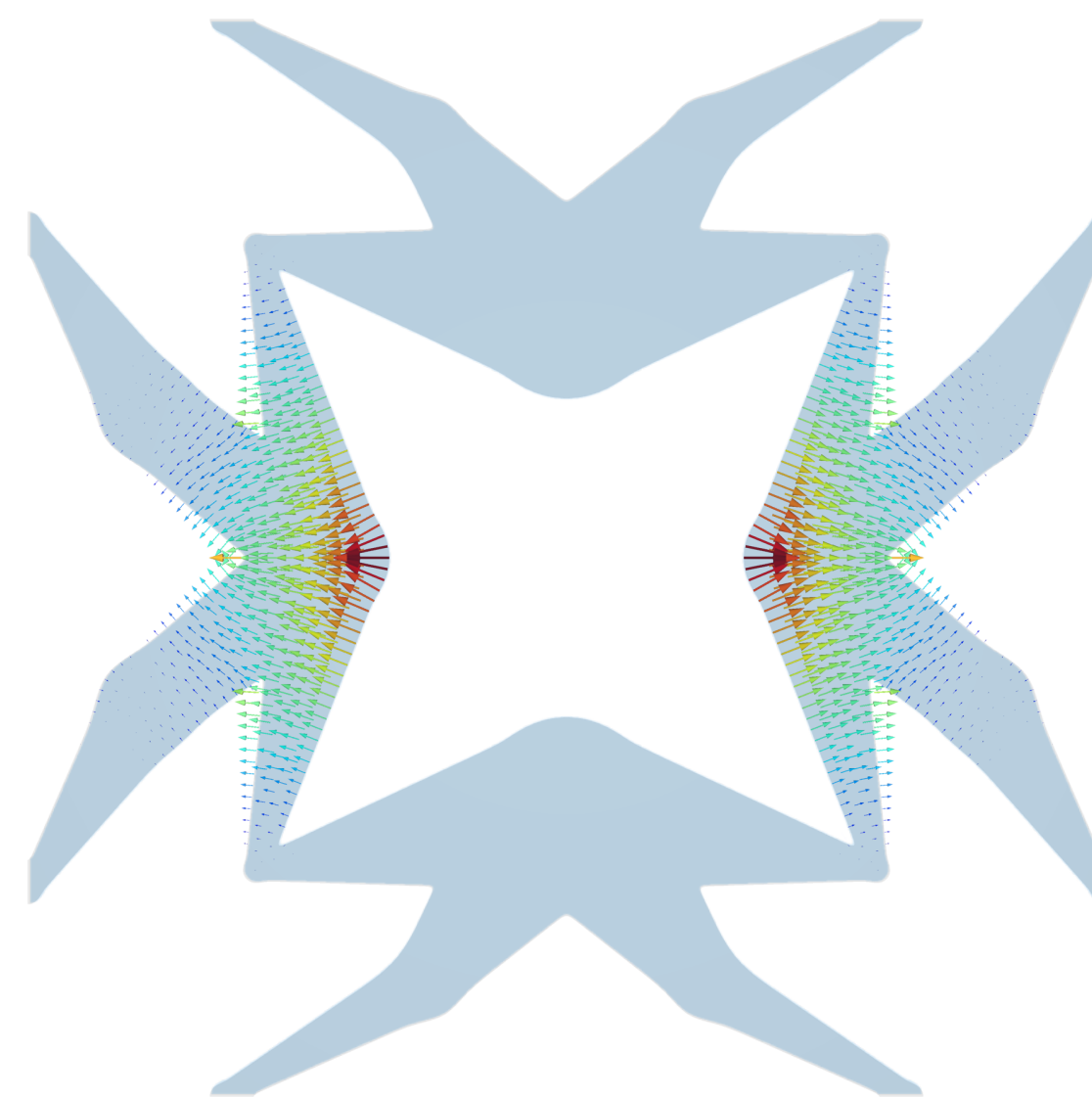
Derivative of fitting objective
with respect to mesh node positions



$$\frac{\partial \bar{\psi}(\bar{F})}{\partial \Omega}$$

Topology

Derivative of mesh node positions
with respect to shape parameter



$$\mathbf{v} = \frac{\partial \Omega(p)}{\partial p}$$

Gauss-Newton algorithm

analytical gradient

$$\left\langle \frac{\partial \bar{\psi}(\bar{F})}{\partial \mathbf{X}}, \mathbf{v} \right\rangle = \int_{\Omega} G_{\bar{\psi}} : \nabla \mathbf{v} d\mathbf{X}$$

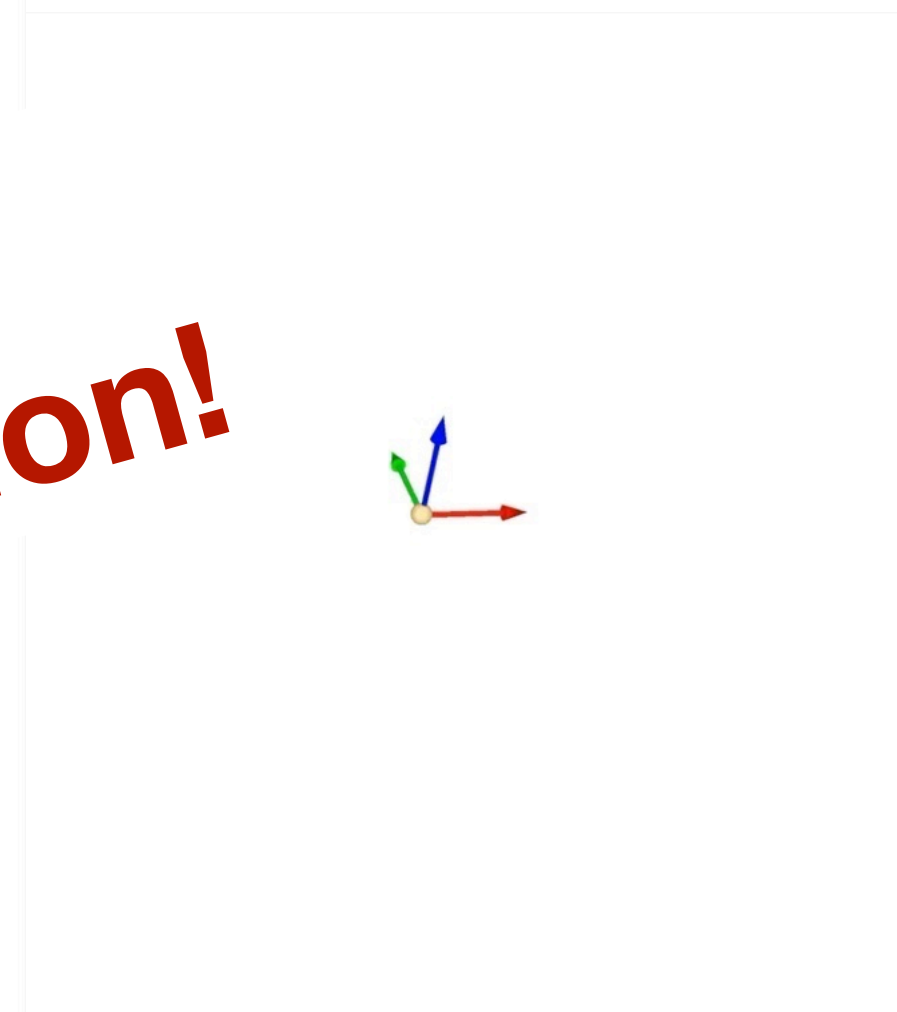
$$\left\langle \frac{\partial \bar{\sigma}_{ij}(\bar{F})}{\partial \mathbf{X}}, \mathbf{v} \right\rangle = \int_{\Omega} G_{\bar{\sigma}_{ij}} : \nabla \mathbf{v} d\mathbf{X}$$

DESIGN PROBLEM

Collision Removal

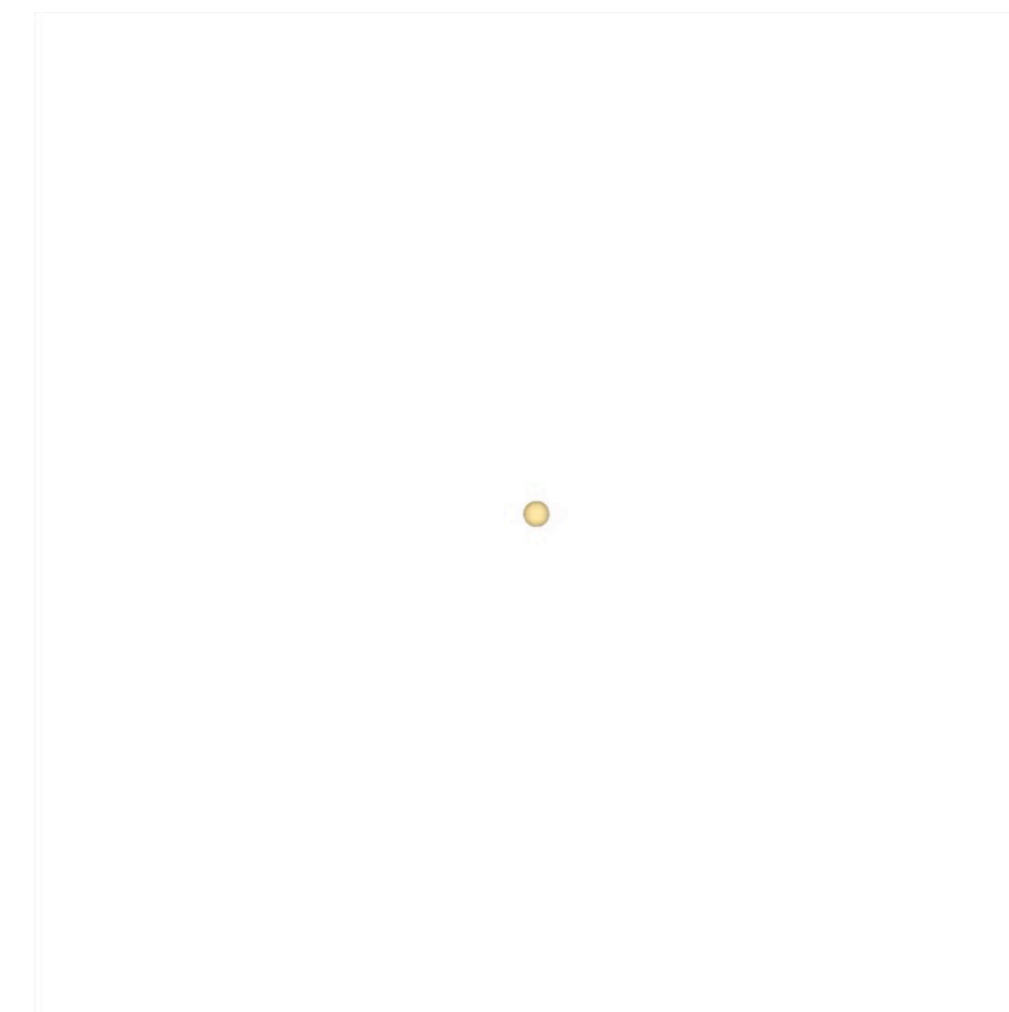
$$\min_{\Omega} \frac{1}{|\mathcal{F}|} \int_{\mathcal{F}} \mathbf{w}_e \left(\bar{\psi} - \bar{\psi}_{tgt} \right)_{rel}^2 + \mathbf{w}_{\sigma} \left\| \bar{\psi}' - \bar{\psi}'_{tgt} \right\|_{rel}^2 d\bar{F}$$

Avoid collisions!
No contact simulation!



Strain Domain

Red for high collision area



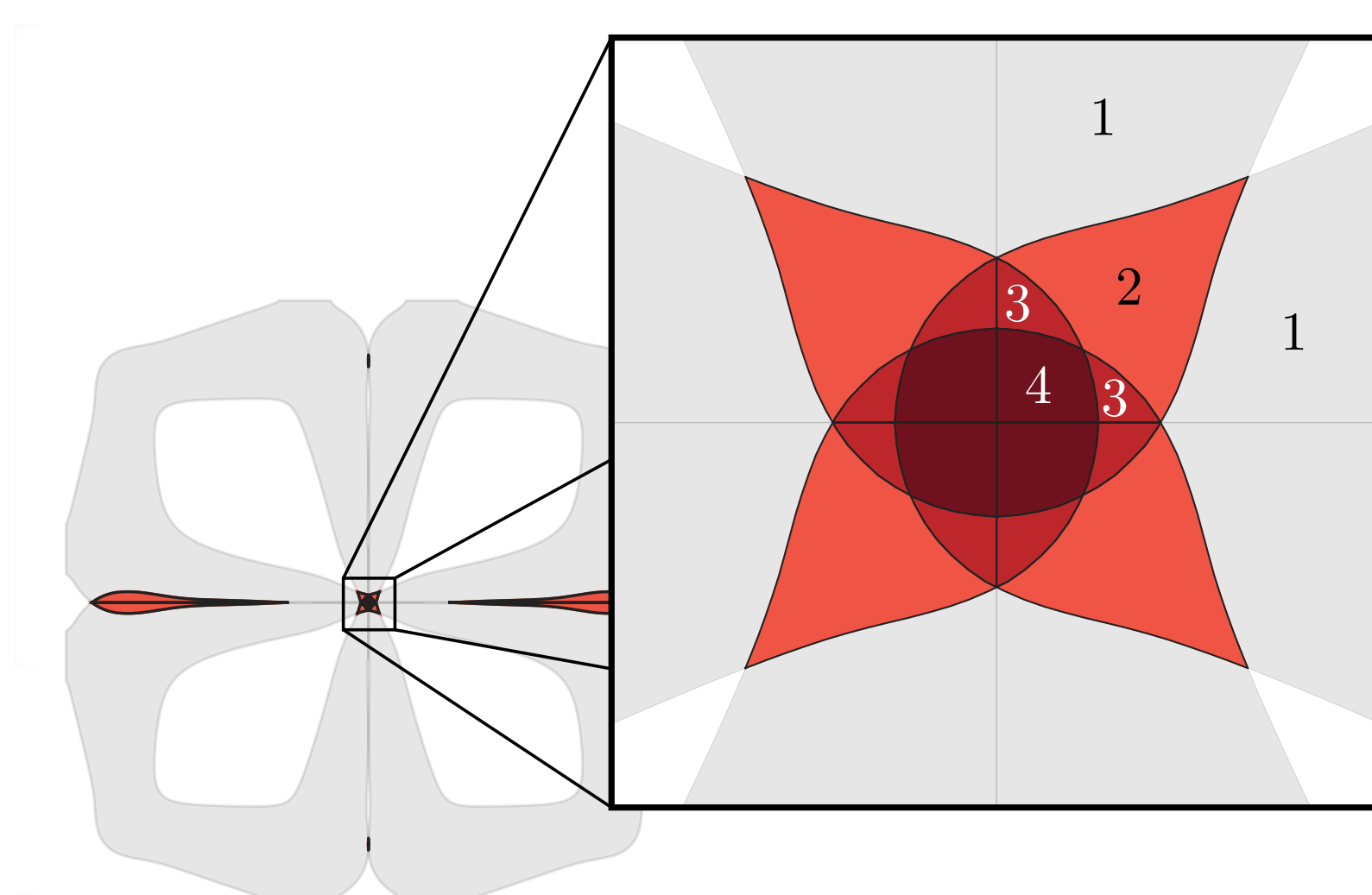
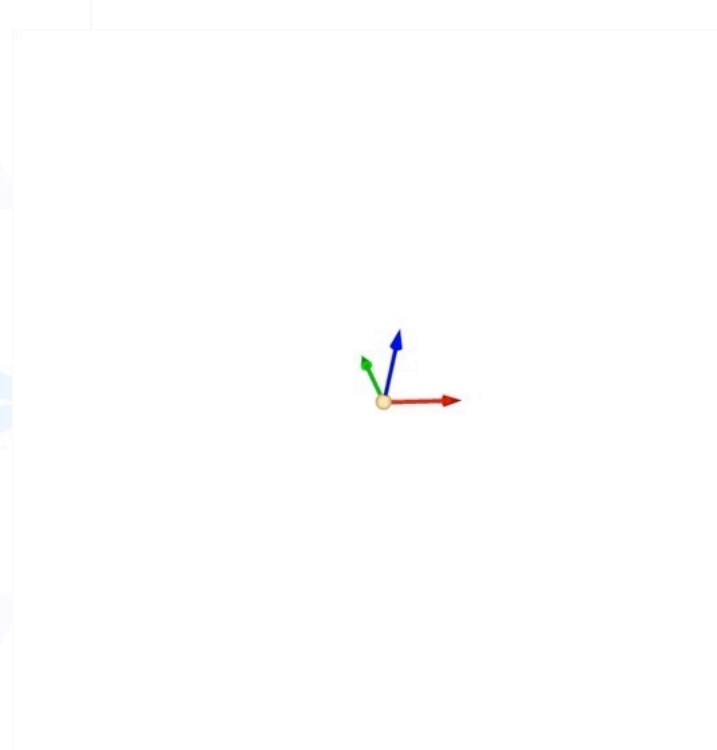
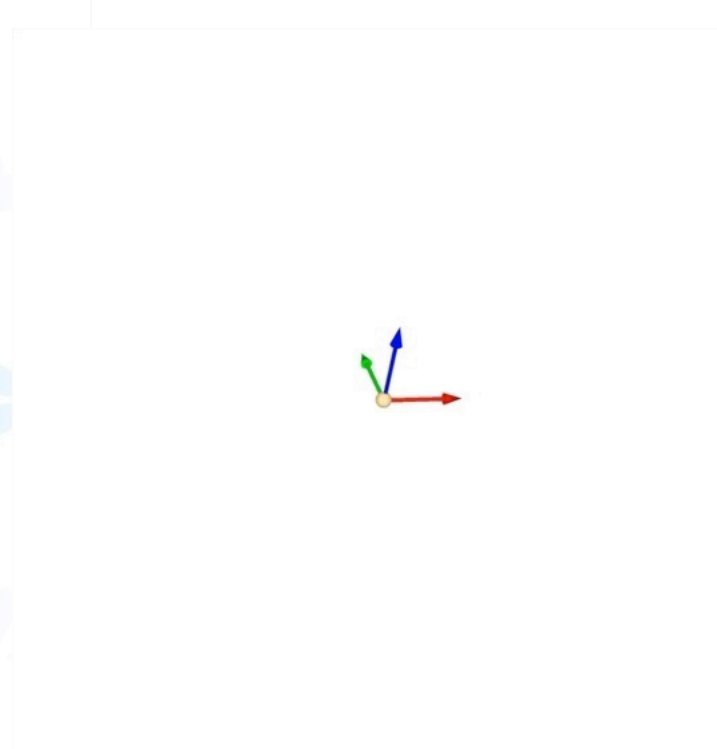
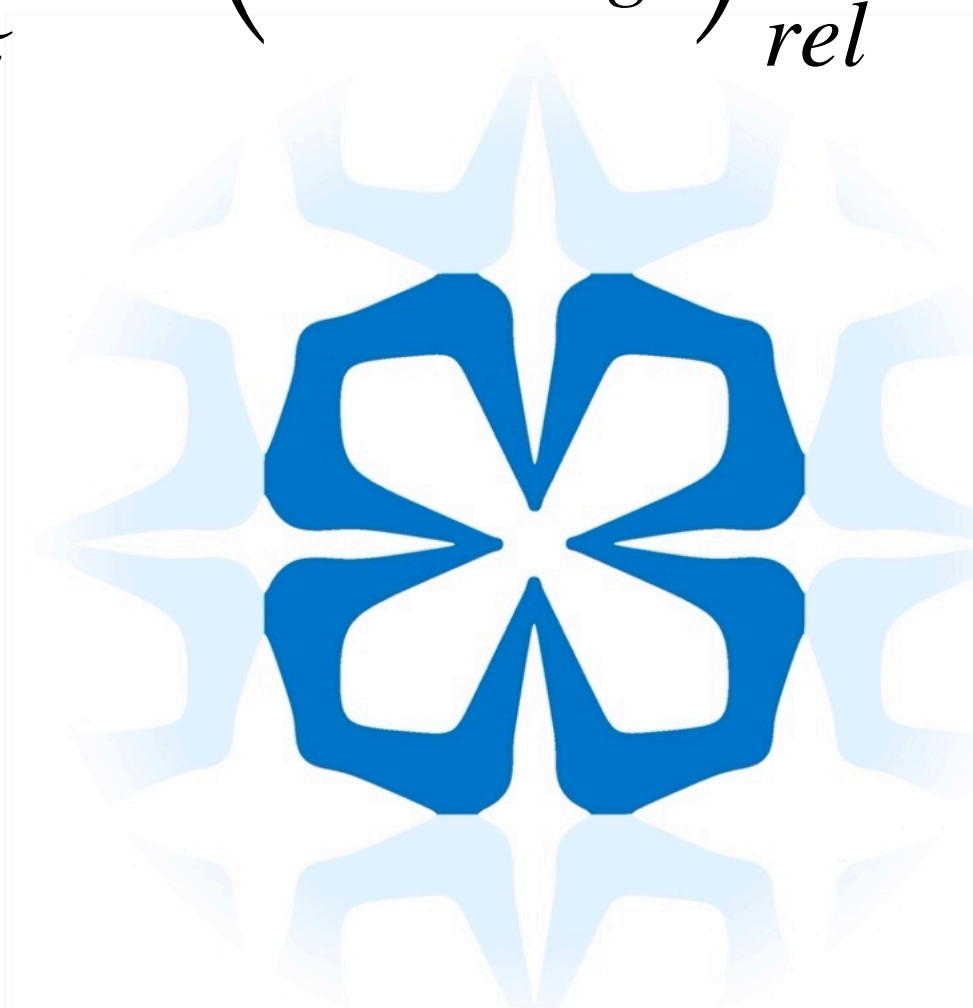
Stress Domain

Blue for low relative error

DESIGN PROBLEM

Collision Removal

$$\min_{\Omega} \frac{1}{|\mathcal{F}|} \int_{\mathcal{F}} \mathbf{w}_e \left(\bar{\psi} - \bar{\psi}_{tgt} \right)_{rel}^2 + \mathbf{w}_\sigma \left\| \bar{\psi}' - \bar{\psi}'_{tgt} \right\|_{rel}^2 + \mathbf{w}_c \left[\mathbf{A}(\omega^*(\bar{\mathbf{F}}) + \bar{\mathbf{F}}\mathbf{X}; \Omega) \right]^2 d\bar{\mathbf{F}}$$

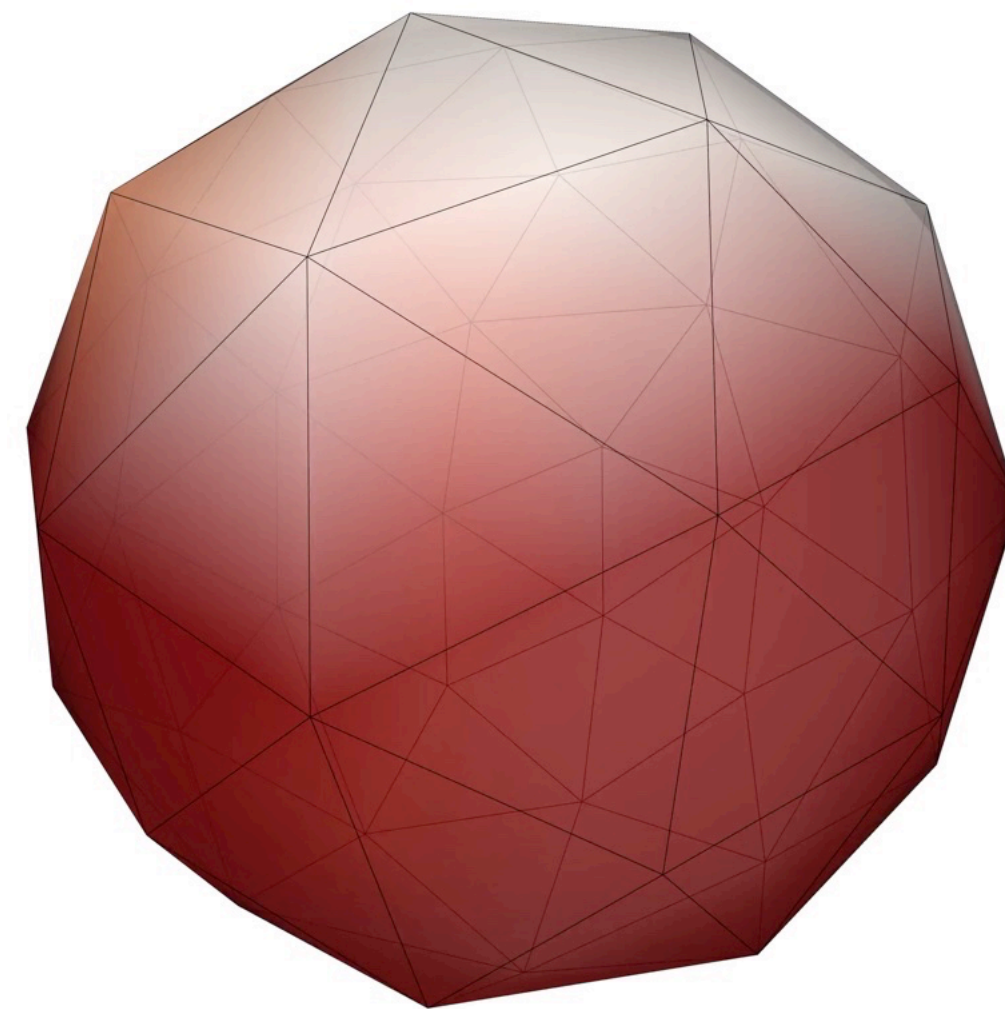


$$A(\Phi; \Omega) = \int_{\Phi(\Omega)} (\text{wind}(\mathbf{x}, \Phi(\partial\Omega)) - 1)_+ d\mathbf{x}$$

DESIGN PROBLEM

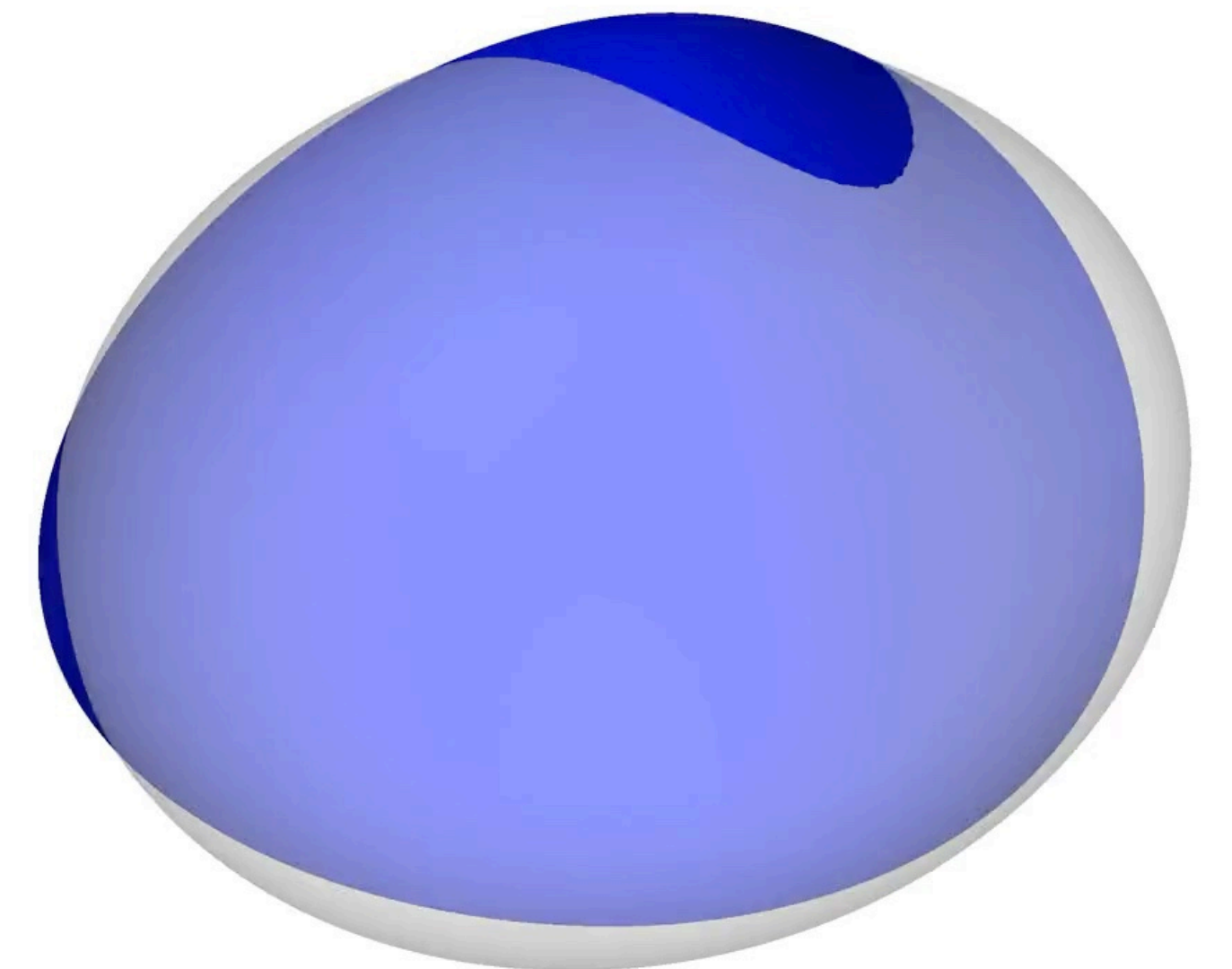
Collision Removal

$$\min_{\Omega} \frac{1}{|\mathcal{F}|} \int_{\mathcal{F}} \mathbf{w}_e \left(\bar{\psi} - \bar{\psi}_{tgt} \right)_{rel}^2 + \mathbf{w}_\sigma \left\| \bar{\psi}' - \bar{\psi}'_{tgt} \right\|_{rel}^2 + \mathbf{w}_c \left[\mathbf{A}(\omega^*(\bar{\mathbf{F}}) + \bar{\mathbf{F}}\mathbf{X}; \Omega) \right]^2 d\bar{\mathbf{F}}$$



Discrete Strain Domain

Red for high collision area

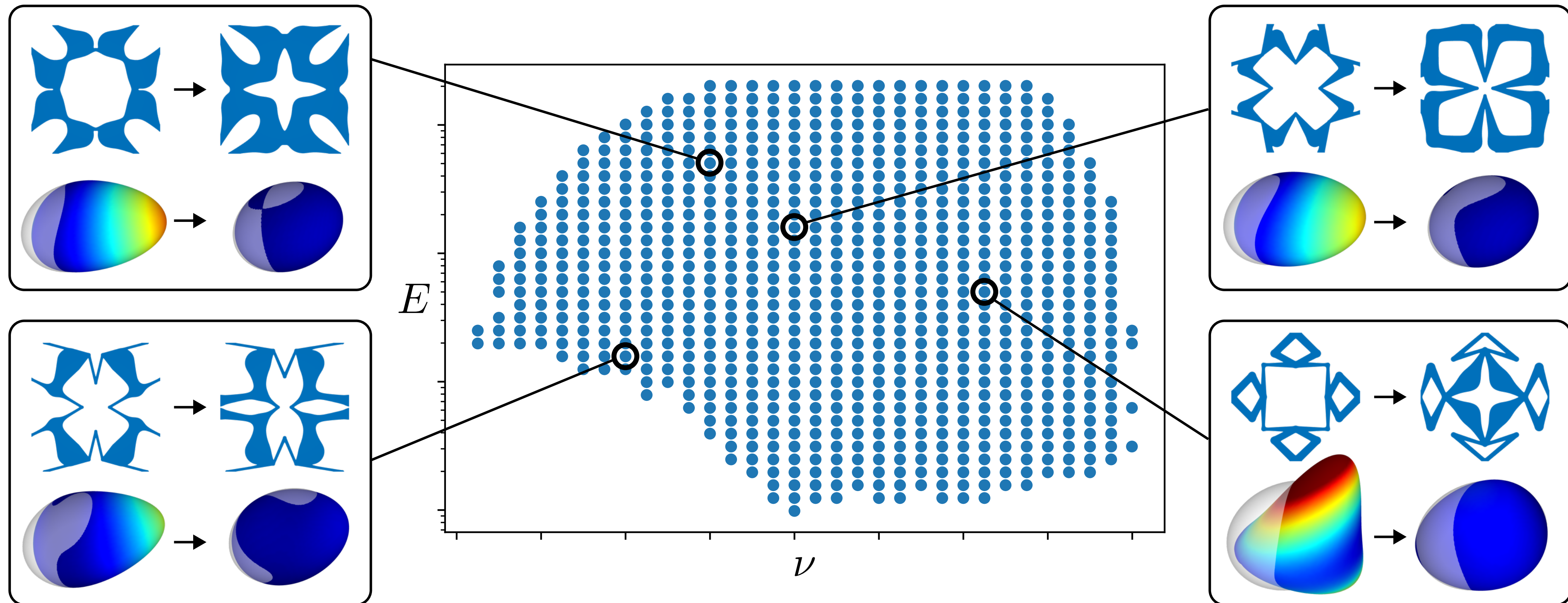


Stress Domain

Blue for low relative error

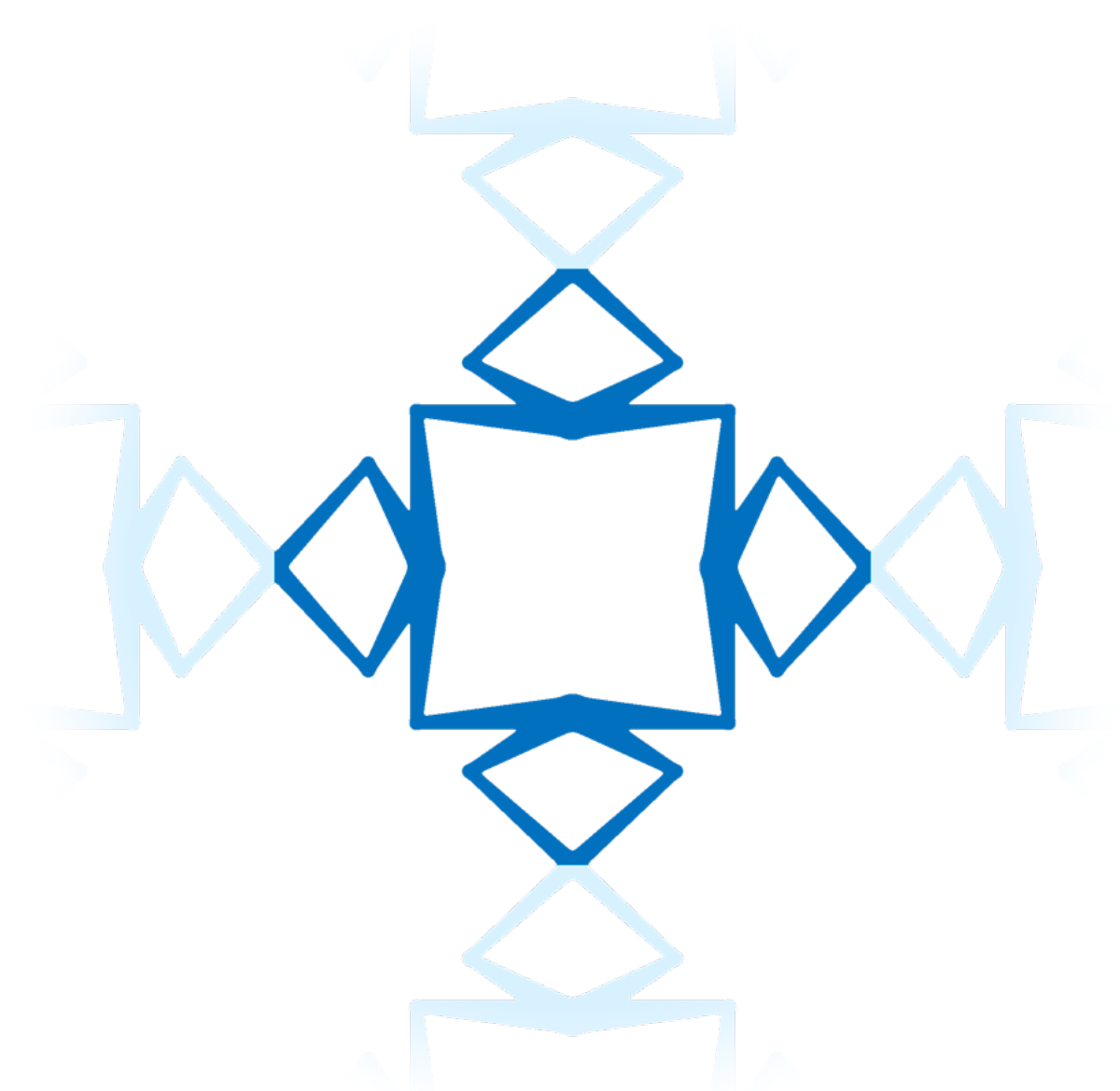
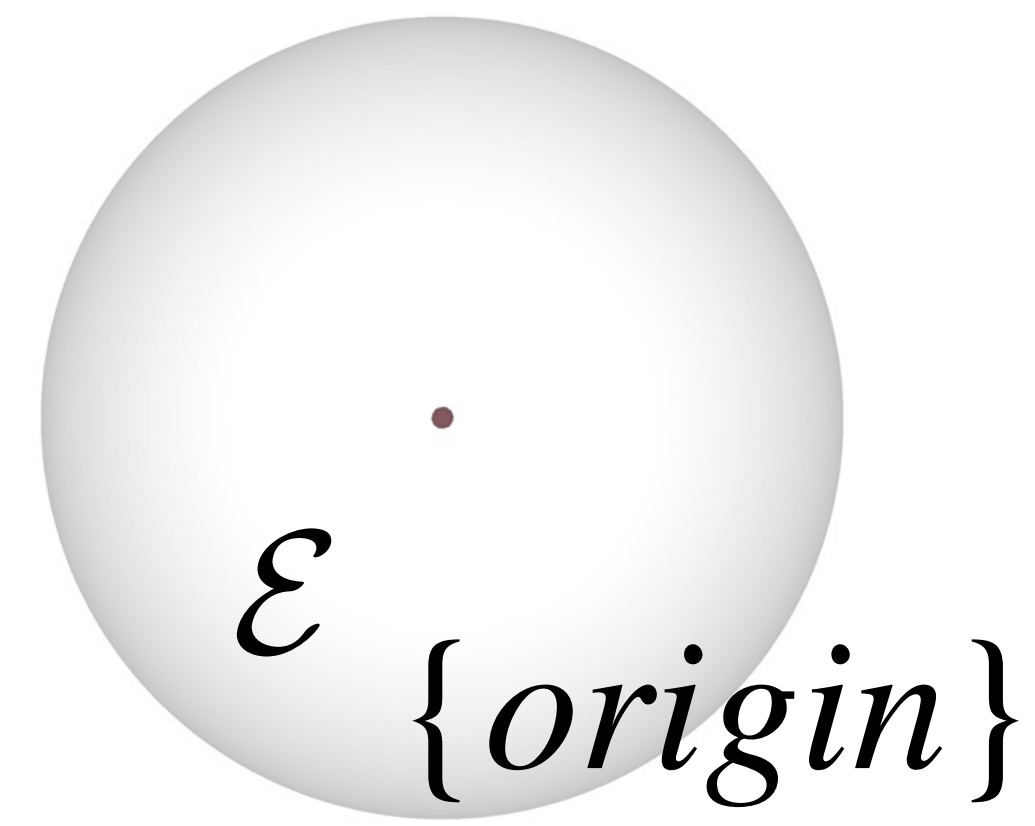
LARGE-SCALE VALIDATION

Isotropic Linear Material Design



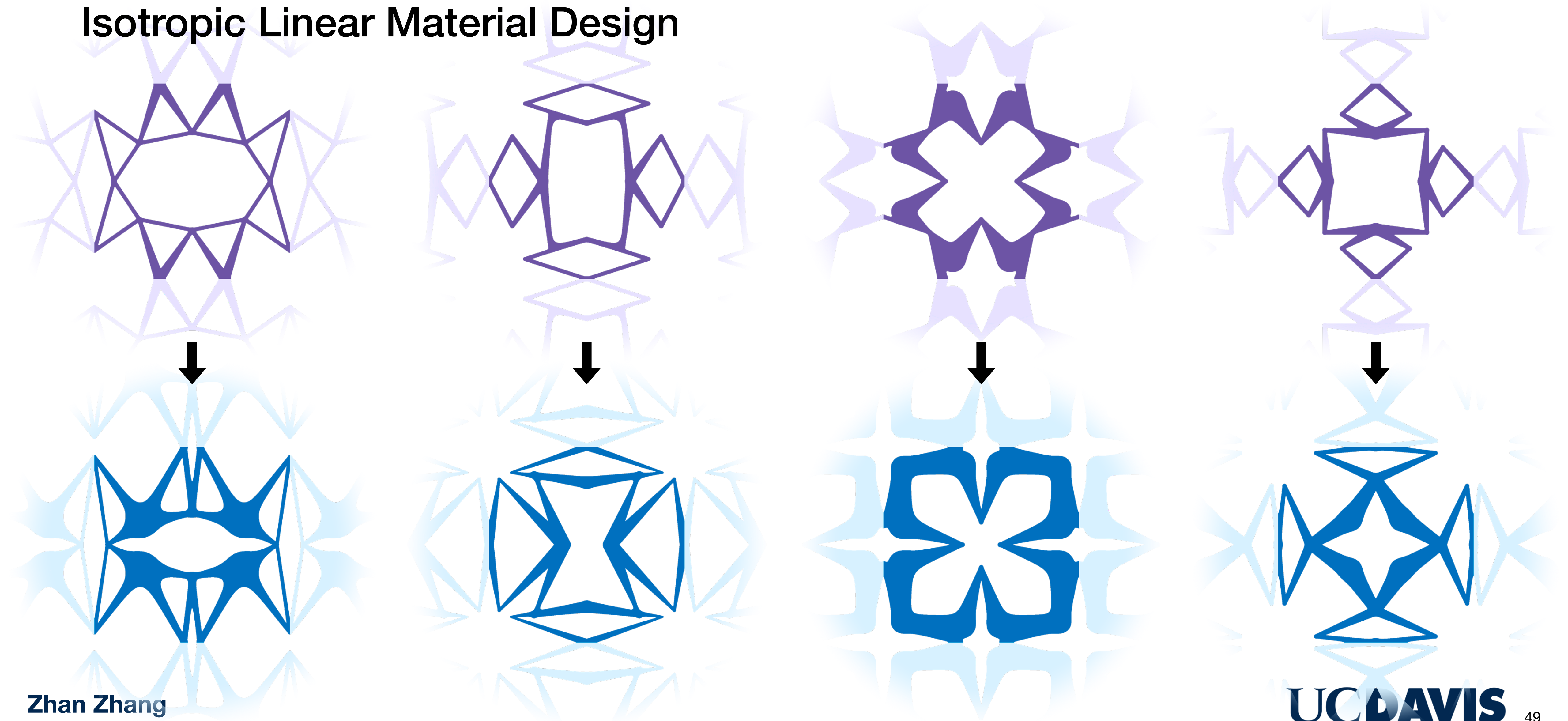
LARGE-SCALE VALIDATION

Isotropic Linear Material Design

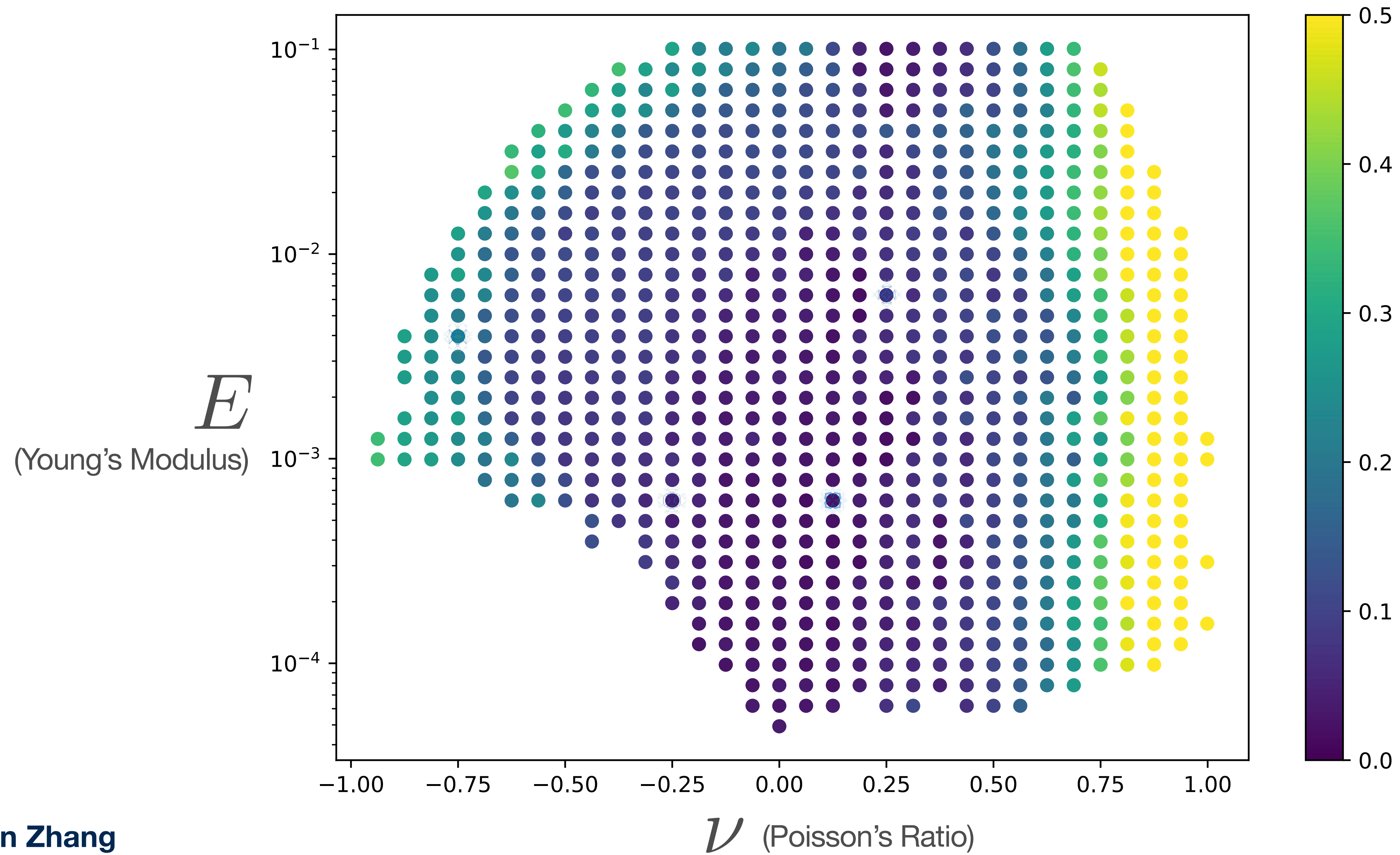


LARGE-SCALE VALIDATION

Isotropic Linear Material Design



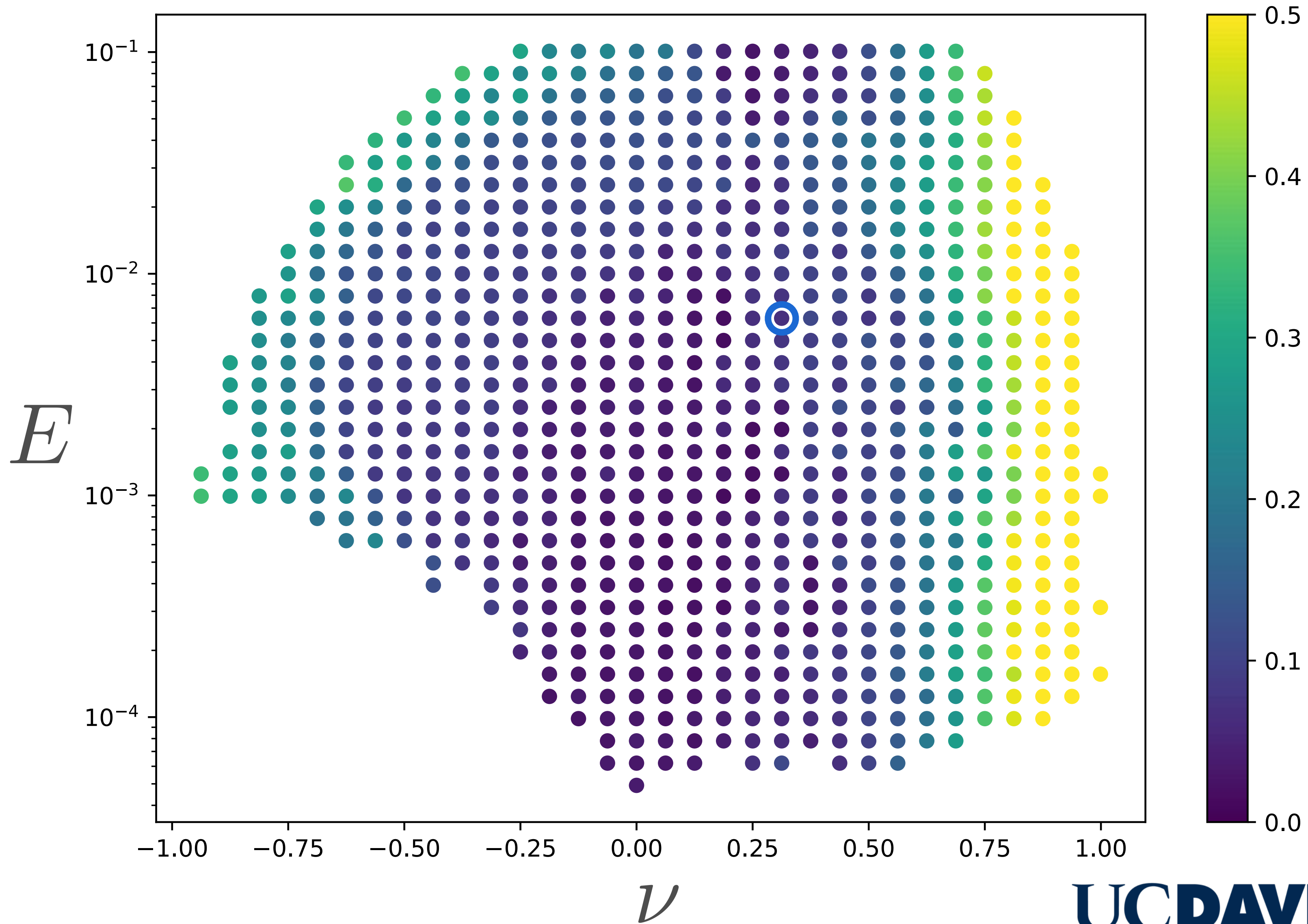
LARGE-SCALE VALIDATION



RESULTS



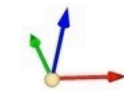
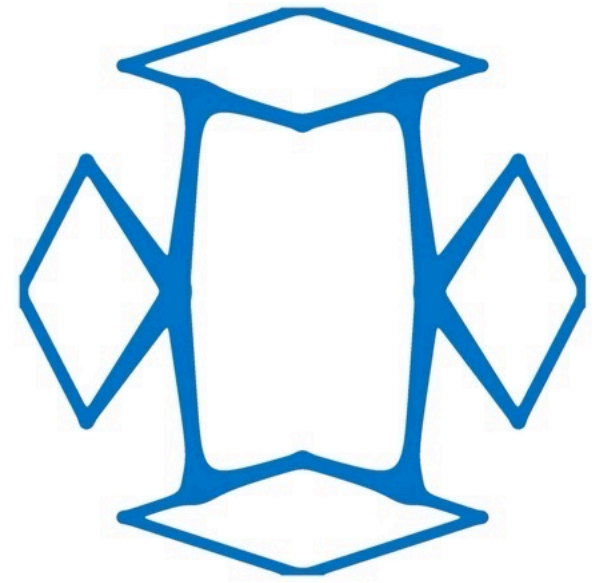
Zhan Zhang



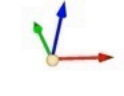
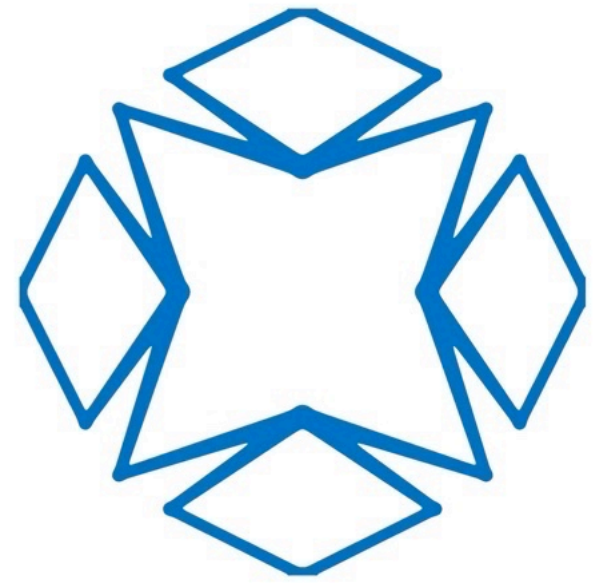
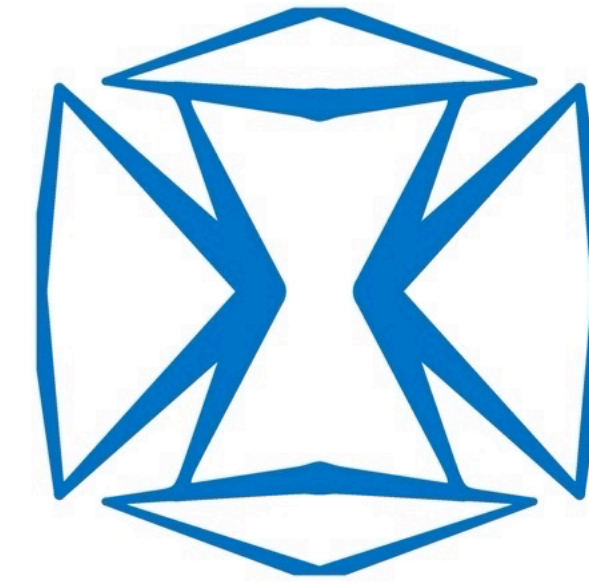
UCDAVIS

RESULTS

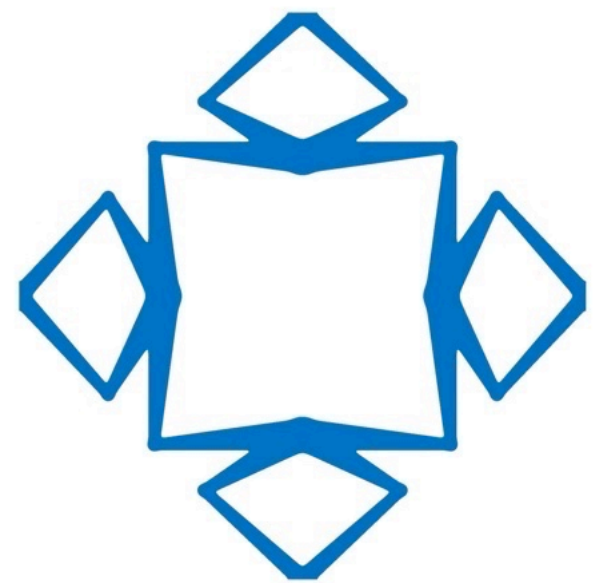
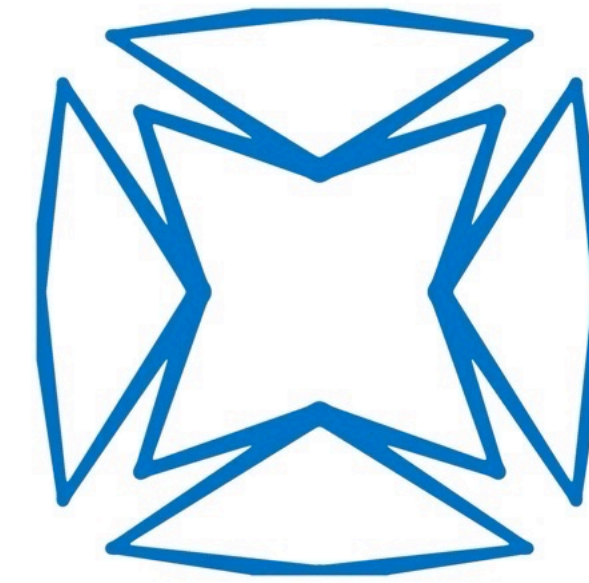
Different Poisson's Ratio



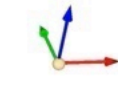
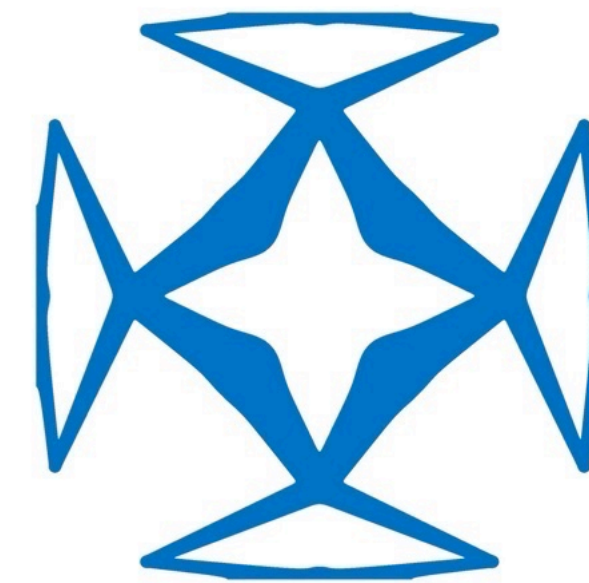
-0.25



0.00

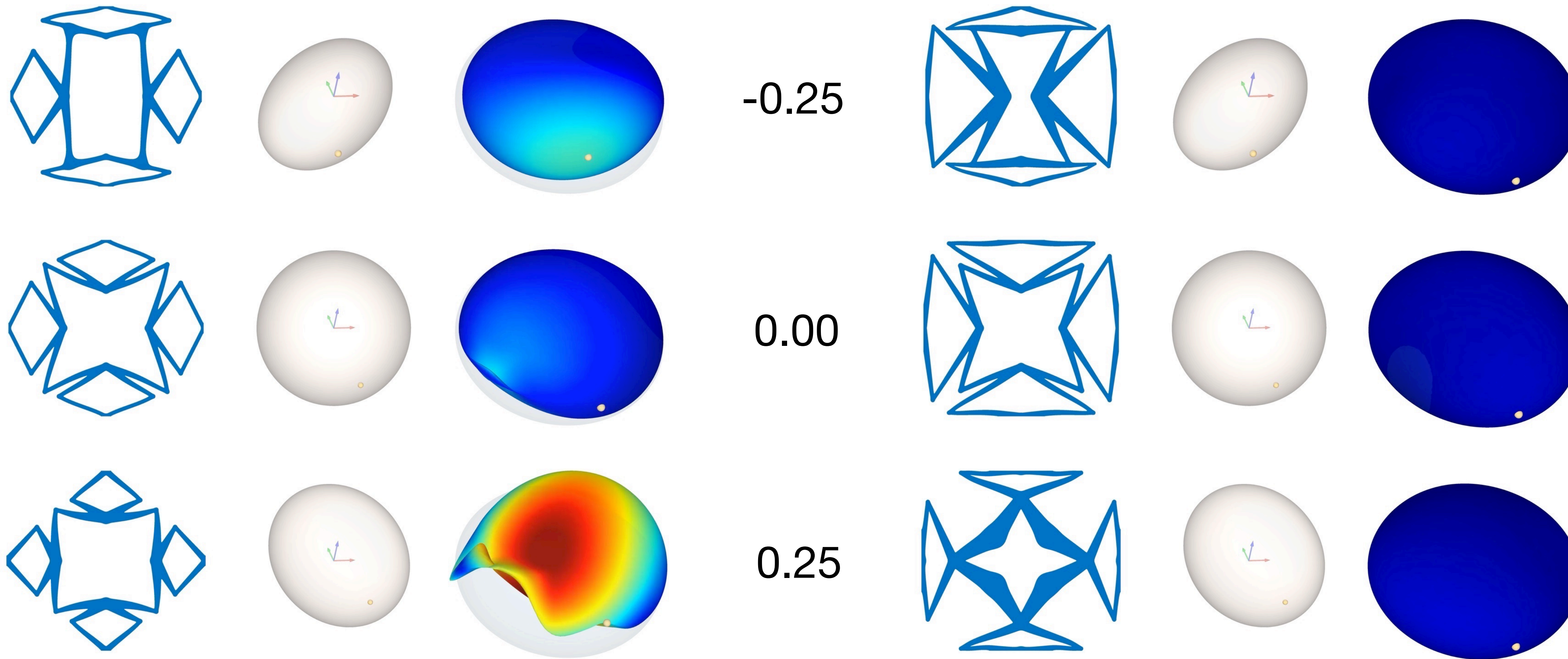


0.25



RESULTS

Different Poisson's Ratio



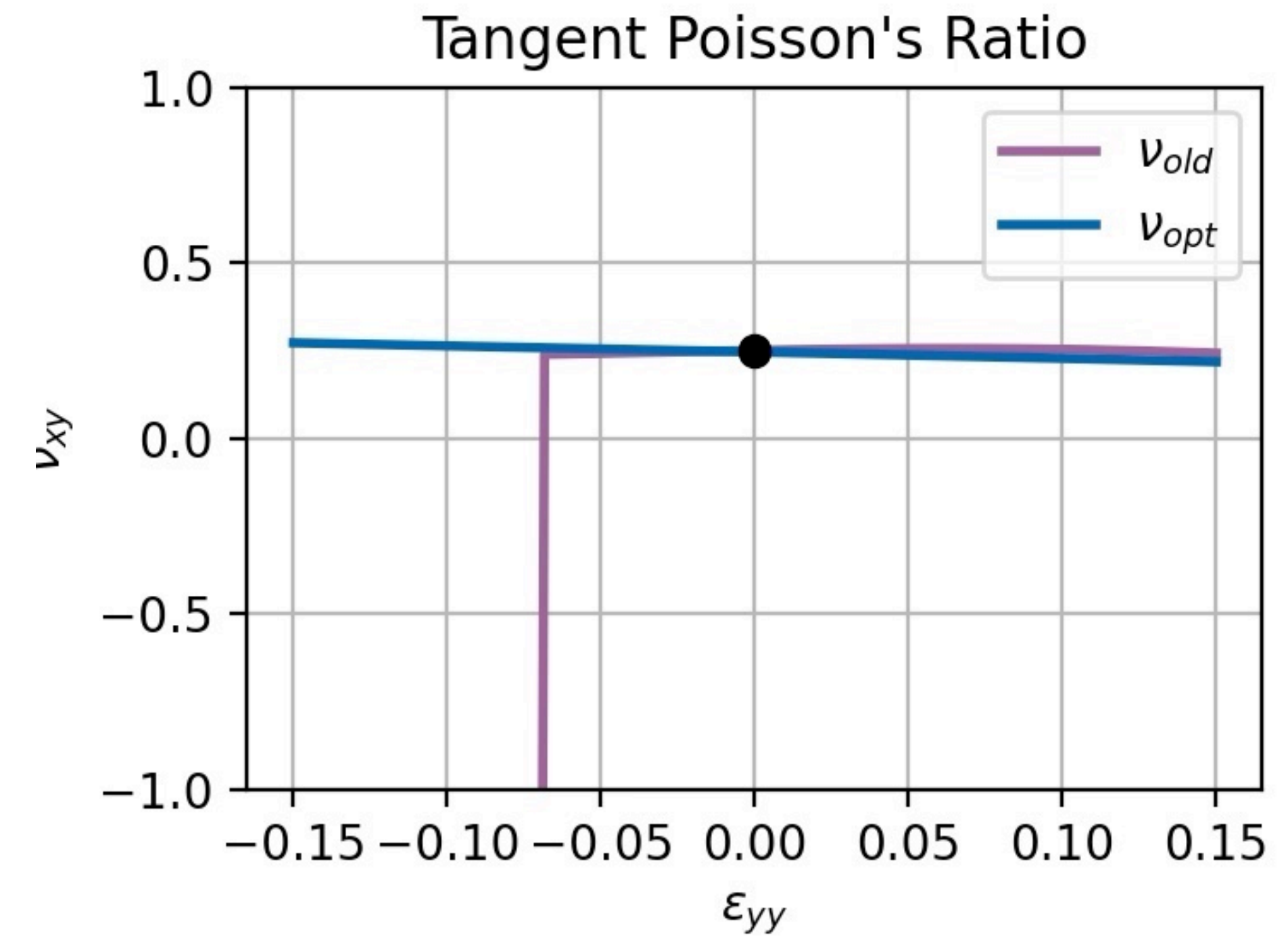
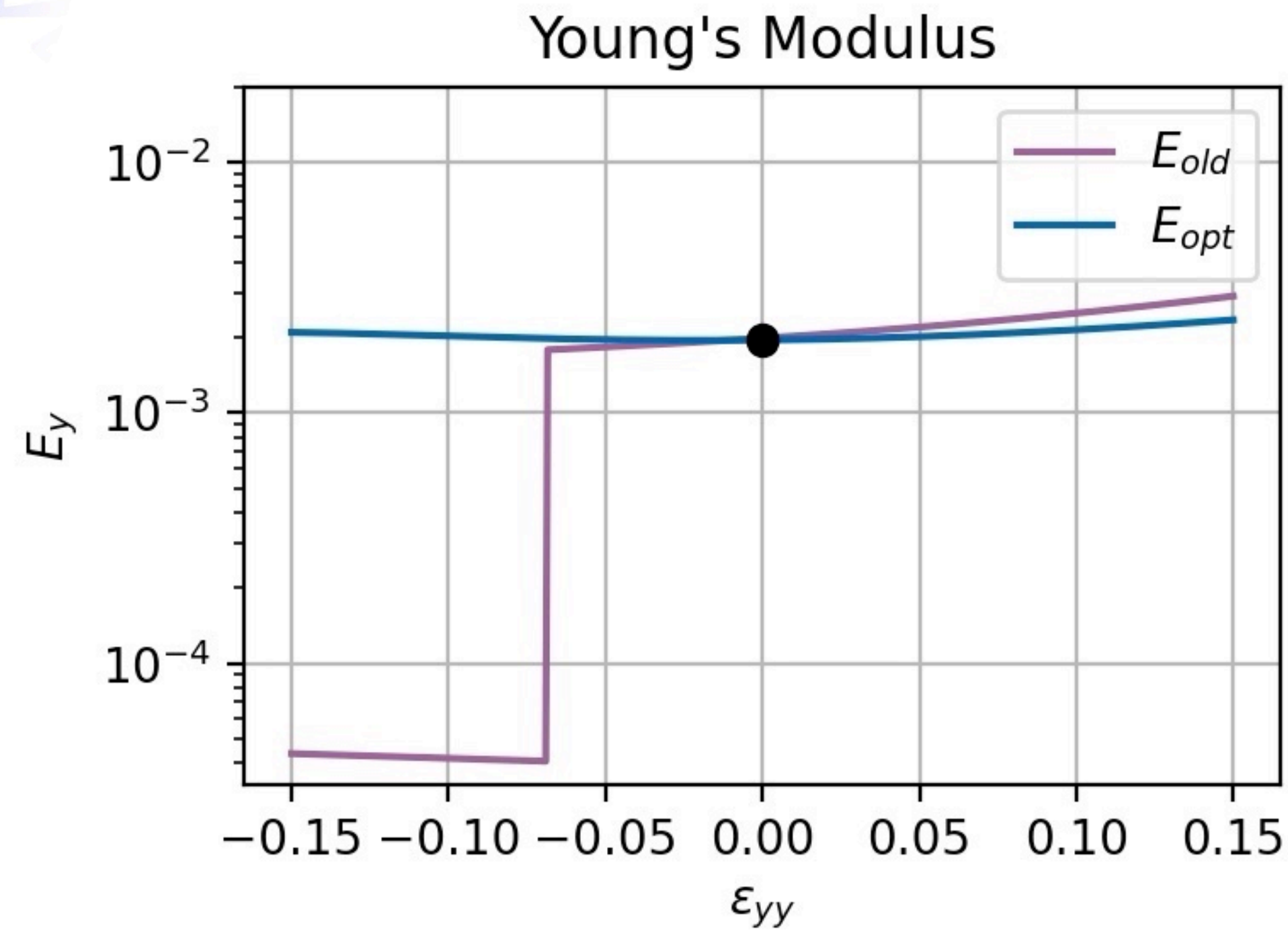
-0.25

0.00

0.25

RESULTS

Young's Modulus & Poisson's Ratio

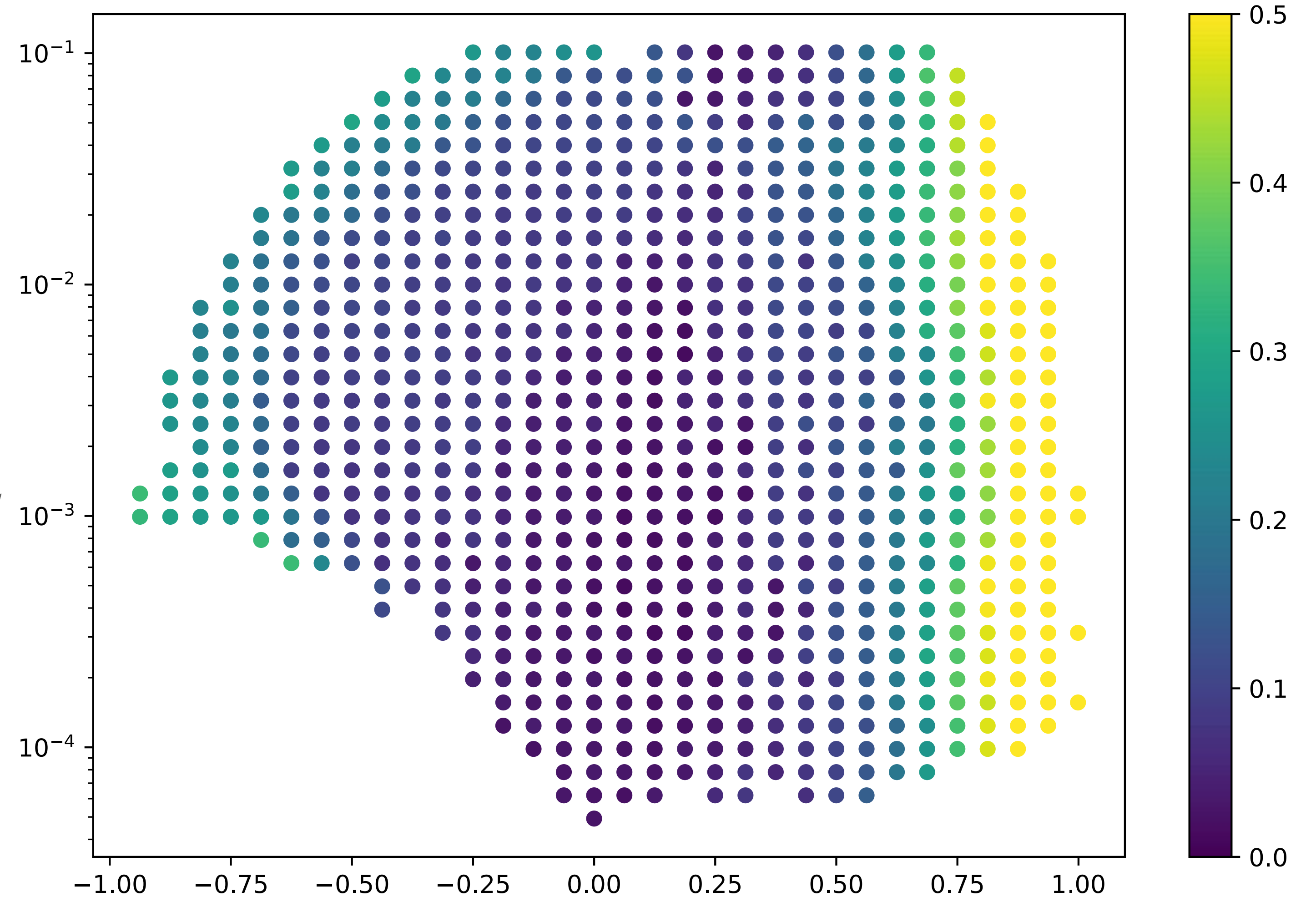


RESULTS

Max Relative Error

$$\text{eval}_{opt} = \max_{\bar{F} \in \mathcal{F}} \left\| \bar{\psi}'(\bar{F}) - \bar{\psi}'_{tgt}(\bar{F}) \right\|_{rel}$$

Ignoring Collisions



Results for 10% Strain

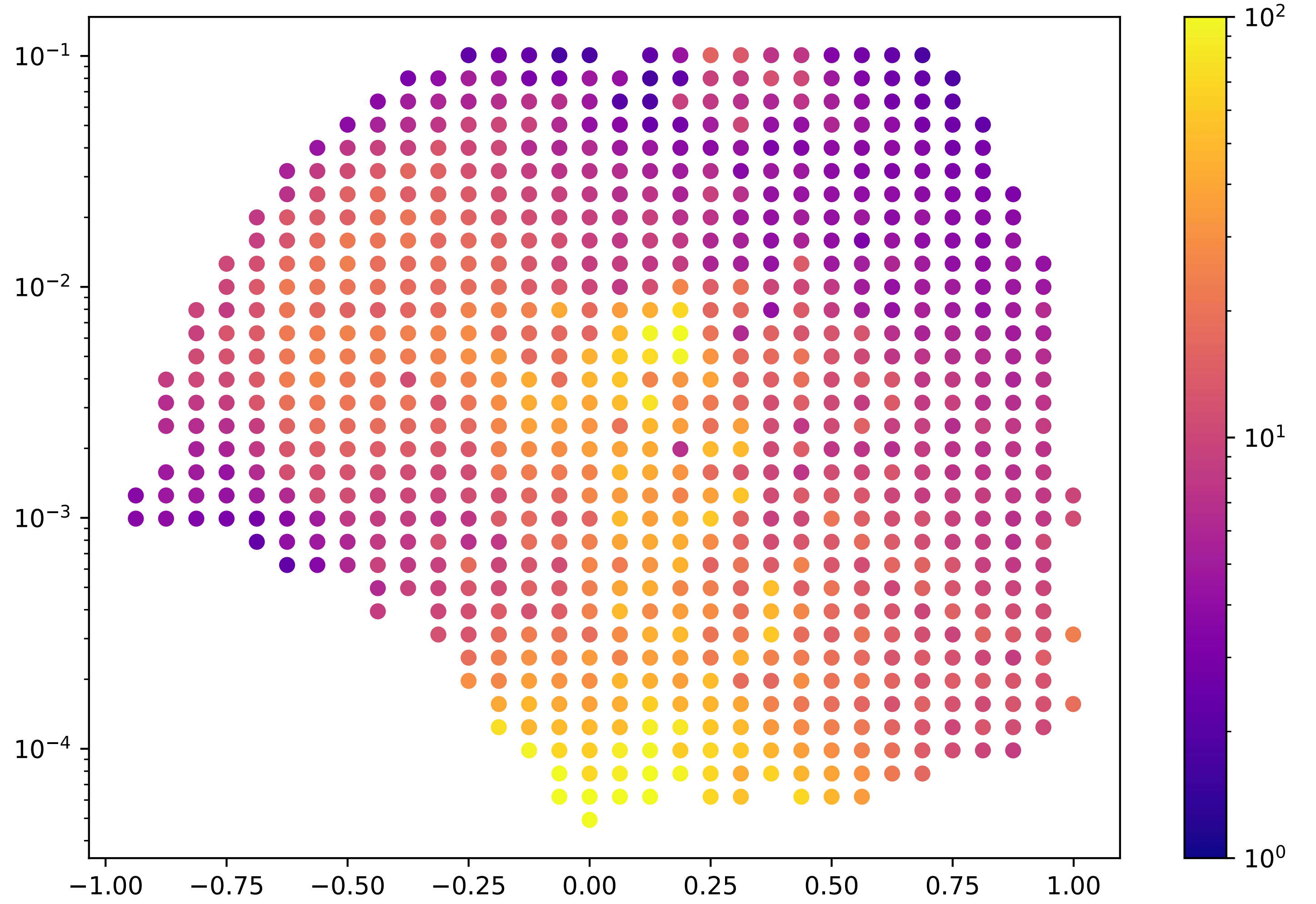
RESULTS

Improvement

$$\frac{\text{eval}_{old}}{\text{eval}_{opt}}$$

Ignoring Collisions

100X 



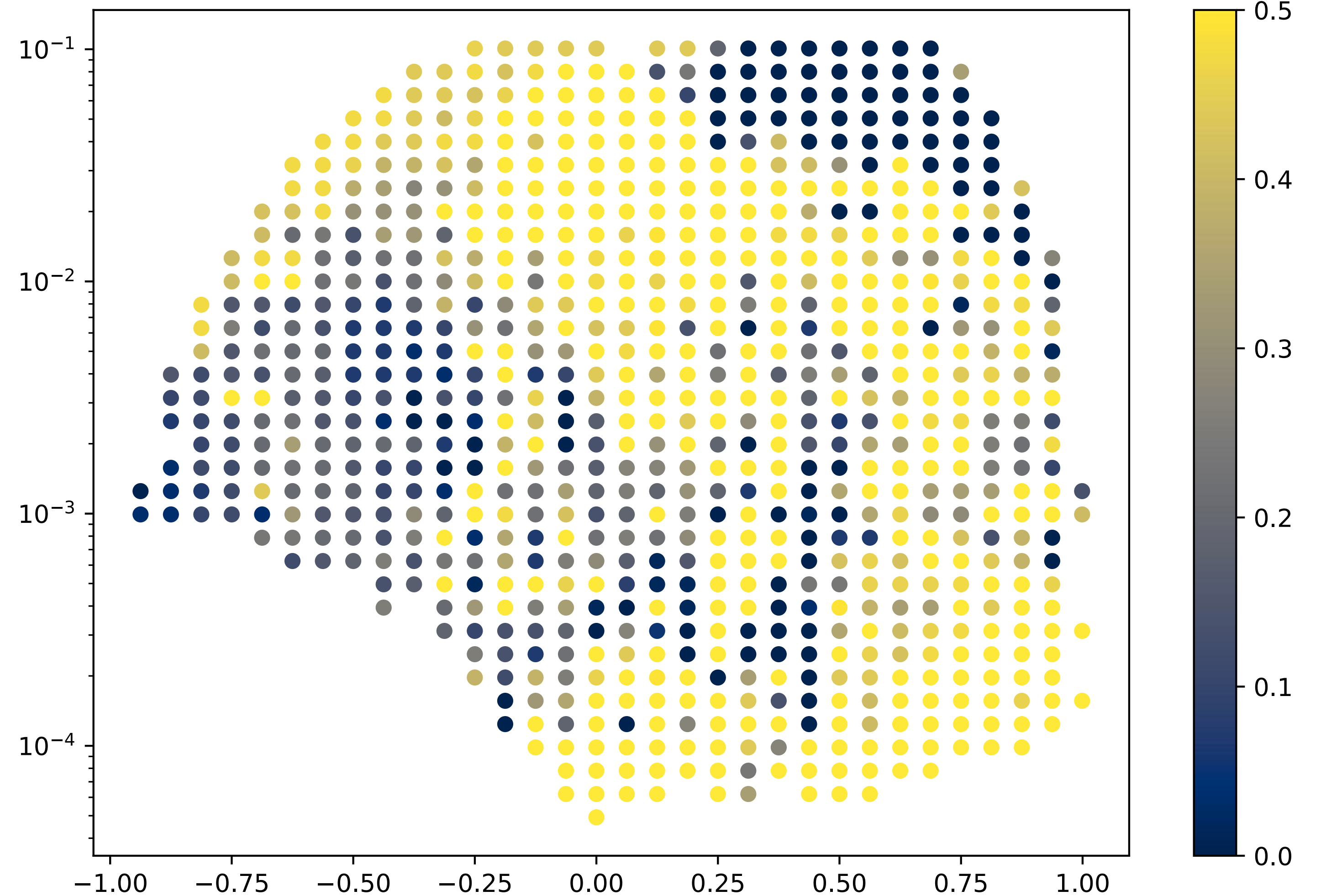
RESULTS

Collision Ratio

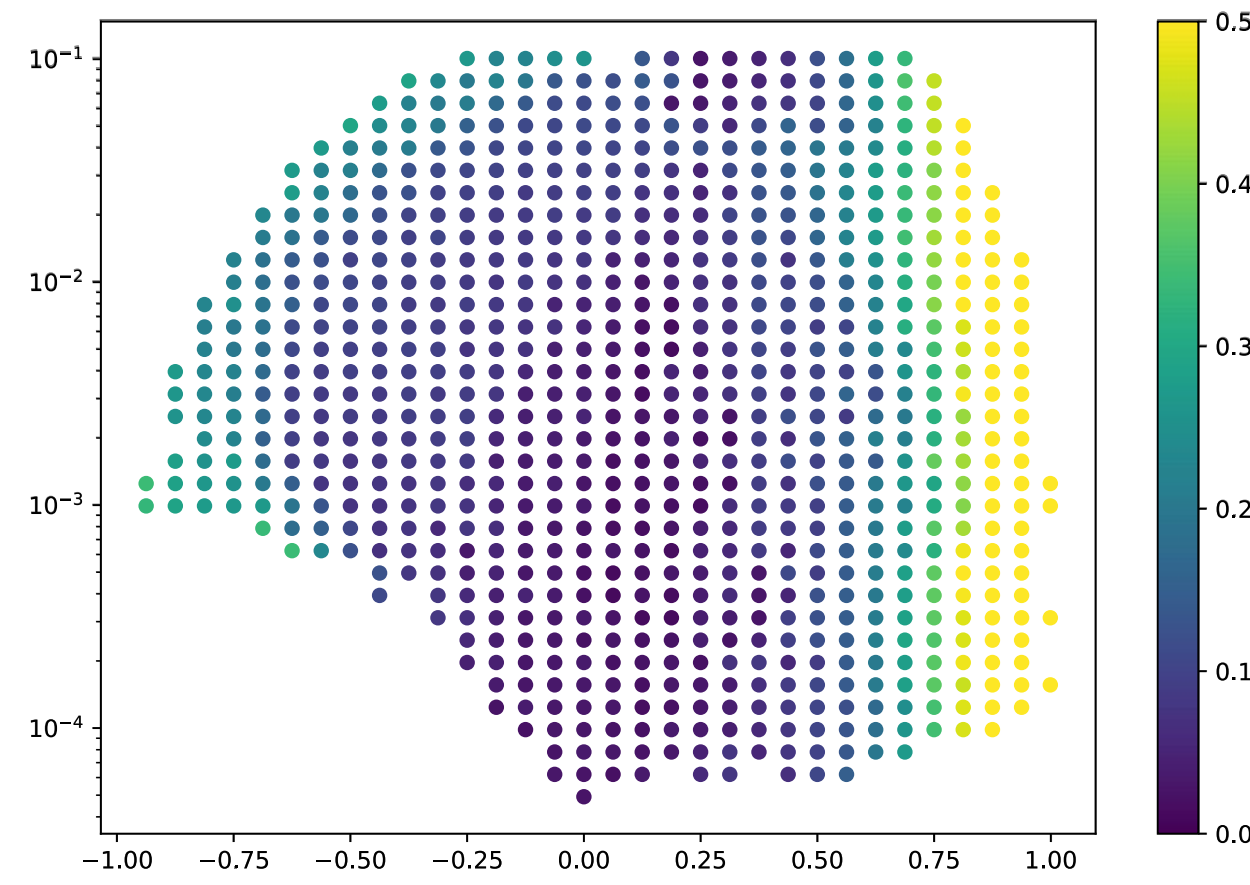
$$\frac{V_{collision}}{V_{domain}}$$

Ignoring Collisions

50% of strains
have collision

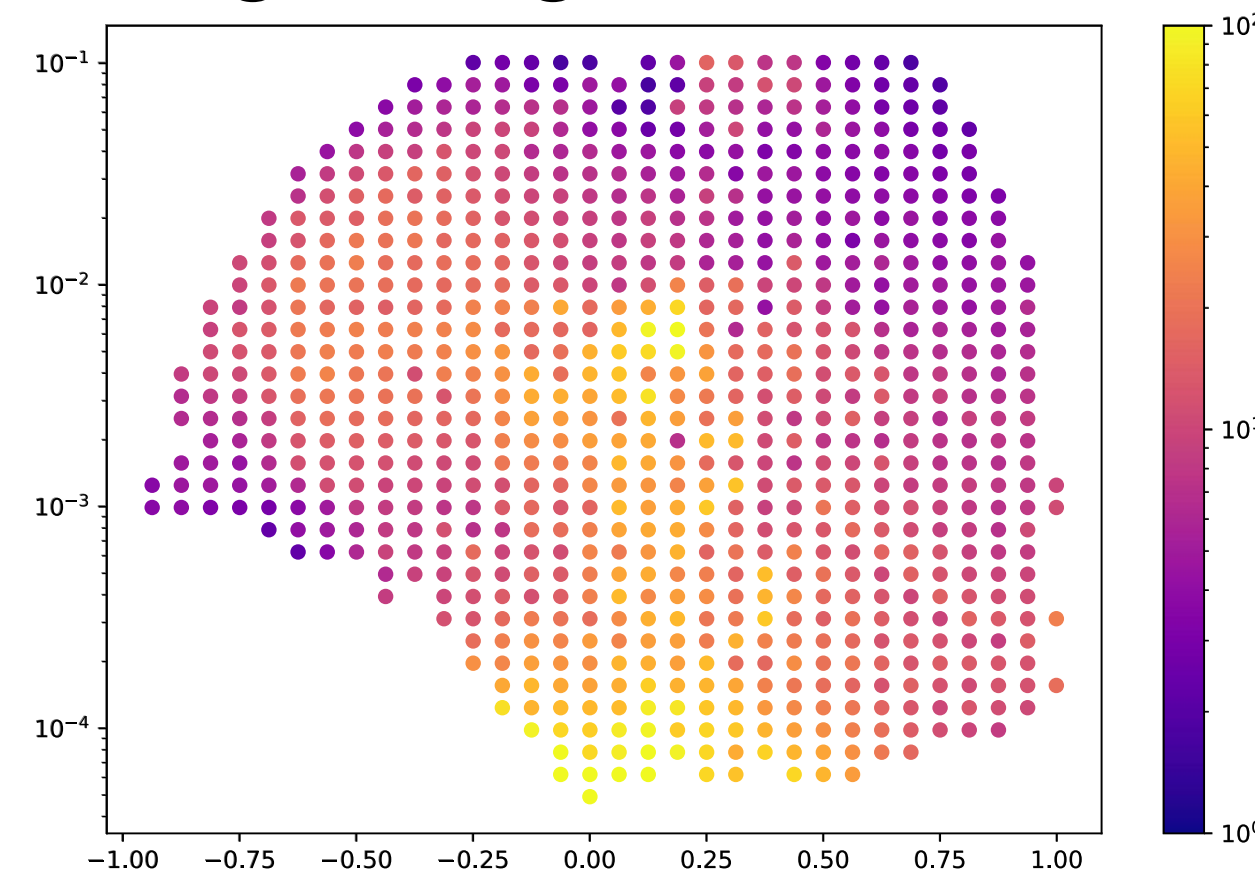


RESULTS

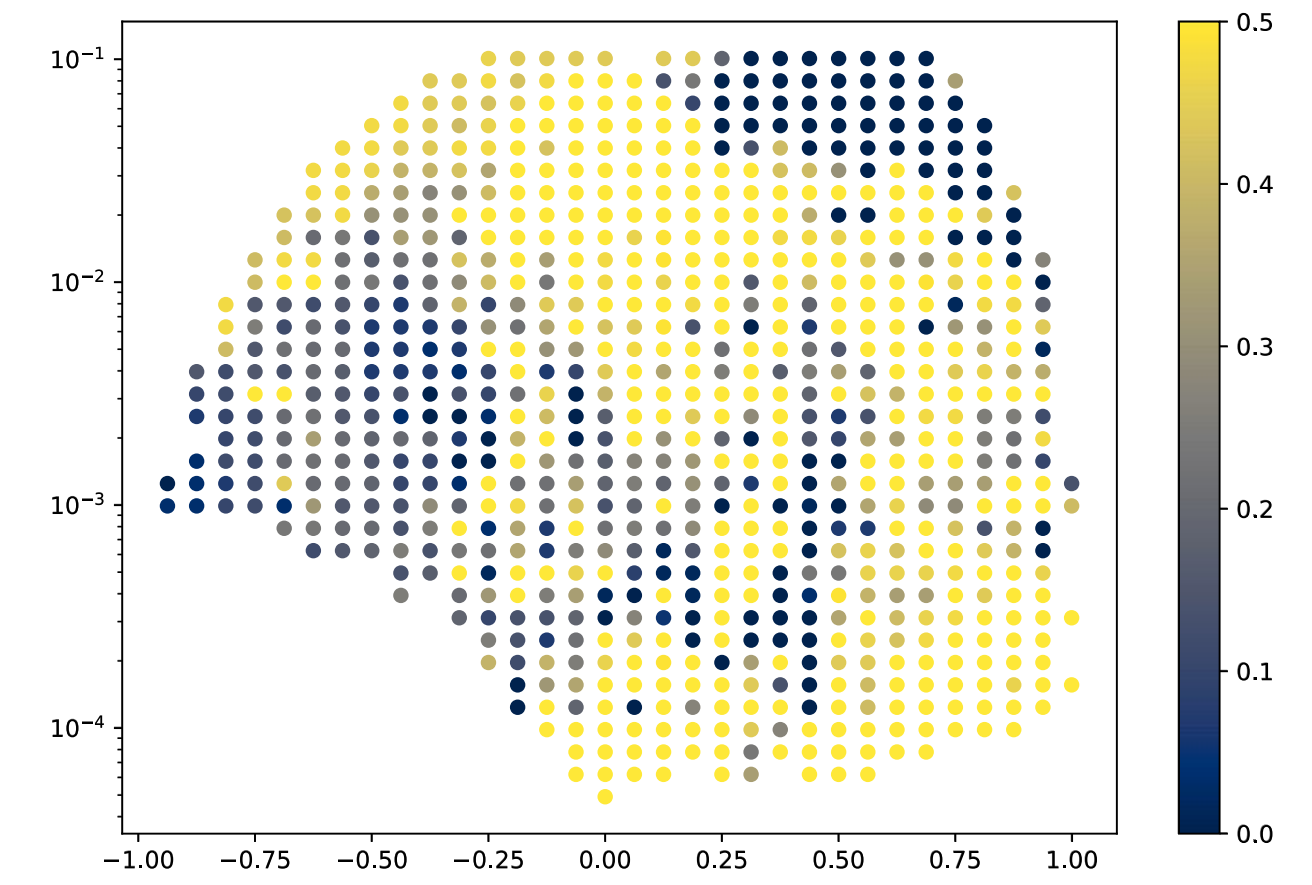


Relative Error

Ignoring Collisions

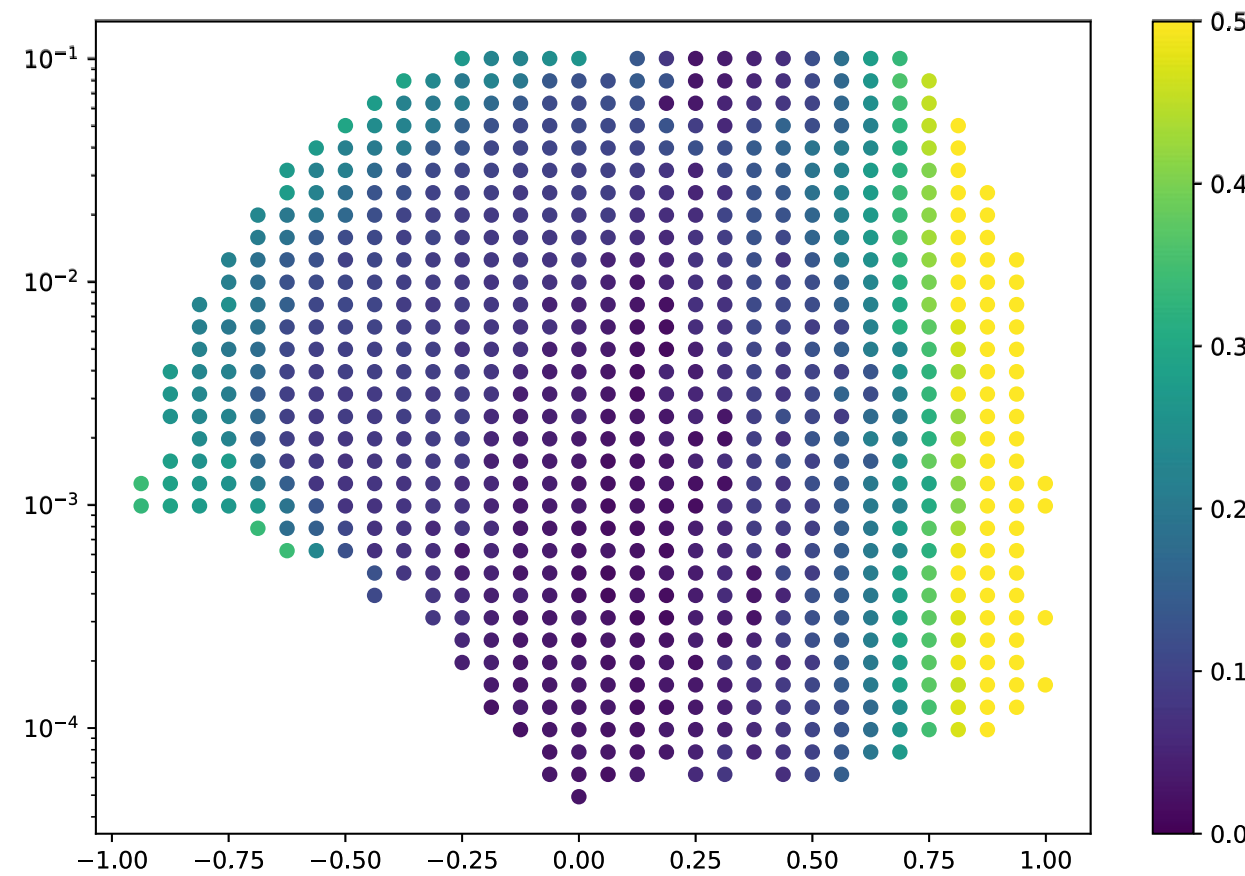


Improvement

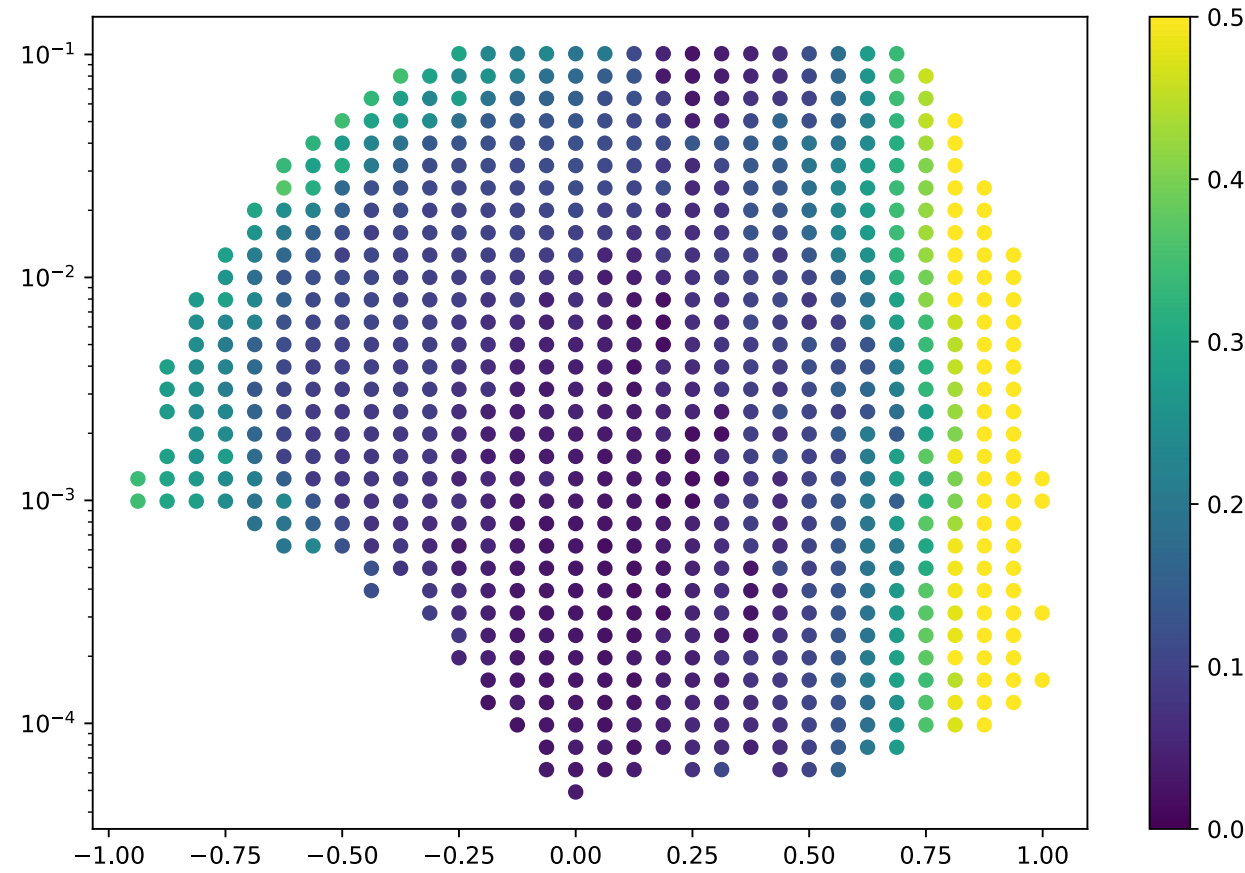
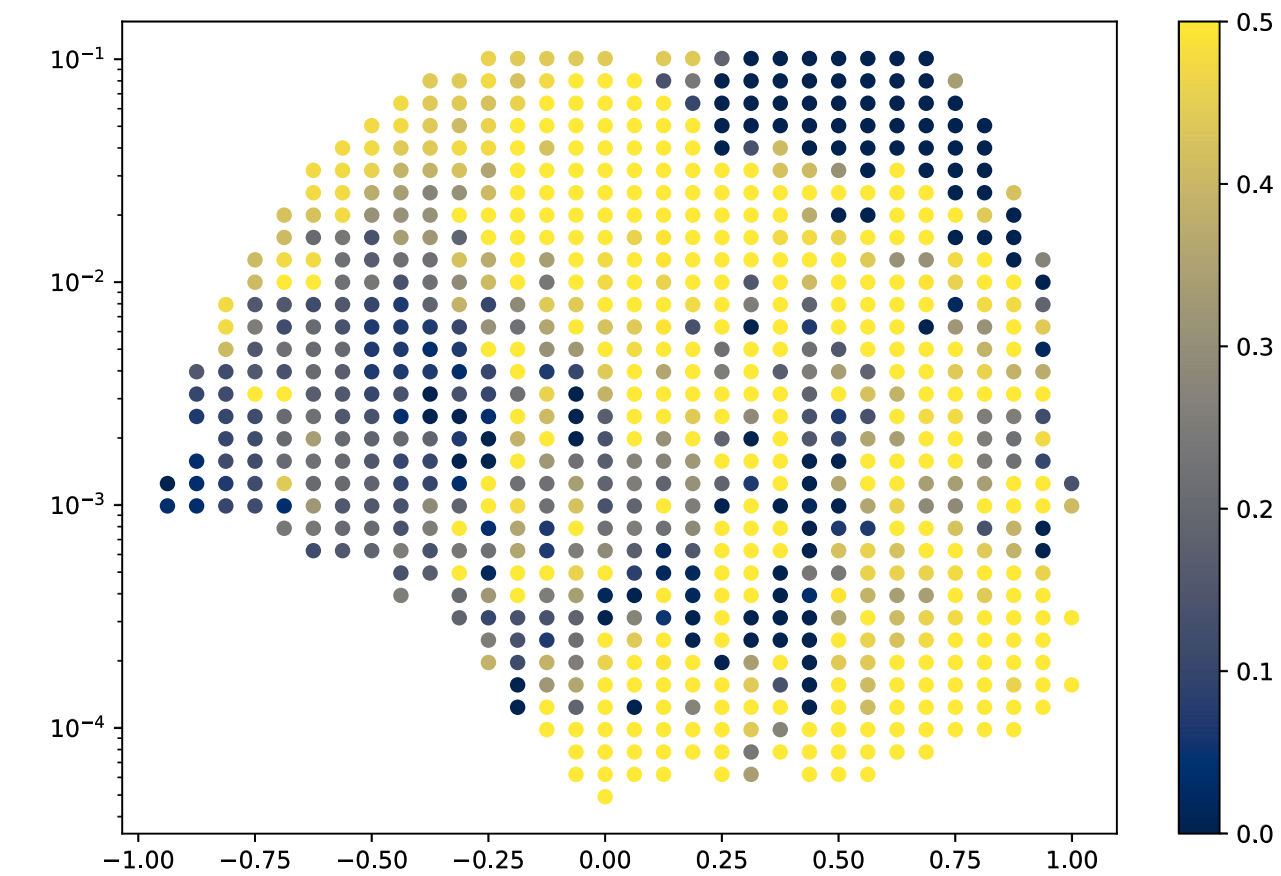
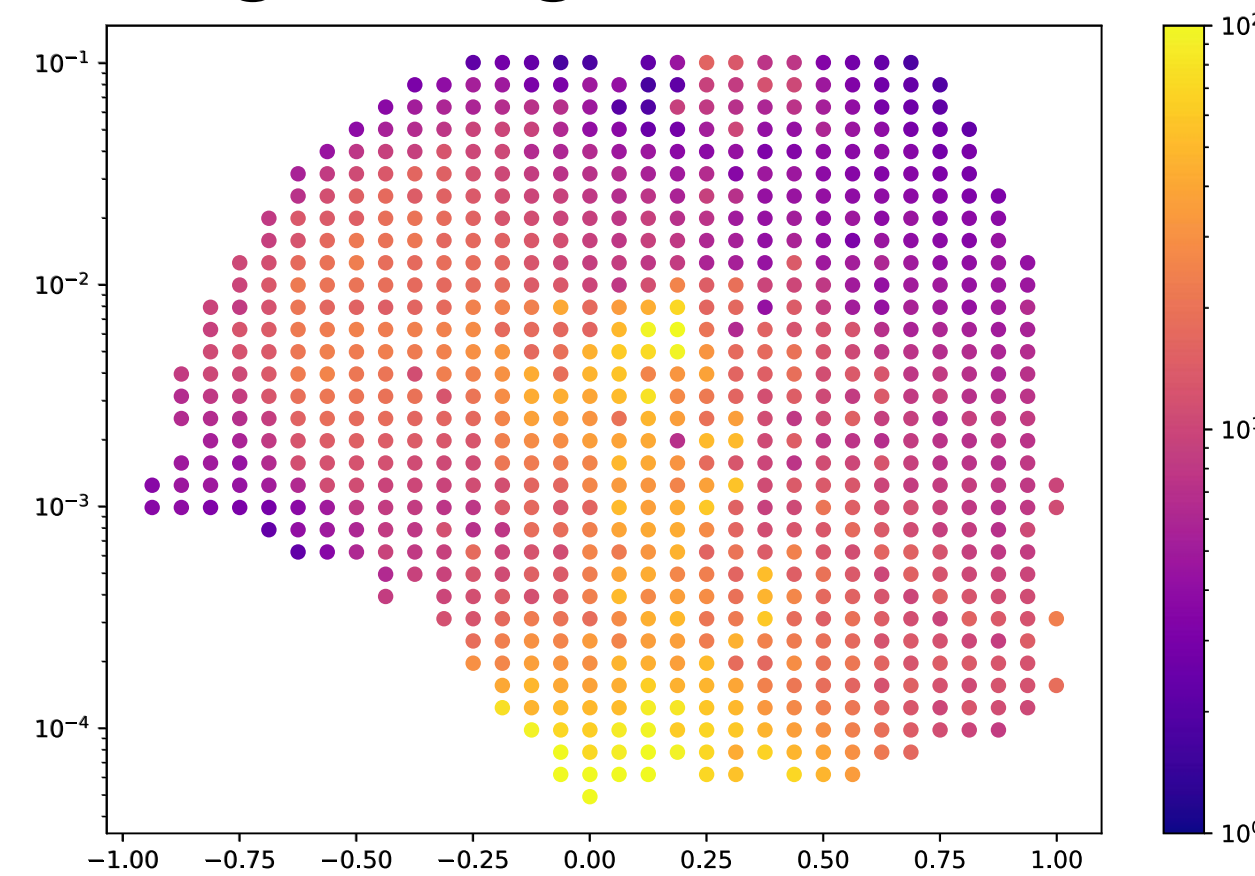


Collision Ratio

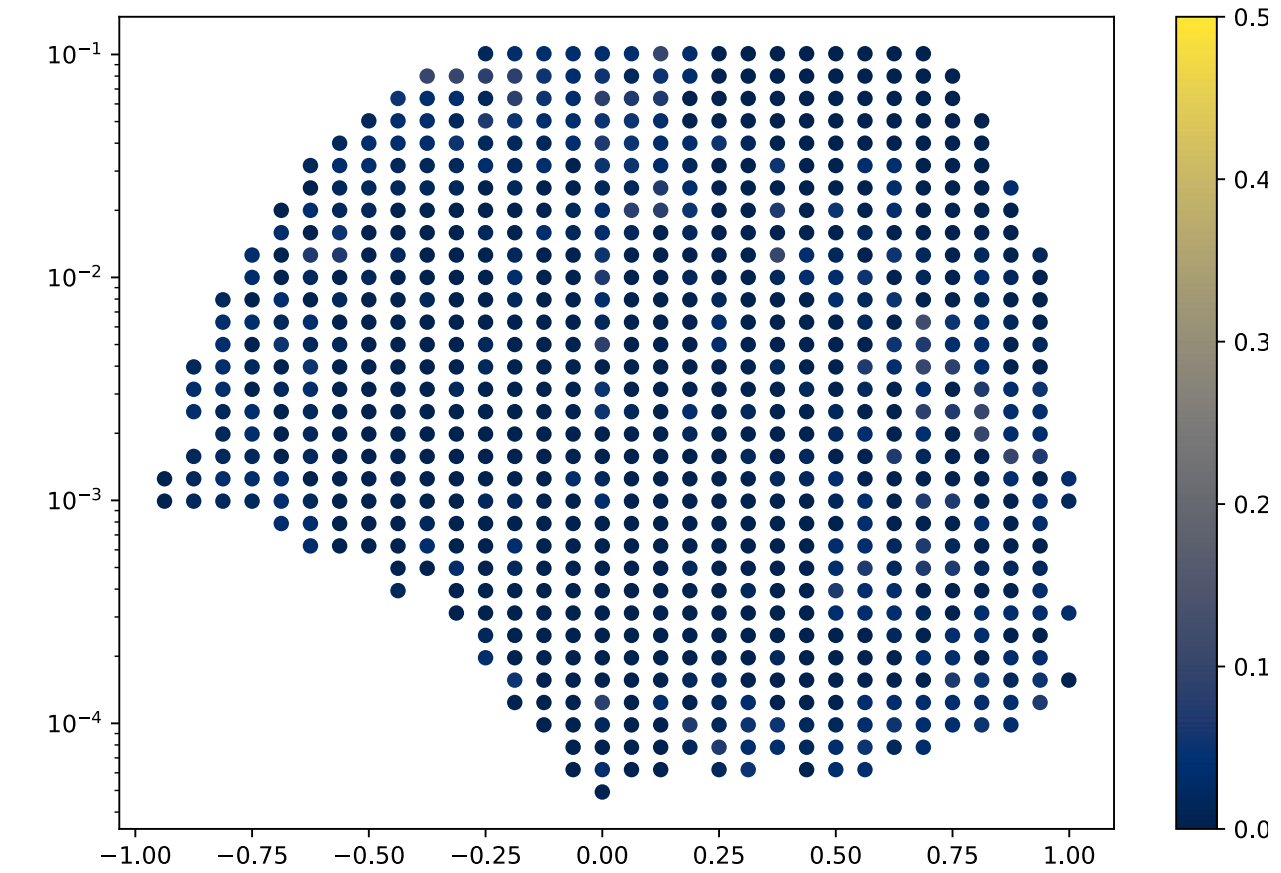
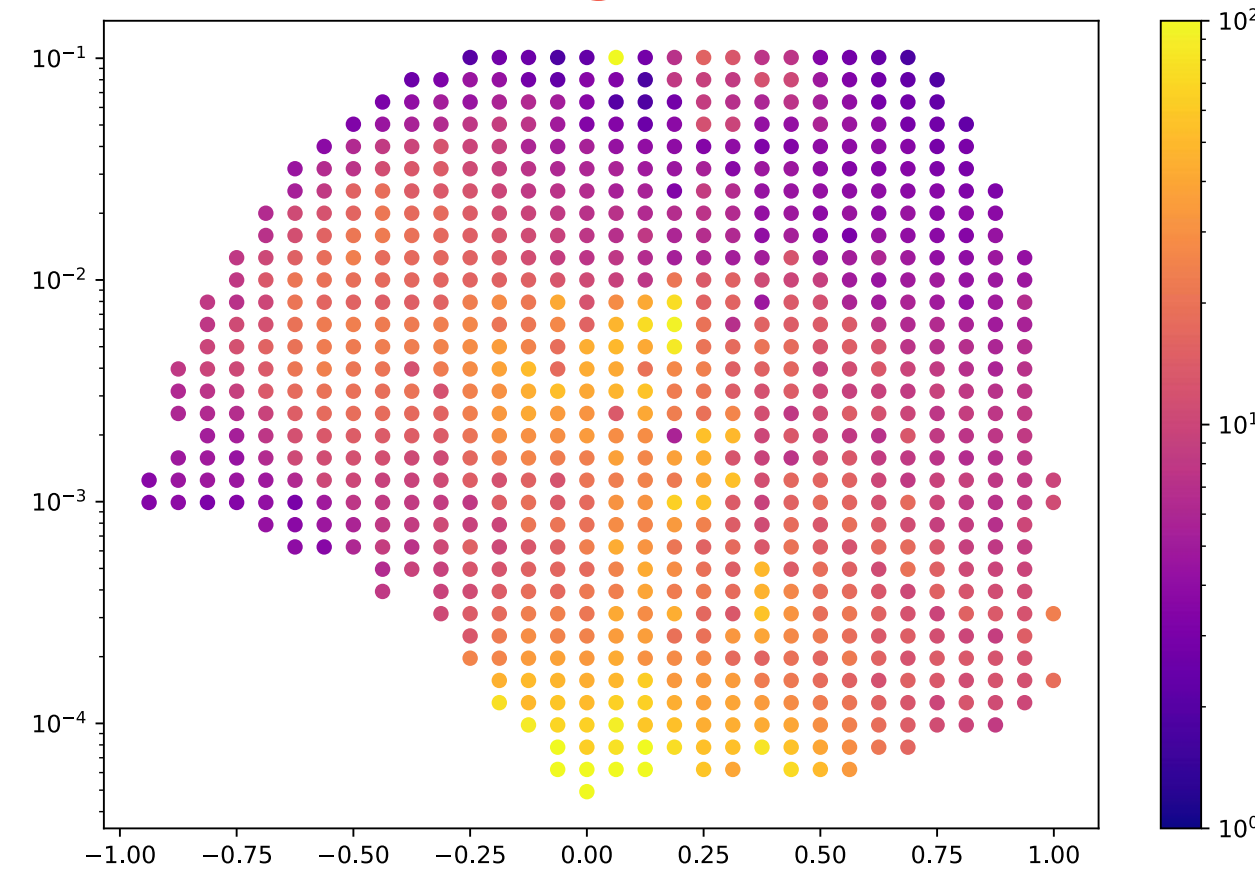
RESULTS



Ignoring Collisions



Removing Collisions



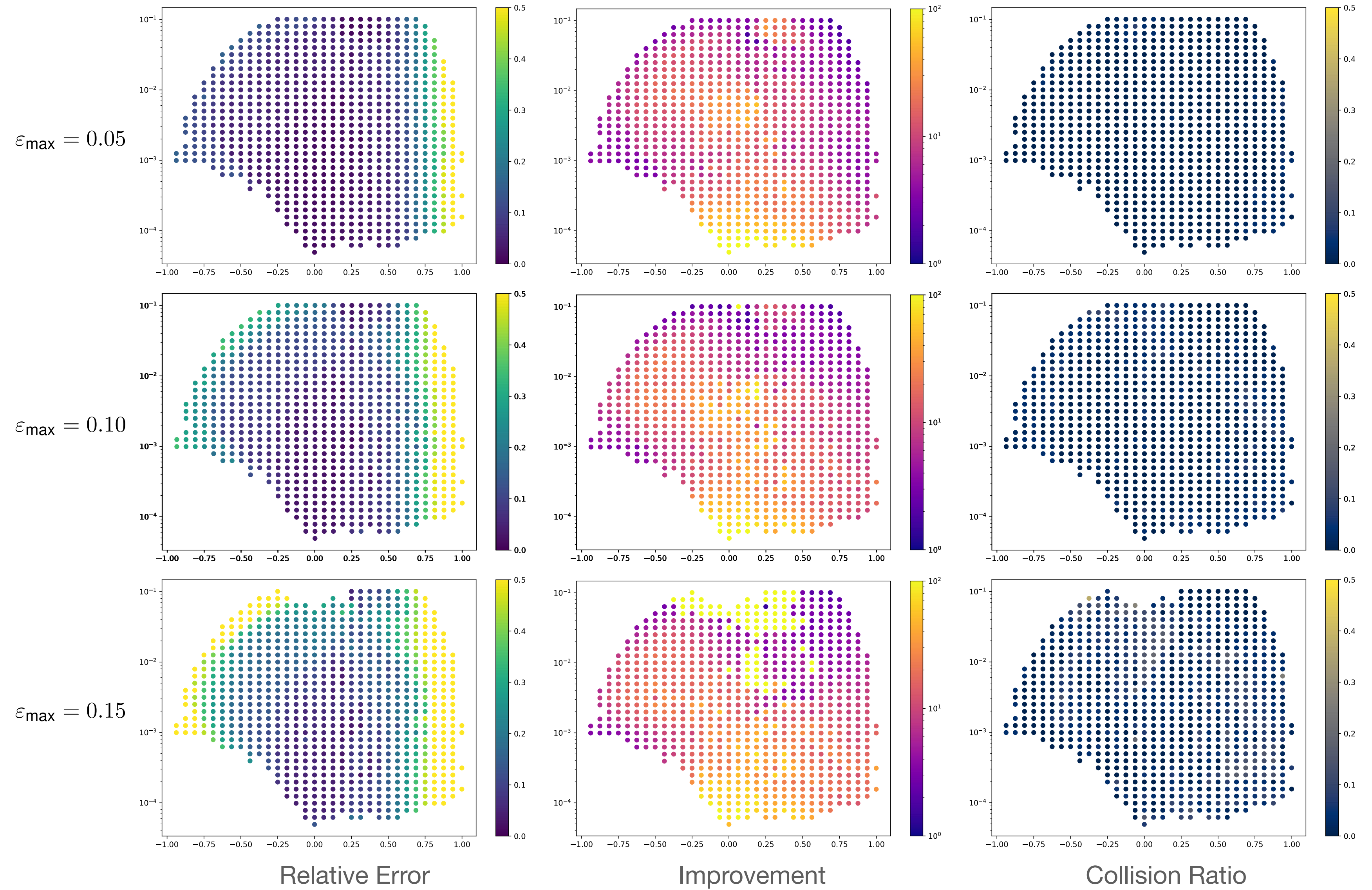
Relative Error

Improvement

Collision Ratio

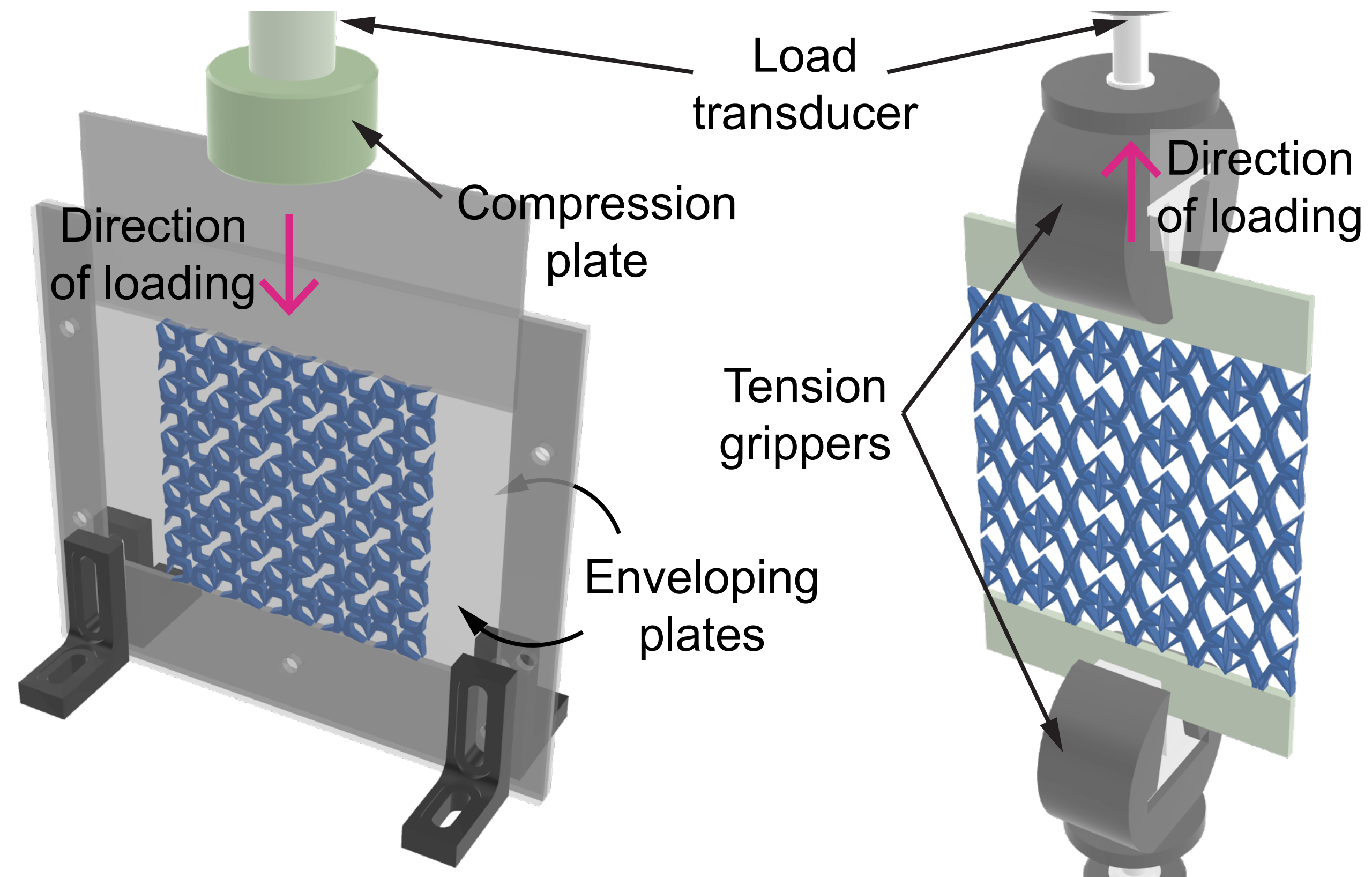
RESULTS

Removing collisions



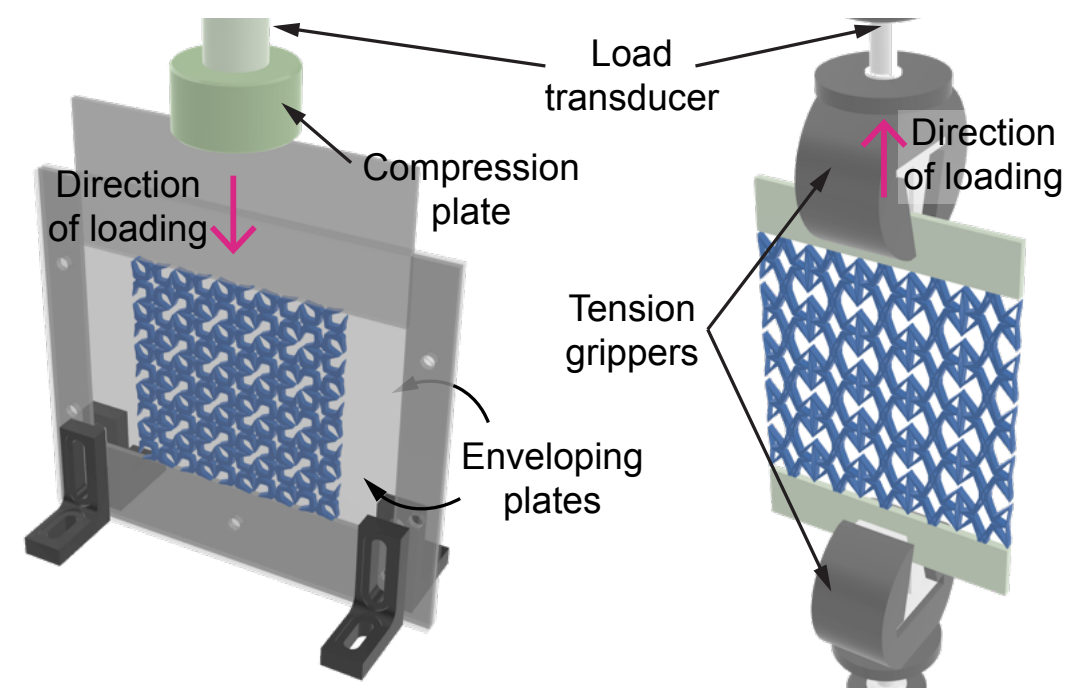
VALIDATIONS

Physical Tests



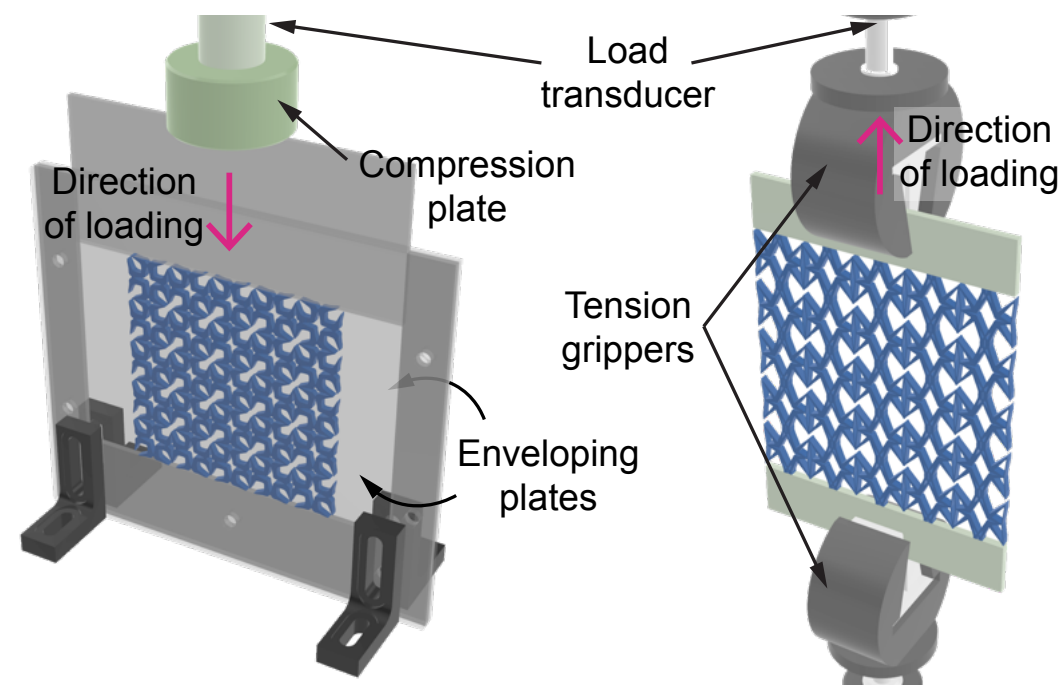
VALIDATIONS

Physical Tests

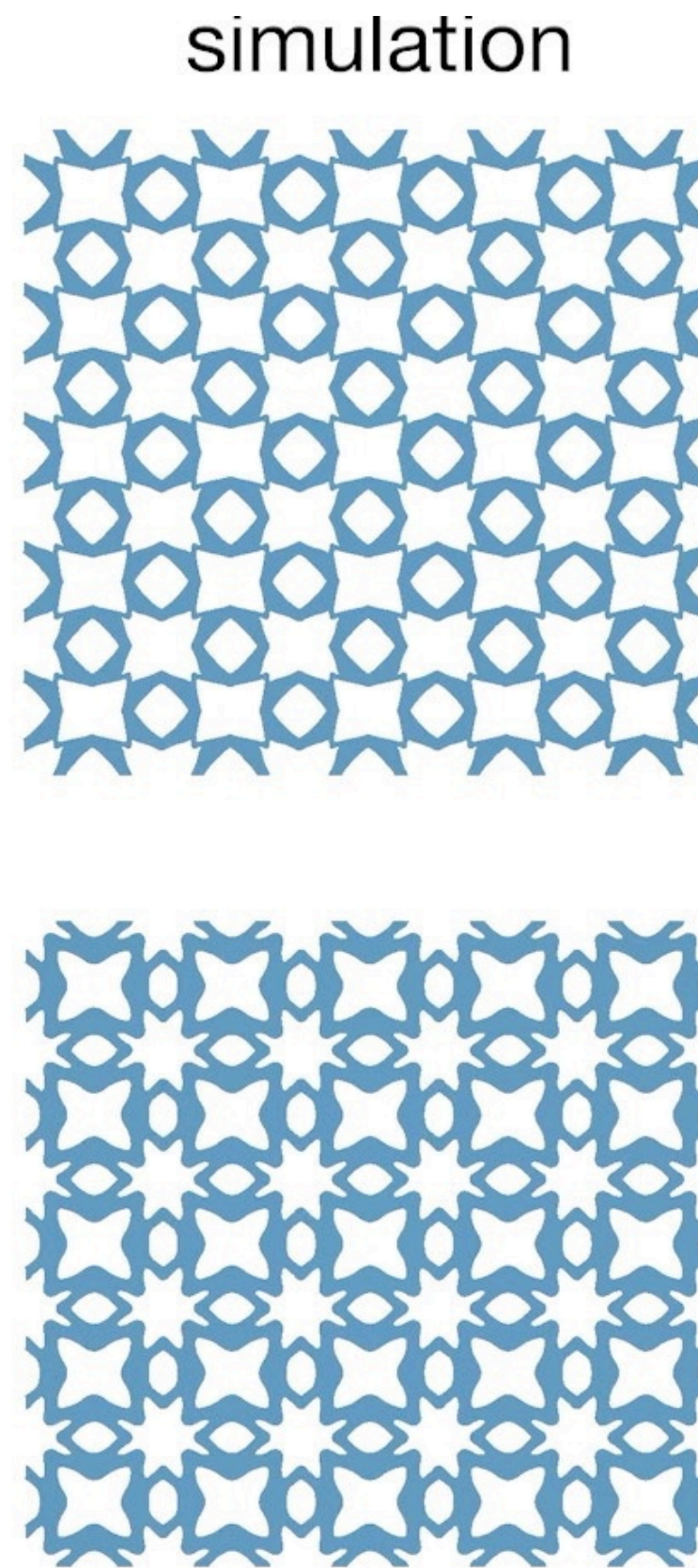


VALIDATIONS

Physical Tests

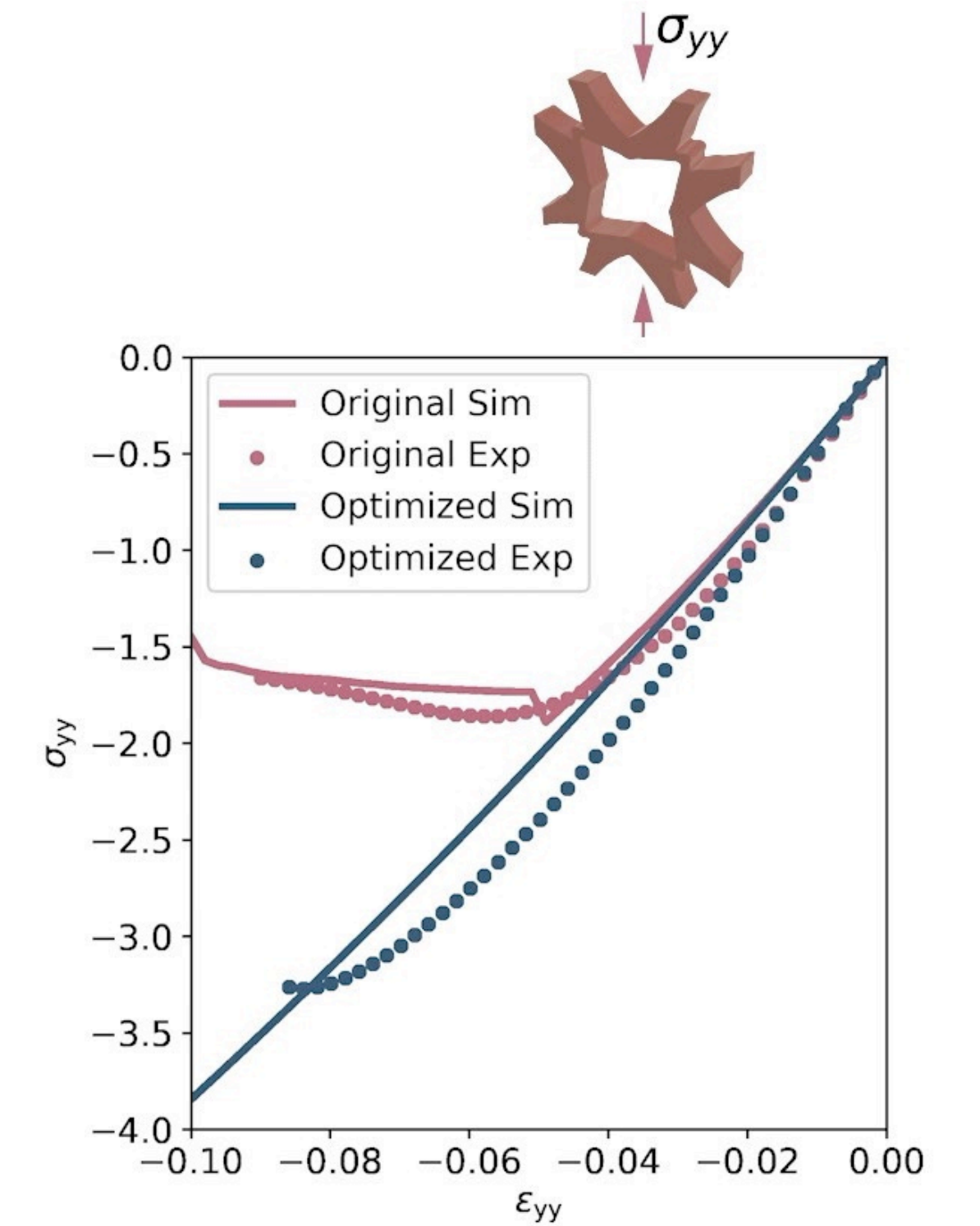


Original



Optimized

experiment

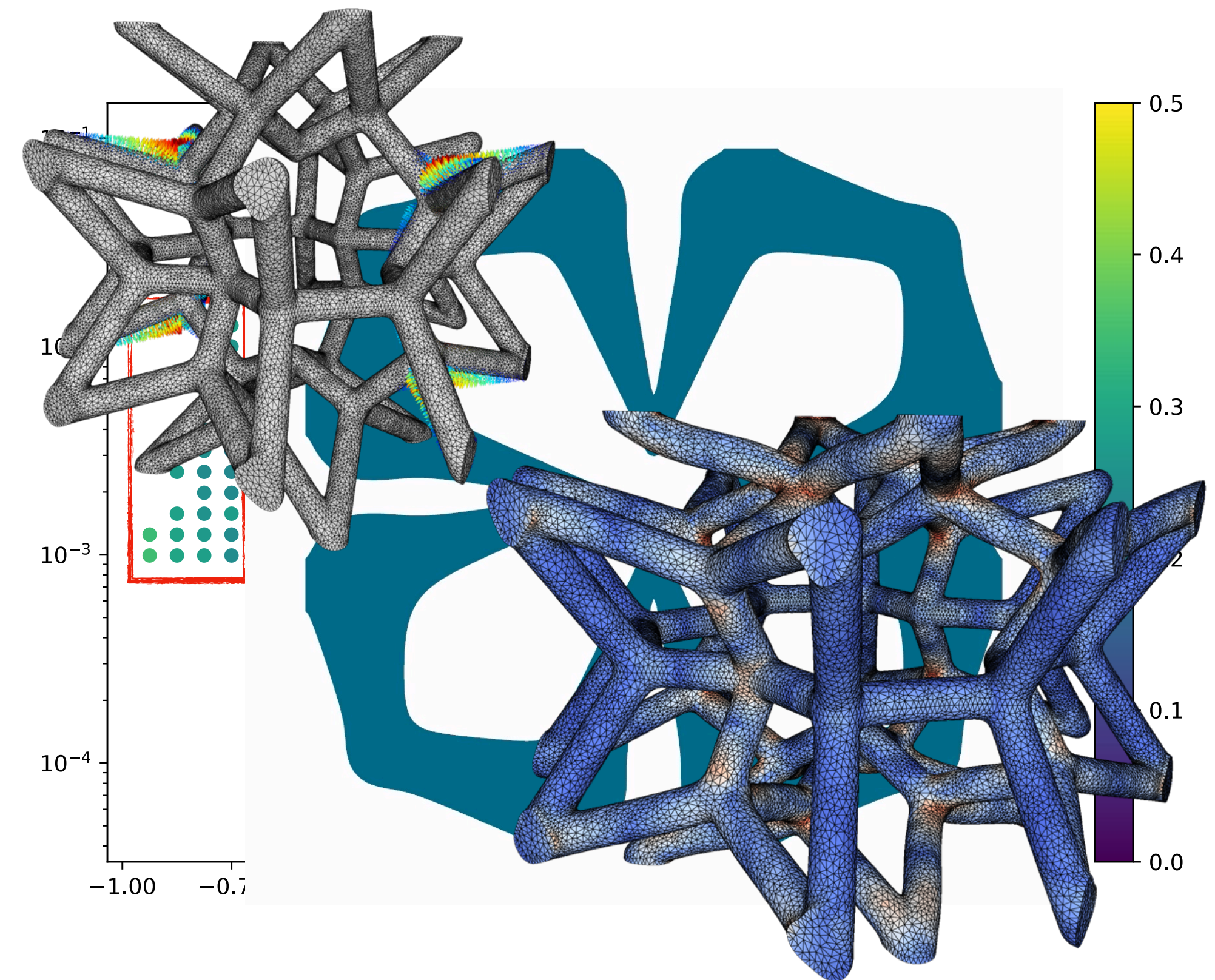


CONCLUSIONS

- An adaptive, data-accelerated nonlinear homogenization (with high-order interpolation)
- A shape design algorithm for nonlinear collision-free planar microstructures

Future Work:

- Better understand the shape parameters space
- Expand the range of achievable material properties
- Homogenize with differentiable contact simulation
- Develop 3D computational design framework



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University of Houston, Architected Intelligent Matter Laboratory



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Thanks!

